Slope sensitive dip move-out correction

Shuki Ronen

Abstract

Spatial aliasing reduction by dip move-out (DMO) is rederived and discussed in both time and frequency domains. A synthetic example in which an aliased zero offset section is compared with a stack after DMO is given. The DMO reduces most of the aliasing noise; the remaining noise results from undersampling in the offset and in midpoint directions. Theoretically the aliasing in the offset direction should not be a problem, this is shown in a synthetic example. A way to reduce the remaining midpoint aliasing noise by using a spatial derivative is described.

Aliasing reduction by DMO

The relation between the Fourier transform of a common offset section after normal move-out (NMO) $P_h(k,\omega)$ to the Fourier transform of the zero offset section $P_0(k,\omega)$ is approximately given by

$$P_0(k) \;=\; \exp\left[-i\frac{h^2k^2}{2\omega t}\right] P_h(k) \tag{1}$$

The appearance of ω and t together in the all-pass-like operator indicates that it is time dependent. A derivation of relation (1) is reviewed by Ronen, this report.

Suppose we have a few common offset sections $P_h^{(s)}(y,t)$. The superscript (s) implies that these sections are spatially aliased.

$$P_h^{(s)}(k) \;=\; \sum_n P_h(k - nk_0) \tag{2}$$

where $k_0 = 2\pi/\Delta y$ is twice the Nyquist frequency. Let $P_{0(h)}^{(s)}(k)$ be the section after applying NMO and DMO to the aliased common offset section. It is exactly the zero offset section if there is no aliasing.

$$P_{0(h)}^{(s)}(k) = \exp\left[-i\,\frac{h^2k^2}{2\omega t}\right]\cdot\sum_n P_0(k-nk_0)\exp\left[i\,\frac{h^2(k-nk_0)^2}{2\omega t}\right] \qquad (3)$$

$$= \sum_n P_0(k-nk_0)\exp\left[i\,\frac{h^2k_0^2}{2\omega t}n\left[n-\frac{2k}{k_0}\right]\right]$$

The stack is the sum over offsets

$$Stack(k) = \sum_n P_0(k-nk_0)\sum_h \exp\left[i\,\frac{h^2k_0^2}{2\omega t}n\left[n-\frac{2k}{k_0}\right]\right] \qquad (4)$$

The signal, ($n = 0$ term), stacks in; while the aliasing noise, ($n \neq 0$ terms), does not, except for some problematic frequencies. One kind of noise is when $k = nk_0/2$ and for all ω; the spatial frequency is an integer multiplication of the Nyquist frequency. This noise appears in Figure 3 as ridge 0, it will remain regardless of the number of offsets as can be seen in equation (4). Another group of noisy frequencies is when the argument of the exponent is $i2\pi$ times an integer for all offsets. This defines a group of spatial and temporal frequencies that appear as "ridges -2,-1,1,2 " in Figure 3. The "$2\pi j$ noise" disappears when the sampling rate in the offset direction increases. A more complete discussion is given by Ronen and Rocca in SEP-32. The improvement in spatial resolution is mentioned by Bolondi et al, 1982, and by Salvador and Savelli, 1982. Figure 1 describes stacks and migrations for a model of a single point diffractor, with and without DMO. Without DMO the hyperbola on the stack is poorly sampled; in comparison after DMO the hyperbola is interpolated, but some noise remains. The remaining noise is the "Nyquist noise"; it appears on the stack as bristles on the hyperbola and migrates to periodical artifacts which are just remnants of the aliasing noise we see on the migrated section without DMO.

In the time-space domain the transfer function of a point diffractor using data from one common midpoint gather is a "V" pattern having one side on the expected zero-offset hyperbola and the other side on a symmetric hyperbola. (Rocca and Ronen, 1982). Figure 2 shows two V patterns. For each V the arrival time from one point diffractor to one particular midpoint were calculated for a number of offsets, then normal moved-out, then dip moved-out. In both V's, the range of offsets was up to the depth of the point diffractor. The difference is that the left V has more offsets. The figures were obtained with finite differencing DMO (Ronen, this report); hence, part of the noise is evanescent, such as that above time 180.

