A short note on constant velocity migrations

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The correct choice of migration velocities is often crucial for the accurate migration of seismic data. As discussed by Chun and Jacewitz (1978), constant velocity migrations, generated over a range of velocities, are a helpful aid in the precise determination of a migration velocity function. These individually migrated sections allow relatively confident determination of migration velocities as a function of x and τ, removing much of the guesswork that might otherwise have been necessary.

Once determined, the correct velocity function is then often used for a subsequent, variable velocity migration. This note, however, investigates an alternative, direct approach: instead of remigrating, one may effectively synthesize a variable velocity migration from the original cube of data, $p(x,\tau,v)$, by extracting needed values along the v-axis. This will be possible when the v-axis is sampled with sufficient density. We derive here a suitable sampling criterion for Δv.

Choosing Δv for $p(x,\tau,v)$

Our objective is to define Δv so that the the movement of energy on the (x,τ) grid is limited to a specified Δx and $\Delta \tau$, chosen to avoid aliasing along the v-axis. The result will depend on dip. If the steepest dip of interest contains spatial and temporal frequencies up to the Nyquist limit, then we will want to limit Δx and $\Delta \tau$ to their respective sampling intervals on the (x,τ) section.

The model for analysis is a point scatterer at (x_0,τ_0). The diffraction is expressed by

$$p(x,\tau,t=0) \;=\; \int\int e^{ik_x(x-x_0)\,+\,i\omega_\tau(\tau-\tau_0)}\,dk_x\,d\omega_\tau \;. \tag{1}$$

Extrapolation in time yields

$$p(x,\tau,t) \;=\; \int\int e^{-i\omega(k_x,\omega_\tau)t}\,e^{ik_x(x-x_0)\,+\,i\omega_\tau(\tau-\tau_0)}\,dk_x\,d\omega_\tau \;, \tag{2}$$

where $\omega = \sqrt{\omega_\tau^2 + v^2 k_x^2}$. The location of the energy at any time t can be found by using the method of stationary phase. Following Claerbout (1982), we denote the phase of the integrand by φ,

$$\varphi \ = \ -\omega(k_x,\omega_\tau)t \ + \ k_x x \ + \ \omega_\tau \tau \ , \tag{3}$$

and note that the energy is located at those points where the partial derivatives of φ with respect to ω_τ and k_x equal 0, or, in other words, where the phase is "stationary." Thus we write

$$\frac{\partial \varphi}{\partial k_x} \ = \ -t\frac{\partial \omega}{\partial k_x} + x - x_0 \ = \ 0 \tag{4a}$$

$$\frac{\partial \varphi}{\partial \omega_\tau} \ = \ -t\frac{\partial \omega}{\partial \omega_\tau} + \tau - \tau_0 \ = \ 0 \ . \tag{4b}$$

The coordinates (x,τ) of the resulting diffraction can then be represented parametrically as

$$(x - x_0, \tau - \tau_0) \ = \ t \left[\frac{\partial \omega}{\partial k_x}, \ \frac{\partial \omega}{\partial \omega_\tau} \right] \ . \tag{5}$$

If the extrapolation is performed with velocity $v_0 \neq v$, then the location of the energy is determined with the same calculation, but with $\hat{\omega} = \sqrt{\omega_\tau^2 + v_0^2 k_x^2}$. We can then find the resulting errors, Δx and $\Delta \tau$, by subtraction:

$$\Delta x \ = \ \hat{x} - x \ = \ t \ \frac{\partial}{\partial k_x}(\hat{\omega} - \omega) \tag{6a}$$

$$\Delta \tau \ = \ \hat{\tau} - \tau \ = \ t \ \frac{\partial}{\partial \omega_\tau}(\hat{\omega} - \omega) \ . \tag{6b}$$

Making use of the relation $\sin\theta = \frac{v k_x}{2\omega}$, and defining $V = \frac{v_0}{v}$, we find, after some algebra,

$$\frac{2\Delta x}{v \tau_0 \tan\theta} \ = \ V^2 \left[\cos^2\theta + V^2 \sin^2\theta\right]^{-1/2} - 1 \tag{7a}$$

$$\frac{\Delta \tau}{\tau_0} \ = \ \left[\cos^2\theta + V^2 \sin^2\theta\right]^{-1/2} - 1 \ . \tag{7b}$$

