Finite differencing dip move-out

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Abstract

A method to correct dip move-out by finite differencing in (k,t) domain is described. The shape of the impulse response is found to be a gaussian which is an approximation to the exact elliptical shape. It is shown how this method can be adapted to operate before normal move-out and to handle vertical and horizontal velocity variations.

Introduction

Bolondi et al, 1981, considered the differential equation

$$\frac{\partial^2 P}{\partial h \partial t} = -\frac{h}{t} k^2 P \tag{1}$$

For any t_0 we can change variables by

$$\log\left(\frac{t}{t_0}\right) = \frac{\tau}{t_0} \tag{2}$$

and obtain the equation

$$\frac{\partial^2 P}{\partial h \partial \tau} = -\frac{h}{t_0} k^2 P \tag{3}$$

which is solved by

$$P_0(k,\omega) = \exp\left(-i\,\frac{h^2 k^2}{2\omega t_0}\right) P_h(k,\omega) \tag{4}$$

where ω is the frequency associated with τ.

Equation (4) describes a time dependent operator that extrapolates in offset. $P_h(k,\omega)$ is the two dimensional Fourier transform of a section having common half offset h, $P_0(k,\omega)$ is the transform of the zero offset section. To calculate the impulse response of (4) consider input of $P_h(\tau,x) = \delta(\tau)\delta(x)$. The output will be

$$P_0(x,\tau;0,0) \;=\; \int dk \int d\omega \, e^{i(\omega\tau - kx)} \exp\left[-i \, \frac{h^2 k^2}{2\omega t_0}\right] \tag{5}$$

The phase of the integrand is

$$\Phi \;=\; \omega\tau - kx - \frac{h^2 k^2}{2\omega t_0}$$

The phase is stationary when

$$0 \;=\; \frac{\partial \Phi}{\partial k} \;=\; -x - \frac{h^2 k}{\omega t_0}$$

$$0 \;=\; \frac{\partial \Phi}{\partial \omega} \;=\; \tau + \frac{h^2 k^2}{2\omega^2 t_0}$$

Therefore the impulse response is mainly on the parabola

$$\tau(x) \;=\; -\, t_0 \, \frac{x^2}{2h^2}$$

Now use (2) to find the shape of the response of (1)

$$t_f(x) \;=\; t_0 \exp\left[-\frac{x^2}{2h^2}\right] \tag{6}$$

This is an approximation to the ellipse

$$t_d \;=\; t_0 \left[1 - \frac{x^2}{h^2}\right]^{\frac{1}{2}}$$

which is the exact shape of the impulse response of the move-out operator, obtained by osculation of diffraction hyperbolas from a non zero offset migration ellipse shown by Figure 1. (Deregowski and Rocca, 1982).

The error in (6)

$$t_d^2 - t_f^2 \;=\; t_0^2 \left[1 - \frac{x^2}{h^2} - \exp\left[-\frac{x^2}{h^2}\right]\right]$$

is small since the ellipse is truncated for

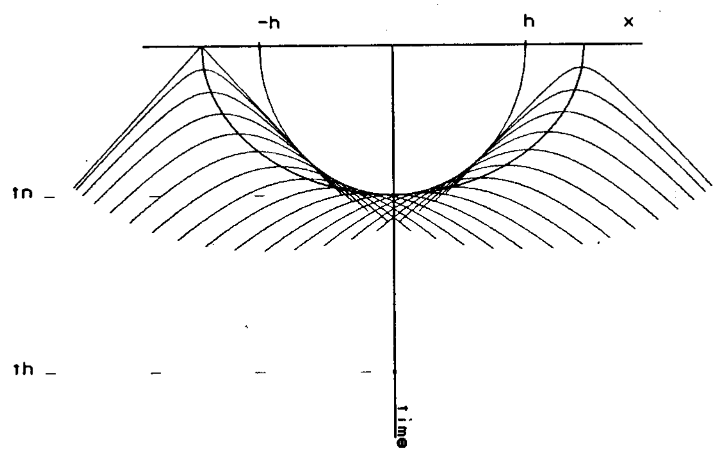


FIG. 1. Impulse response of DMO.

$$\left| \frac{x}{h} \right| < \frac{2h}{vt_h}$$

The right hand side cannot exceed 1. If the mute allows NMO stretch of s then we have

$$\frac{2h}{vt_h} < \left[1 - \frac{1}{s^2} \right]^{\frac{1}{2}}$$

If this error is a problem it can be handled in a way described below.