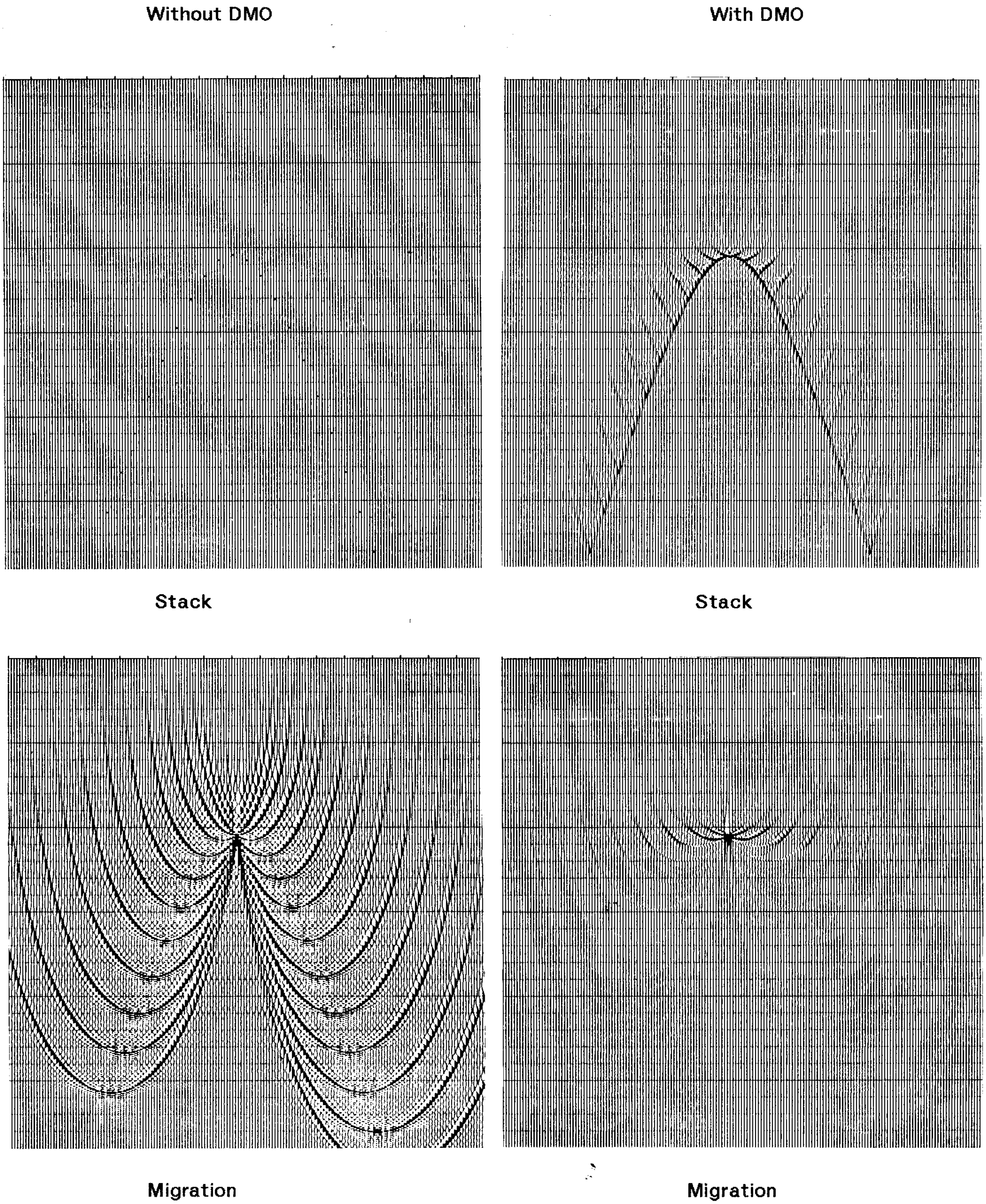

FIG. 1. Spatial aliasing reduction. One point diffractor.