We can solve each of the above equations for $V = 1 + \frac{v_0 - v}{v}$ in terms of Δx and $\Delta \tau$, to obtain the maximum sampling interval, $\Delta v = v_0 - v$, that is allowable if we wish to limit the movement of a parcel of energy to Δx and $\Delta \tau$. In terms of

$$X \ = \ 1 + \frac{2\Delta x}{v \tau_0 \tan\theta} \tag{8a}$$

and

$$T = 1 + \frac{\Delta\tau}{\tau_0} \tag{8b}$$

these limits are

$$\left(\frac{\Delta v}{v}\right)_{\Delta x} = \frac{1}{\sqrt{2}}\left(X^2\sin^2\theta + \sqrt{X^4\sin^4\theta + 4X^2\cos^2\theta}\right)^{1/2} - 1 \tag{9a}$$

$$\left(\frac{\Delta v}{v}\right)_{\Delta\tau} = \frac{1}{T\sin\theta}\left(1 - T^2\cos^2\theta\right)^{1/2} - 1 \tag{9b}$$

respectively. Note that Δv increases with velocity, but decreases with depth. In practice, the smaller of the two Δv's would be chosen. Graphs of both equations are pictured in Figure 1. For small dip, velocity increments of about 5% are acceptable, but substantial dip requires a much smaller increment, possibly less than 1%.

Example

Figure 2 shows data that has been migrated by the extraction of needed points from a suite of constant velocity Stolt migrations. An 8 point, tapered sinc interpolator was used when the correct velocity fell between velocity panels. The migration velocities ranged from 1650 m/sec. to 2000 m/sec., varying both laterally and with depth. Due to the steep (40 degree) dips present in the data, a velocity increment of approximately 1% was required; a constant 15 m/sec. increment was used. The unmigrated input and other migration results are displayed in Figures 4-8 of "Residual migration," by Rothman, Levin, and Rocca, also in this volume. Comparison with the other migrations shows this extracted, synthesized migration to be of similar quality.

Nota bene

Variable velocity migration by the method outlined here is only valid insofar as rms velocities accurately describe diffractions. Ray bending resulting from velocity inhomogeneity is ignored, but as for Kirchhoff migration, a reasonable degree of migration accuracy is expected when lateral variations are slight (Schneider, 1978). Migration by the recomposition of constant velocity migrations is, in principle, much like Kirchhoff migration - summation paths are hyperbolic. One must not be misled, however, by the ease with which $v(x,\tau)$ is specified. Complex geology poses problems beyond the limitations of constant velocity migrations, and, indeed, time migration in general.

ACKNOWLEDGMENTS

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REFERENCES

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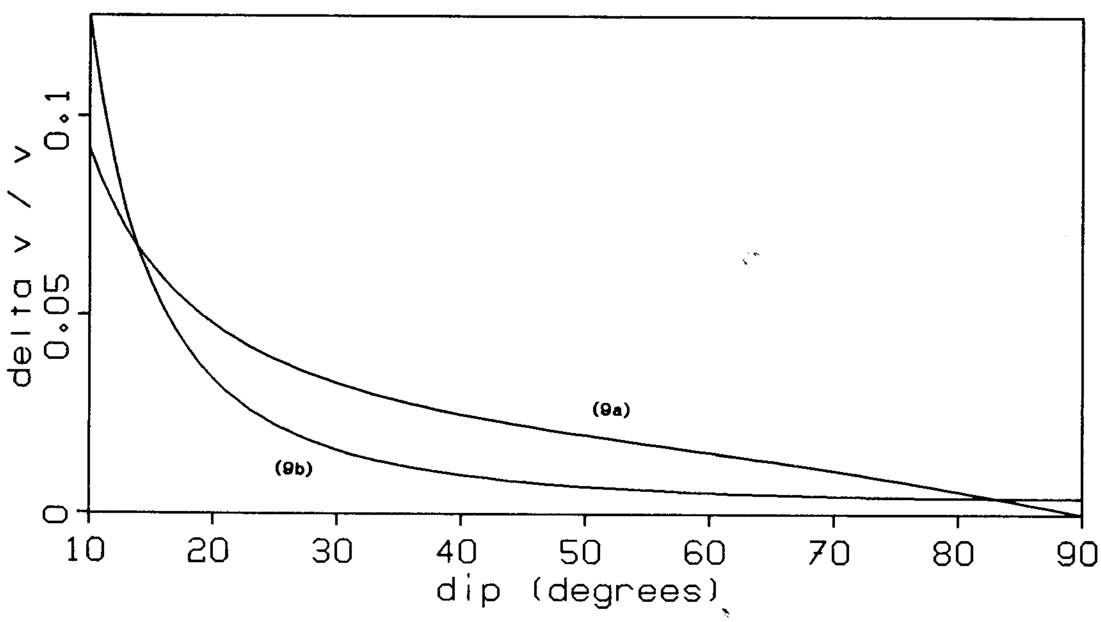