Finite differencing dip move-out

Consider equation (1) as an initial value problem. A way to solve (1) is by finite differencing. The difference equation

$$P_{h_j}^{t} = \frac{1 - \alpha}{1 + \alpha} \left[P_{h_{j+1}}^{t} + P_{h_j}^{t+1} \right] - P_{h_{j+1}}^{t+1} \qquad (7)$$

with

$$\alpha = \frac{h_{j+1}^2 - h_j^2}{8t / \Delta t} k^2$$

gives a way to extrapolate the data in offset direction. (Bolondi et al, 1982; Salvador and Savelli, 1982.) For every common offset section, starting from the initial value of $P_{h_N}(x,t)$, transform over x, then for every k apply equation (7) to find P_{h_j} from $P_{h_{j+1}}$, for all times.

Figure (2) was obtained by applying (7) with impulsive initial value.

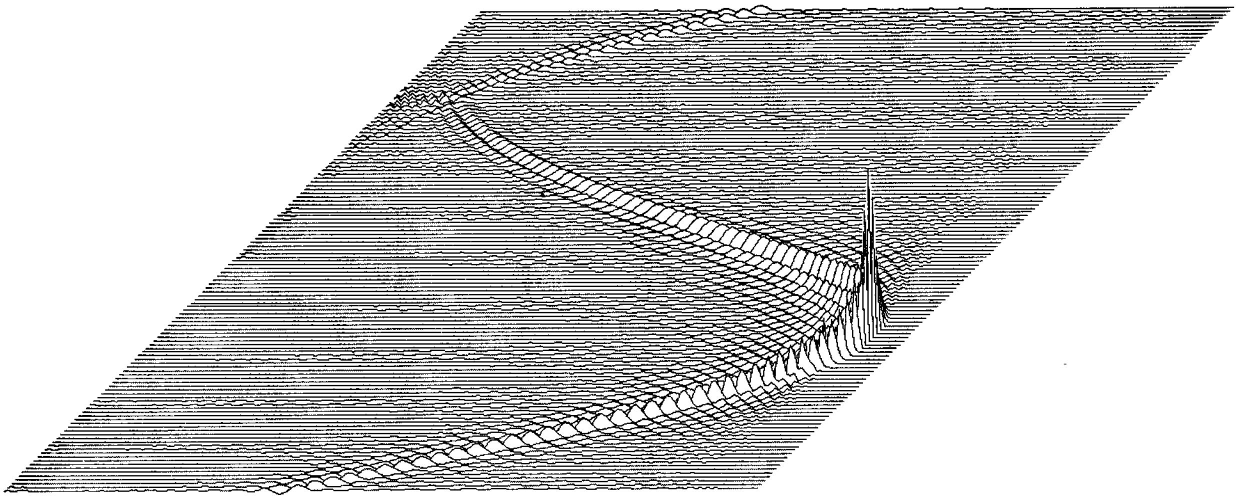


FIG. 2. Impulse response of the finite differencing DMO done in the $k-t$ domain. No truncation was done so the gaussian shape is clearly visible. The temporal dispersion may be avoided by using a more elaborate numerical method.

Dip Move-Out before Normal Move-Out

Move-out is the operator that transforms a common offset section into a zero offset section, its response to an impulse at time t_h is the bottom of the ellipse

$$\frac{t^2}{t_n^2} + \frac{x^2}{h^2} = 1 \tag{8}$$

where t_n is the NMO time:

$$t_n^2 = t_h^2 - \frac{4h^2}{v^2} \tag{9}$$

The ellipse (8) is truncated at

$$\left| x \right| \leq \frac{2h^2}{vt_h}$$

Moving-out a common offset section to a zero offset section can be done in two steps:

(1) NMO with equation (9). This operator is velocity dependent but it involves just a time stretch applied to each trace separately.

(2) DMO expands every impulse into the ellipse of equation (8), bottoming on the normal moved out impulse. The DMO involves more than one trace at a time, it is time dependent and velocity independent except for the truncation of the ellipse which is done anyway when the evanescent field is removed after stack, either by dip filtering or by migration.

