Invertible velocity analysis

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The proposition and the problem

By transforming a common-shot or common-midpoint gather to x^2-t^2 space, one can take a slant stack, unstretch the time axis, and treat each resulting trace as a constant velocity stack. Each transform inverts easily; events selected on the basis of velocities and coherence may be restored to $x-t$ space. To make such a process useful, however, one must avoid making the x^2-t^2 stretches explicitly.

The invertible transform and its use

The transforms are simple. Transforming to x^2-t^2 space maps hyperbolic events, corresponding to reflections from flat horizons, into straight lines. The moveout of such events is given by the equation

$$t^2 = x^2 / v^2 + 4 \cdot h^2 / v^2 \tag{1}$$

where x is the offset from shot to geophone, and h the depth to the reflector. The slope of the linear x^2-t^2 events is $1/v^2$ A slant stack maps straight lines into points by summing along lines of constant slope.

$$Q'(p,t) = \int P'(x,t + px) dx \ , \tag{2}$$

where $P'(x,t)$ is being transformed. Thus a hyperbola with a peak at t_0 and a corresponding velocity of v should, after both transforms, map to a single point (t_0, $p = 1/v^2$).

An invertible velocity stack allows signal/noise separation. Slant stacks may be inverted by frequency filtering ("rho" filtering) and other slant stacks. Stretched axes may be unstretched. Points in p-t space should map back into hyperbolas in x-t space with the original amplitude distributions. With an invertible stack one could mask out multiple reflections in p-t space, events which show anomalously low velocities for their arrival times. One might extract high amplitude events in p-t space, representing strong, coherent events in x-t space, invert them, subtract them from the original data set, and then proceed to estimate the strongest uncorrelated noise. (See Harlan, SEP-32, for a similar process with slant stacks alone.)

Implementation

One would not want to transform to x^2-t^2 space explicitly. The early samples of a t^2 trace would possess only low frequencies with gross oversampling. To avoid aliasing the latest waveforms, one would need to extend a trace with n samples to $2n^{3/2}$. A one-step version of the forward transforms may be written almost immediately.

$$Q(p,t) = \int P(x^2, (\sqrt{t} + px)^2) dx \ . \tag{3}$$

Let $\chi = x^2$; $d\chi = 2\sqrt{\chi} dx$.

$$Q'(p,t) = \int P(\chi, t + 2\sqrt{t\chi}p + p^2\chi) \frac{1}{2\sqrt{\chi}} d\chi \tag{4}$$

The integration over the data set now implicitly stretches and unstretches it. The $1/2\sqrt{\chi}$ factor weights the near offsets rather unequally, but to remove it would make inversion less obvious.

The inversion of equation (2) is essentially the inverse of the radon transform. In the time domain this inverse appears (cf. Thorson, SEP-14)

$$P'(x,t) = \int Q'{*}M_S(p,t - ph) \, dp \tag{5}$$

$$M_S(t) \equiv \frac{S \sin 2\pi S t}{\pi t} - \frac{\sin^2 \pi S t}{\pi^2 t^2} \tag{6}$$

where $Q'{*}M_S$ is a convolution over time of $Q'(p,t)$ with $M_S(t)$. S is the highest temporal frequency, the Nyquist frequency. The Fourier conventions of Bracewell, 1978, are used. The convolution by M_S corresponds to "rho" filtering--multiplication by $|\omega|$ in the frequency domain. To invert equation (4) then requires

$$P(x,t) = \int Q{*}M_S(p,t - 2\sqrt{tx}p + p^2 x) \, dp \tag{7}$$

where now

$$Q{*}M_S(p,t) = \int Q(p,\tau) \cdot M_S(t - \sqrt{\tau}) \frac{1}{2\sqrt{\tau}} d\tau \tag{8}$$

Just as for the forward transform, the integrations may be made directly over unstretched data. Because of the implicit stretch, S should represent the highest temporal frequency of $Q(x,t^2)$. For efficiency, S should vary with t as though the sampling in t^2 did not greatly exceed the corresponding sampling in t. M_S could be tapered to zero after say ten terms.

##

The following facts about the town of Podunk are true:

(1) No two inhabitants have exactly the same number of hairs.

(2) No inhabitant has exactly 518 hairs.

(3) There are more inhabitants than there are hairs on the head of any one inhabitant.

What is the largest possible number of inhabitants of Podunk?

##

Two American coins add up to thirty cents, yet one of them is not a nickel. What coins are they?

##

Is it more correct to say "The yolk is white" or "The yolk are white"?

##

A train leaves from Boston to New York. An hour later, a train leaves from New York to Boston. The trains are going at exactly the same speed. Which train will be nearer to Boston when they meet?

##

A certain street contains 100 buildings. A sign-maker is called to number the houses 1 to 100. Without using pencil and paper, how many 9's will he need?

##

You are on the island of Princes and Pawns. Princes always tell the truth, and pawns always tell lies. You cannot tell them apart from appearance. You come across two people, A and B. A says: "At least one of us is a pawn."

What are A and B?

##

You are on the island of Princes and Pawns. Princes always tell the truth, and pawns always tell lies. You cannot tell them apart from appearance. You come across three natives: A, B, and C. You ask A: "How many princes are among you?"

You could not hear what A said, so you turn to B and ask: "What did A say?"

The second native replies: "A said that there is one prince among us."

C spoke up and said: "Don't believe B; he is lying!"

What are B and C?

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