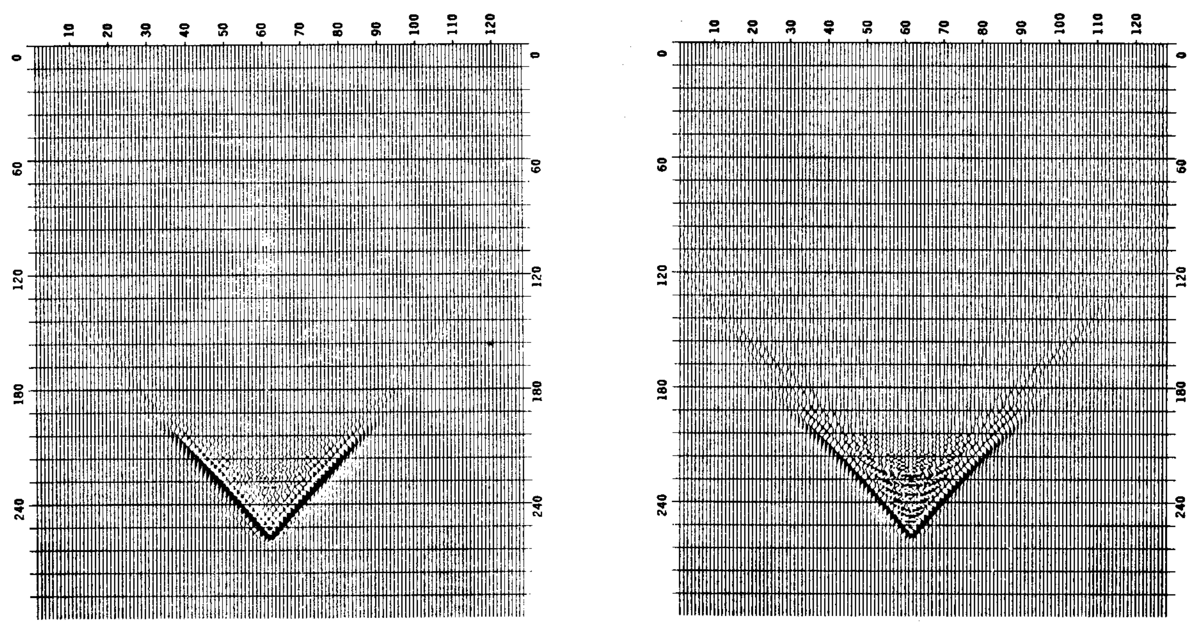

FIG. 2. The V pattern. Left: well sampled offset axis; 30 offsets. Right: poorly sampled offset axis; 9 offsets.

The noise in the middle of the "V" pattern corresponds to the "$2\pi j$ noise"; it depends on how the offset axis is sampled, getting stronger if the offset is poorly sampled. Uniform offset squared sampling (Ronen and Rocca, SEP-32) is optimal with respect to this kind of noise. When the number of offsets increase, the "$2\pi j$ noise" is reduced as shown at Figure 2. Theoretically, seismic data contain enough offsets to avoid the "$2\pi j$" noise, except for very shallow events.

The more severe aliasing noise is the "Nyquist noise". In the time domain it is identified with the "wrong" side of the "V" pattern. It does not depend on the offset sampling and vanishes only when the data are not aliased or in another way described below.

First derivative

The "Nyquist noise" stems from the fact that for every common midpoint gather it is impossible to tell which is the wrong side of the "V". However, if we have the in-line spatial derivative of the wavefield, we are able to tell the right side of the "V" within a single common mid-point.

If the derivative is to be recorded directly it means doubling the number of phone groups, each station having two close in-line groups that will be summed to give the wavefield and subtracted for the derivative. There may be a way to calculate the spatial derivative without special recording, with the wave equation, using the well sampled temporal derivatives and the aliased spatial derivative.

Suppose we have the spatial derivative of the wavefield $P'(x,t)$, in addition to the wavefield itself $P(x,t)$. The Fourier transform of the sampled derivative is

$$P_h'^{(s)}(k) = \sum_n i(k - nk_0)\, P(n - nk_0) \tag{5}$$

Define a composite common offset section:

$$Q_h(k) = \frac{1}{2}\left[P_h(k) + \frac{1}{ik} P'_h(k)\right] \tag{6}$$

Problems at $k=0$ can be avoided either by integration on x or by adding a k dependent weight $w(k)$ such that $w(k)/k$ is $1/k$ for big k and vanishes at $k=0$, where we do not have aliasing problems anyway.

