Separation of Non-Stationary Signals and Noises

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Despite the many sophisticated papers published on deconvolution, we still have no way to solve single channel problems that humans solve all the time. You hear several voices, and after a while you are able to distinguish several speakers. This paper is an attempt to formulate the problem in a very simplified way.

Let d denote some data which has not been AGCed. Imagine a white noise component ξ in d. The white component has an unknown time-variable variance. The signal will be denoted by a. We have $d = \xi + a$. To begin, we take half of d to be in a and half to be in ξ. Then we construct initial AGC functions with

$$D_\xi^2 \quad = \quad diag\left(\frac{1}{\ll | d-a |^2 \gg} \right) \tag{1}$$

$$D_a^2 \quad = \quad diag\left(\frac{1}{\ll | a |^2 \gg} \right) \tag{2}$$

The signal will be assumed to be adequately sampled, so that its spectrum drops off near the Nyquist. Thus we take the power spectrum of the signal after scaling by D_a, to be proportional to $1/(\omega_0^2 + \omega^2)$. The covariance matrix of the inverse of this spectrum is the familiar tridiagonal matrix encountered in migration programs. Now we set out to minimize

$$E \ = \ (d-a)^* D_\xi^2 (d-a) \ + \ a^* D_a \, T \, D_a \, a \tag{3}$$

Set $\partial E / \partial a^*$ to zero.

$$-D_\xi^2 (d-a) + D_a \, T \, D_a \, a \ = \ 0 \tag{4}$$

$$(\, D_\xi^2 \, + \, D_a \, T \, D_a \,) \, a \ = \ D_\xi^2 \, d \tag{5}$$

Equations (1), (2), and (5) constitute an algorithm which can be iterated. To capture big spikes in the residual, I imagine that we would want to have a short time constant in the smoothing $\ll \gg$ in (1) but a much longer smoothing window in (2).

Sparse Signals

Now suppose that we have been too cavalier in writing the inverse covariance matrix of the signal as $D_a \, T \, D_a$. Perhaps the signal a is produced by a sparse spike train which is smoothed through some wavelet. Maybe the inverse covariance matrix should be represented as $B^* D_a^2 B$ where B is a deconvolution matrix. A tricky point, that I frequently gloss over is how to define D_a^2. I guess it should be the square of the AGC function for $B \, a$. I can also imagine a slow AGC before decon and a fast one afterward.

There are very many unresolved issues of whether and how iterations will work.

Accelerating the Convergence in the Missing Data Iteration.

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Recall from SEP-26 the problem of a time series with points missing in gaps. Some gaps are large and some are small. The missing data iteration is

$$d \quad = \quad d - M\, F^{-1}\, \frac{1}{<P>} F\, d \tag{1}$$

In this expression, M is a square matrix which multiplies missing data by one and known data by zero, F stands for Fourier transform, $<P>$ is a slightly smoothed spectrum which either must be scaled so that its inverse has a maximum of 1 or else we must solve an auxiliary minimization problem for a scaling factor α.

This problem is known from SEP-26 to converge slowly in the large gaps if $<P>$ has a wide dynamic range. To fill in the long gaps quickly, first solve the problem

$$d \quad = \quad d - M\, F^{-1}\, \frac{1}{< \dfrac{P}{\ll P \gg} >} F\, d \tag{2}$$

By adjusting the time constant in the heavy smoothing $\ll \, \gg$, the dynamic range of the weights can be compressed, thereby achieving rapid convergence to the solution of a different problem. On a large scale of ω, the new problem is different from the old. But on a small scale of ω the problems are the same. The uncertainty relation tells us that at long t we are solving the same problem as before. But we get the answer more quickly.

A glossary for research reports

It is well known	I want you to feel bad if you have a question about it
Of great theoretical and practical importance	Interesting to me
It has not been possible to provide definite answers to these questions.	I couldn't clear all the bugs but I put it in the report anyway.
It sometimes works	Usually it doesn't make any sense.
The Ant-Arctic data was chosen for detailed study	The method didn't work anywhere else
Typical results shown	Best results shown
The agreement with the theory is:	
excellent	fair
good	poor
satisfactory	doubtful
fair	imaginary
as good as could be expected	non-existent
These results will be reported later	I might possibly take the time to write it down
It is believed that...	I think
It is generally believed that...	A couple of guys think so too.
It might be argued that...	I have such a good answer to this objection that I shall now raise it.
It is clear that additional work is required before a complete understanding	This is the only thing clear about it.
Correct within an order of magnitude	Wrong
It is hoped that this work will stimulate further work in this subject	This paper isn't very good but neither are any of the others in this miserable field.

Adapted from Metal Progress 71, 1957. By C. H. Graham, Jr.