FIG. 1. Δv / v as a function of dip, as described by equations (9a,b). The cases shown are for Δx = 25m, $\Delta\tau$ = -4 msec., τ_0 = 1 sec., and v = 1500 m/sec.

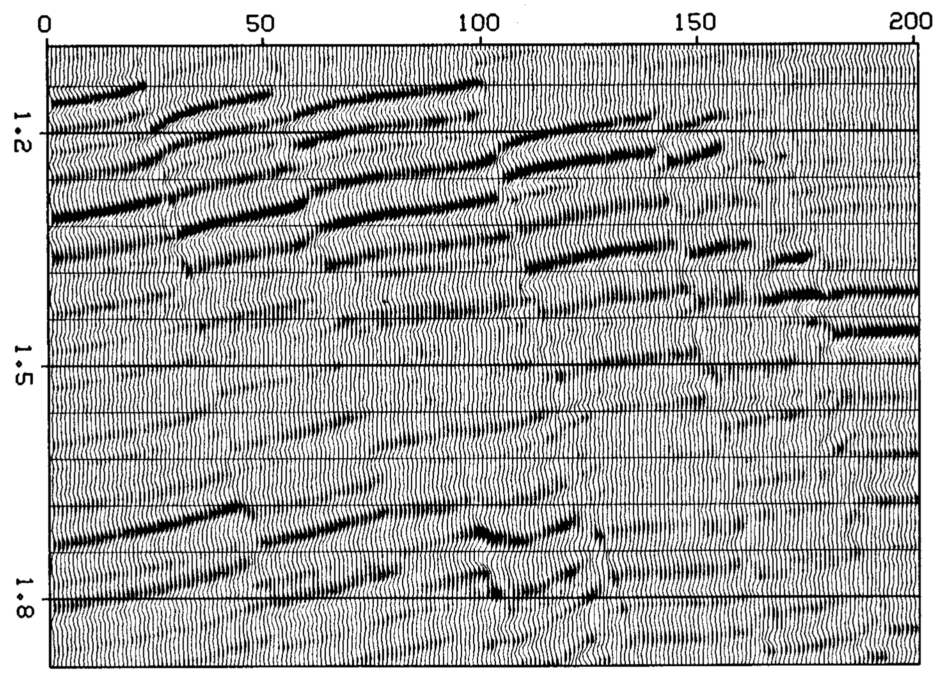

FIG. 2. Migration by the extraction of needed points from a suite of constant velocity migrations.

"Therein" contains 11 smaller English words, and you do not have to rearrange any letters. "There" is one; "I" is another. Find at least eight others.

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The six letters in the word "chesty" can be transposed into only one other English word. What is it?

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The eight letters in the word "pictures" can be rearranged into only one other common word. What is it?

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Fill in the question marks below with consonants to form the longest word with alternating consonants and vowels.

?e?i?i?i?i?u?e?

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You in a forest in which every inhabitant is either a prince or a pawn. Princes always tell the truth, and pawns always lie. In addition, some of the inhabitants, both princes and pawns, can be werewolves. You meet three inhabitants, A, B, and C, and you know that exactly one of them is a werewolf. They make the following statements:

A: C is a werewolf.

B: I am not a werewolf.

C: At least two of us are pawns.

Questions:

a) Is the werewolf a prince or a pawn?

b) If you had to choose one of them as not being a werewolf, which one would you choose?

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A: I am a werewolf.

B: I am a werewolf.

C: At most one of us is a prince.

What are A, B, and C?

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