In analogy to equation (2), $Q_h^{(s)}$ is the sampled section

$$Q_h^{(s)}(k) = \frac{1}{2}\left(\sum_n P_h(k - nk_0) + \sum_n \frac{k - nk_0}{k} P_h(k - nk_0)\right) \tag{7}$$

$$= \sum_n \left(1 - \frac{nk_0}{2k}\right) P_h(k - nk_0)$$

After DMO we would have

$$Q_{0(h)}^{(s)}(k) = \exp\left(-i\,\frac{h^2k^2}{2\omega t}\right) Q_h^{(s)}(k) \tag{8}$$

$$= \sum_n P_0(k - nk_0)\left(1 - \frac{nk_0}{2k}\right) \exp\left[i\,\frac{h^2k_0^2}{2\omega t} n \left(n - \frac{2k}{k_0}\right)\right]$$

The stack is

$$Stack(k) = \sum_n P_0(k - nk_0)\left(1 - \frac{nk_0}{2k}\right) \sum_h \exp\left[i\,\frac{h^2k_0^2}{2\omega t} n \left(n - \frac{2k}{k_0}\right)\right] \tag{9}$$

The noise at $nk_0/2$ is eliminated. In Figure 3, the function multiplying the $n = 1$ term $P_0(k - k_0)$ in equations (4) is compared to that of equation (9). 3a corresponds to (4) and 3b to (9). If this function is zero then aliasing is avoided. The "ridge" at $k = k_0/2$ vanishes in Figure 3b. To generate Figure 3b a weight function was used to avoid troubles at

$k = 0$. Looking at Figure 3, one should remember that in the summations (4) and (9) there are conjugate terms $n = -1$ that give symmetric pictures.

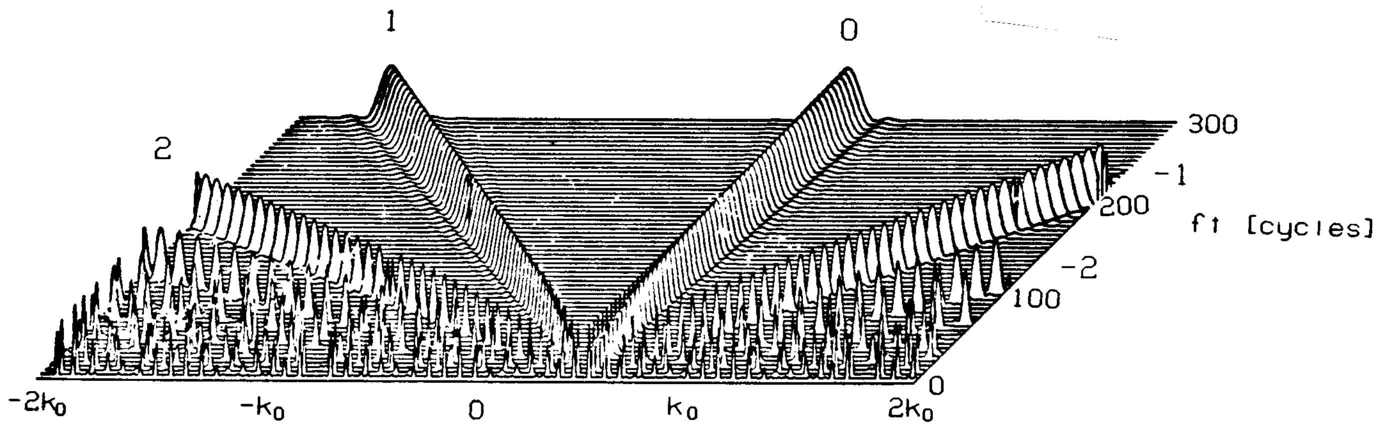

FIG. 3a. Without derivative. [Equation (4)].