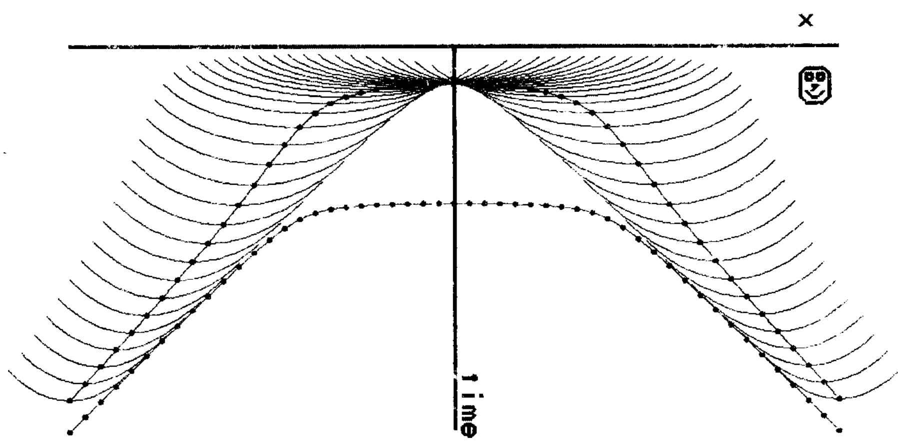


FIG. 3a. Dip move-out after normal move-out. (The right way).

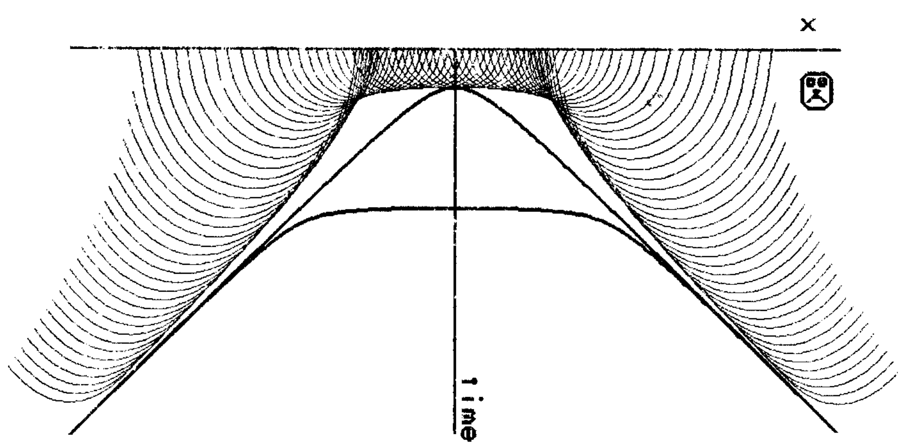


FIG. 3b. Doing dip move-out before normal move-out. (The wrong way.)

Figure 3a shows how a "flat-top" which is the constant offset section of a point diffractor, is moved-out to the expected zero offset hyperbola by DMO after NMO. In Figure 3b the order was reversed; every point on the flat top was first expanded to a DMO ellipse bottoming on the input spike itself at t_h (instead of on the normal moved out time t_n), then normal move-out was done. The resulting osculation is not the zero offset hyperbola. The stack will be coherent only near the apex and on the asymptotes. Although normal move-out should be done before dip move-out, we would prefer to have an operator DMO' that is applied before normal move-out

$$\mathbf{NMO \cdot DMO' = DMO \cdot NMO}$$

this defines the operator

$$\mathbf{DMO' = NMO^{-1} \cdot DMO \cdot NMO} \tag{10}$$

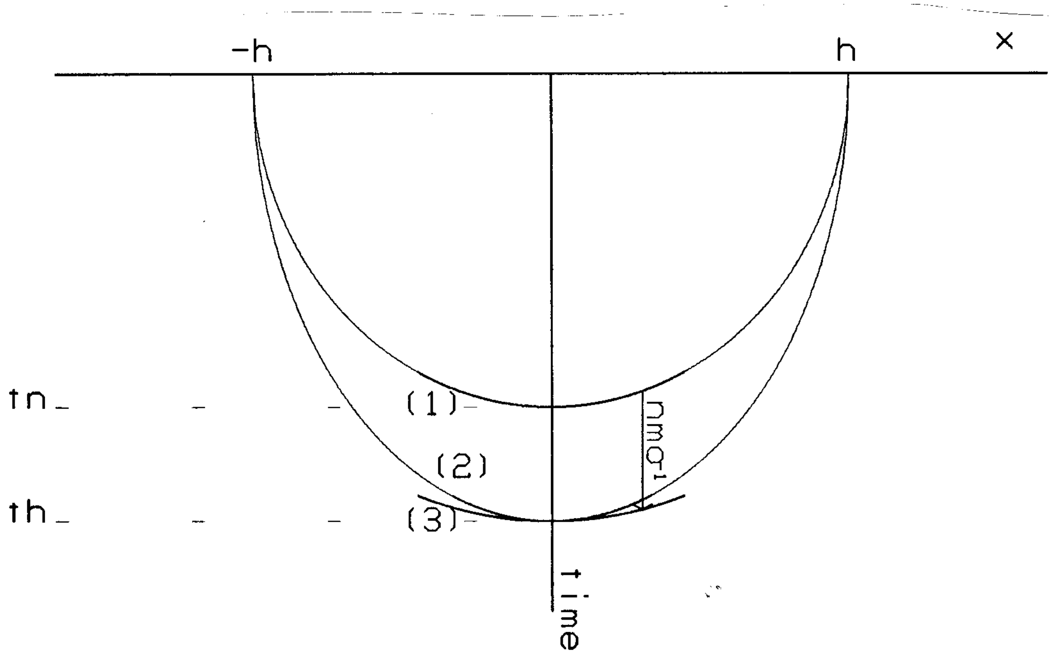


FIG. 4. Finding the response of DMO'.

We can apply this definition to compute the shape of the impulse response of the DMO' at a time t_h. Referring to Figure 4, starting from an impulse $\delta(t - t_h)$, first apply NMO to obtain $\delta(t - t_n)$ at the NMO time t_n,

$$t_n^2 = t_h^2 - \frac{4h^2}{v^2}$$

then expand this impulse with equation (8) to the ellipse $t_d(x)$,

$$t_d^2 = t_n^2 \left[1 - \frac{x^2}{h^2} \right]$$

$$= \left[t_h^2 - \frac{4h^2}{v^2} \right] \left[1 - \frac{x^2}{h^2} \right]$$

which is curve (1) at Figure 4. Finally apply inverse NMO;

$$t_{d'}^2 = t_d^2 + \frac{4h^2}{v^2}$$

$$= \left[t_h^2 - \frac{4h^2}{v^2} \right] \left[1 - \frac{x^2}{h^2} \right] + \frac{4h^2}{v^2}$$

$$= t_h^2 \left[1 - \frac{x^2}{h^2} \right] + \frac{4x^2}{v^2}$$

$t_{d'}(x)$ is the shape of the response of DMO' to an impulse at time t_h on a section with common half offset h. It turns out to be the ellipse

$$\frac{t_{d'}^2}{t_h^2} + x^2 \left[\frac{1}{h^2} - \frac{4}{v^2 t_h^2} \right] = 1 \qquad (11)$$

which is curve (3) at Figure 4. In comparison, applying DMO to an impulse at t_h gives the ellipse

$$\frac{t_d^2}{t_h^2} + x^2 \left[\frac{1}{h^2} \right] = 1 \qquad (12)$$

which is curve (2) at Figure 4. Applying NMO to (11) will give (8) while applying NMO to (12) will give something else.

The difference between (11) and (12) is

$$t_{d'} - t_d = t_d \left[\left(1 + \frac{4x^2}{v^2 t_d^2} \right)^{\frac{1}{2}} - 1 \right] \qquad (13)$$

The correction is small near the apex when $x \ll vt_h$, and on the asymptotes when $vt_h \gg h$ and the extra term in (11) is negligible. Because equation (13) depends on the distance x, it cannot be applied as a correction on a common offset gather. It is simpler to do approximate NMO first and residual NMO after DMO rather then use a correction like equation (13). A better way may be to apply DMO' directly.