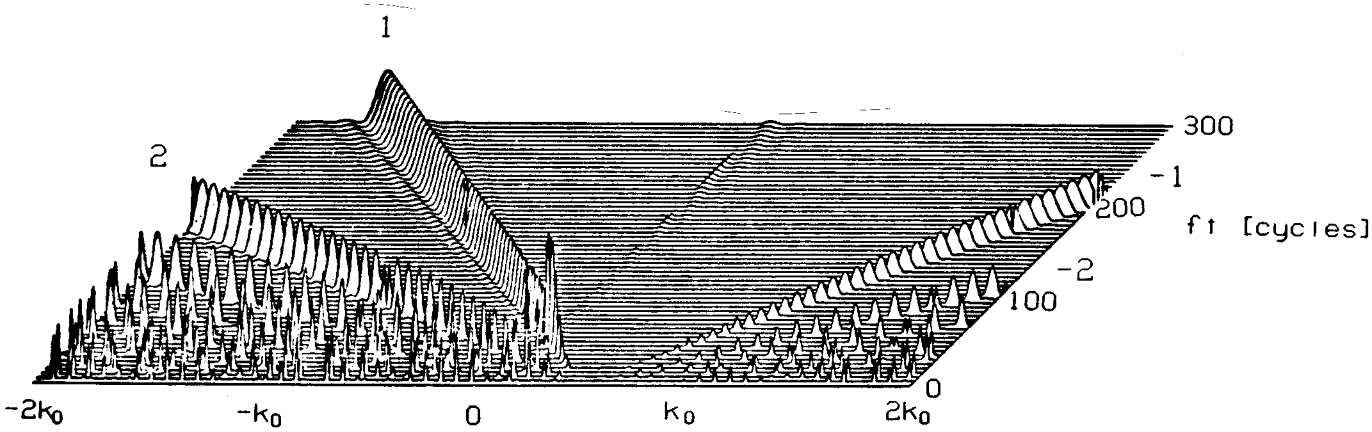

FIG. 3b. With derivative. [Equation (9)].

Higher derivatives

This section is not a practical suggestion but a note that the result (9) is just a special case. In general if we have N derivatives $P_h^{(j)}$ $j = 0, 1, \cdots N$, define

$$Q_h(k) = \sum_{j=0}^{N} \frac{a_j}{(ik)^j} P_h^{(j)}(k) \tag{10}$$

The aliased section will then be

$$Q_h^{(s)}(k) = \sum_{n} \sum_{j=0}^{N} a_j \left(1 - \frac{nk_0}{k}\right)^j P_h(k - nk_0) \tag{11}$$

Let the coefficients a_j solve

$$\sum_{j=0}^{N} a_j \, (1 + x)^j = \left[1 + \frac{x}{2} \right]^N \tag{12}$$

Then the composed common offset section will be

$$Q^{(s)}(k) = \sum_n \left[1 - \frac{nk_0}{2k} \right]^N P(k - nk_0)$$

Using the binomial formula we get from (12)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & \cdot & \cdot & 1 \\ & 1 & 2 & 3 & 4 & \cdot & \cdot & N \\ & & 1 & 3 & 6 & \cdot & \cdot & \cdot \\ & & & 1 & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & \cdot & N \\ & & & & & & & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ \cdot \\ \cdot \\ \cdot \\ a_j \\ \cdot \\ \cdot \\ a_N \end{bmatrix} = \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ 2^{-k} \binom{n}{k} \\ \cdot \\ \cdot \\ 2^{-N} \end{bmatrix}$$

The matrix is upper triangular with Pascal triangle coefficients. The solution is

$$a_j = 2^{-N} \binom{N}{j}$$

The Nyquist noise will be suppressed in the stack

$$Stack(k) = \sum_n P_0(k - nk_0) \left[1 - \frac{nk_0}{2k} \right]^N \sum_h \exp \left[i \, \frac{h^2 k_0^2}{2\omega t} n \left[n - \frac{2k}{k_0} \right] \right]$$

Conclusions

DMO reduces aliasing noise except for two groups of frequencies:

1. Removable noise associated with undersampling in the **offset** direction. The number of offsets in seismic data is probably sufficient to eliminate this noise, although it means that DMO should be applied to many common offset sections.

2. Noise associated with undersampling in the **midpoint** direction. Independent measurement or calculation of the spatial derivative, in addition to the function itself, enables reduction of this noise. This may be considered if the remaining aliasing noise after DMO is strong.

The validity of the assumption in 1 and the feasibility of the suggestion in 2 are still to be checked.

ACKNOWLEDGMENTS

The instruction and suggestions of Fabio Rocca were very helpful.

REFERENCES

Bolondi , G., Loinger, E. and Rocca, F., 1982, Offset continuation of seismic sections, Geophysical Prospecting 30, 813-828.

Rocca, F. and Ronen, S., 1982, Stacking smiles, SEP-32, p. 97-103.

Ronen, S. and Rocca, F., 1982, Why dip move-out ?, SEP-32, p. 81-95.

Ronen, S., 1983, Finite differencing DMO, this report.

Salvador, L. and Savelli, S., 1982, Offset continuation for seismic stacking, Geophysical Prospecting 30, 829-849.