Finite differencing DMO'

We are interested in the DMO' ellipse (11):

$$t_{d'} = t_h \left[1 - x^2 \left(\frac{1}{h^2} - \frac{4}{v^2 t_h^2} \right) \right]^{\frac{1}{2}}$$

We have a program that gives a response with a shape given by (6). Let h in equation (6) be a fictitious offset h_f, different from the real half offset h in (11). The output of the finite differencing will be

$$t_f = t_h \exp \left(- \frac{x^2}{2h_f^2} \right)$$

The error is

$$t_{d'}^2(x) - t_f^2(x) = t_0 \left[1 - x^2 \left(\frac{1}{h^2} - \frac{4}{v^2 t_h^2} \right) - \exp \left(- \frac{x^2}{2h_f^2} \right) \right]$$

The optimal h_f will be slightly bigger than the true half offset h. To have zero error at some $x_0 \neq 0$, in addition to the zero error we always have at $x = 0$ we choose

$$h_f^2 = \frac{x_0^2}{-log \left(1 - x_0^2 \left(\frac{1}{h^2} - \frac{4}{v^2 t_h^2} \right) \right)}$$

The DMO' is velocity dependent through h_f, but a crude velocity model is probably sufficient.

In Figure 5 the error vanishes at the truncation point

$$x_0 = 2h^2 / v t_h$$

and curve 4 approximates curve 2 very well. The same correction can be applied to the DMO if we are too close to the mute; here it was not and so curve 3 slightly deviates from curve 1.

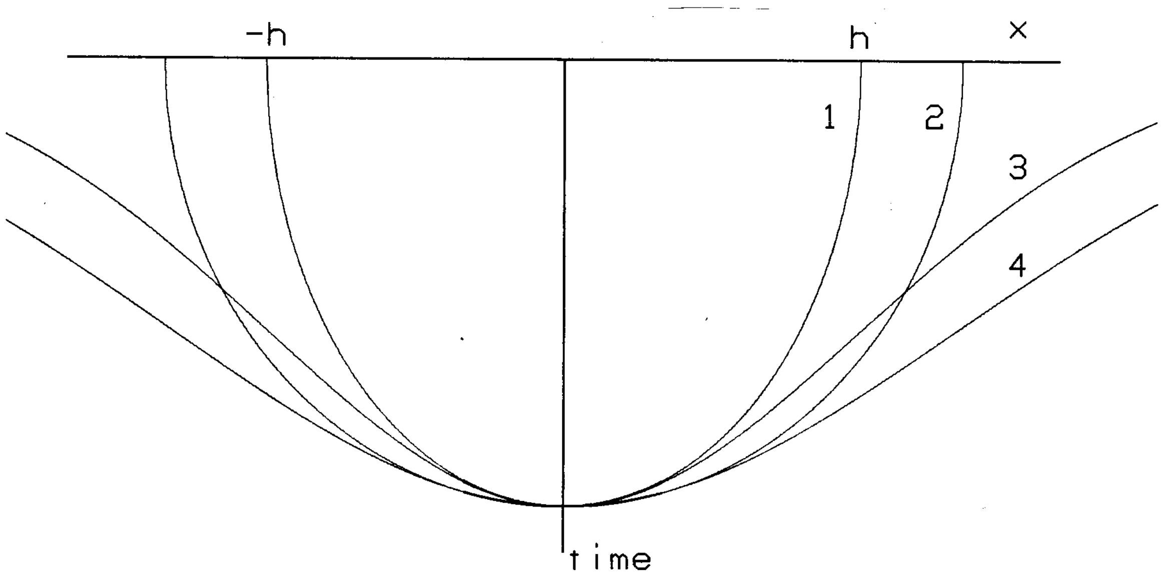


FIG. 5. Curve 1 is the DMO ellipse, 2 is the DMO' ellipse, 3 the finite differencing DMO gaussian and 4 is the finite differencing DMO' gaussian with zero error at the end points.

Velocity variations

The dip move-out operator is independent of the velocity as long as it is constant. Velocity variations, however, do have an effect. Suppose we have a point diffractor at (x_a,t_a), (See Figure 6.) $v_a = v(x_a,t_a)$ is the root mean square velocity of the apex of that event on the zero offset section. Consider a CMP gather at $x \neq x_a$. The point diffractor will appear on the CMP gather as the flat-top marked 1 on Figure 6:

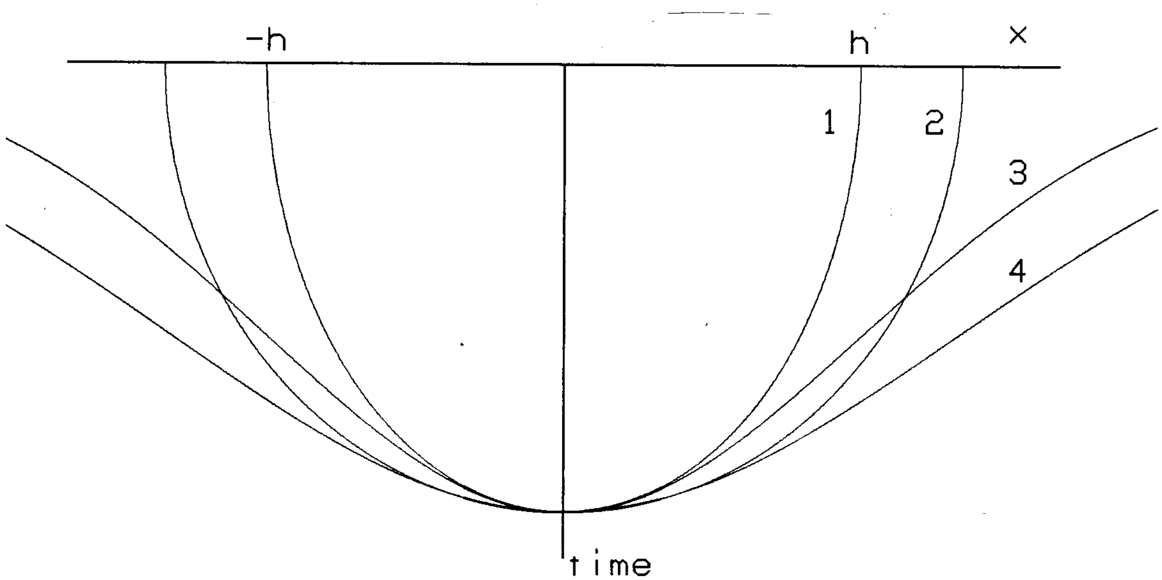
$$t(h) = \frac{1}{v_a}\left[\left(z_a^2 + (x-x_a-h)^2\right)^{\frac{1}{2}} + \left(z_a^2 + (x-x_a+h)^2\right)^{\frac{1}{2}}\right]$$

DMO' transforms the flat top to the hyperbola

$$t(h) = t\left(1 + \frac{4h^2}{v_a^2 t^2}\right)^{\frac{1}{2}}$$

The normal move-out velocity of this hyperbola is v_a, which is different from the velocity at the time $t(h=0)$. On the CMP gather of Figure 6, 3 is the hyperbola of an event with velocity $v(t,x)$ that will stack in, while the hyperbola marked 2 is the DMO' of the flat top 1. It will not stack in unless we correct by

$$\Delta t_c = t\left[\left(1 + \frac{4h^2}{v_a^2 t^2}\right)^{\frac{1}{2}} - \left(1 + \frac{4h^2}{v^2 t^2}\right)^{\frac{1}{2}}\right]$$



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$$\approx \frac{2h^2}{t}\left[\frac{1}{v_a^2} - \frac{1}{v^2}\right]$$

$$\approx \frac{4h^2}{tv^3}\left[v - v_a\right]$$

$$\approx \frac{4h^2}{tv^3}\left[\frac{\partial v}{\partial t}(t - t_a) + \frac{\partial v}{\partial x}(x - x_a)\right]$$

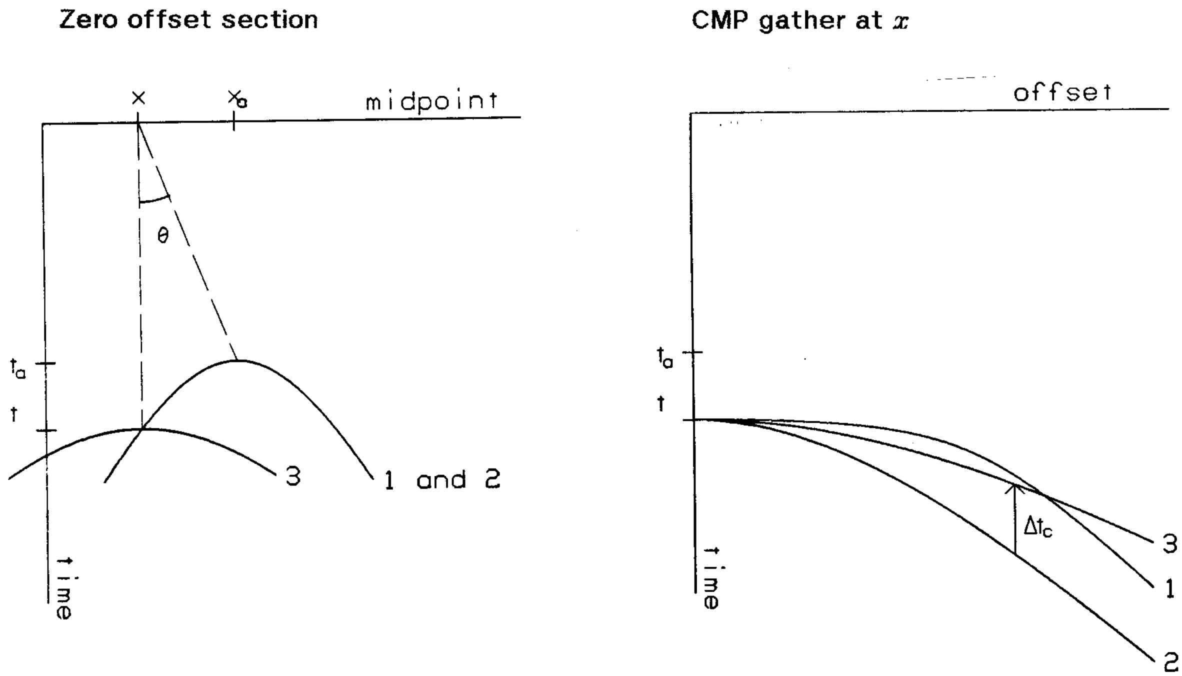

FIG. 6. 1: point diffractor at (x_a,t_a) before DMO'. 2: point diffractor at (x_a,t_a) after DMO'. 3: point diffractor at (x,t).

From Figure 6 we have:

$$\frac{t_a}{t} = \cos\theta$$

$$\frac{x_a - x}{t_a} = tg\,\theta$$

Using this and $\sin\theta = \dfrac{vk}{2\omega}$ we find

$$t - t_a \approx \frac{v^2 t k^2}{8\omega^2}$$

$$x - x_a \approx \frac{v^2 k t}{4\omega}$$

And the correction is approximately

$$\Delta t_c = \frac{h^2 k^2}{2\omega^2 v} \frac{\partial v}{\partial t} + \frac{h^2 k}{v\omega} \frac{\partial v}{\partial x}$$

The corrected dip move-out is

$$\Delta t_D = \frac{h^2 k^2}{2\omega^2 t} \left(1 - \frac{t}{v} \frac{\partial v}{\partial t} \right) - \frac{h^2 k}{\omega v} \frac{\partial v}{\partial x}$$

The operator is:

$$P(h) = exp[\, i\omega \Delta t_D] \cdot P(0)$$

It can be done by the differential equation:

$$\frac{\partial^2 P}{\partial h \partial t} = \left[\frac{hk^2}{t} \left(1 - \frac{t}{v} \frac{\partial v}{\partial t} \right) - 2\frac{h}{v} \frac{\partial v}{\partial x} \frac{\partial}{\partial t} ik \right] P$$

The vertical velocity variation correction is easily incorporated by reducing the fictitious offset h_f by a factor

$$\left(1 - \frac{t}{v} \frac{\partial v}{\partial t} \right)^{\frac{1}{2}}$$

The velocity model should have finite gradient.

The described velocity variations correction should be combined with DMO' (before NMO). There is a slight difference if the correction is introduced in DMO after NMO. Also this treatment ignored bending rays which turn out to introduce a correction with similar magnitude and apposite sign. (Deregowski and Rocca, 1981 and Dave Hale, 1983).

Conclusions

1. The described DMO method has small error below the mute.

2. Time and velocity dependent corrections to the offset enable application of the DMO before NMO and make it capable of handling vertical velocity variations.

3. Lateral velocity variations may be handled by adding a term to the DMO extrapolation equation.

ACKNOWLEDGMENTS

Fabio Rocca suggested how to introduce the vertical velocity gradient correction. Some discussions with Dave Hale and Stew Levin were helpful.

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