Missing Data Analytic Solutions

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Consider moved-out seismic data of any kind. Make the presumption that the spatial spectrum of the data has the form $1/(1+k^2/k_0^2)$. Thus the covariance matrix of the inverse spectrum is tridiagonal, call it T. (The tridiagonal form of an inverse covariance matrix allows us to represent both space variable envelope and space variable spectra.) In matrix formalism, we minimize $d^* T d$, subject to constraints that the data d matches the known data, say d_0. Define K to be a non-square matrix containing ones and zeros arranged to collapse the presumed data vector d to the shorter observed data vector, call it d_0, by the relation $K d = d_0$. The method of Lagrange multipliers or the method of FGDP page 115 shows that the equations to be solved are:

$$\begin{bmatrix} T & K^T \\ K & 0 \end{bmatrix} \begin{bmatrix} d \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ d_0 \end{bmatrix} \tag{1}$$

The solution is an exercise in FGDP, but it will be solved here

$$T d + K^T \lambda = 0 \tag{2}$$

$$K d = d_0 \tag{3}$$

Premultiply (2) by $K\, T^{-1}$

$$K d = -\, K T^{-1} K^T \lambda \tag{4}$$

Insert (3)

$$d_0 = -\, K\, T^{-1} K^T \lambda \tag{5}$$

Solve for λ

$$10\ 11 \tag{6}$$

Insert into (2)

$$Td \quad = \quad K^T (K \, T^{-1} K^T)^{-1} d_0 \tag{7}$$

Matrix inversion is required of a size equal to the number of known data points. In the case of profiles, the matrix size is the number of known channels. Thus exact solution should not be too costly for profiles, and gathers and possibly VSPs, but we will wish to do something iterative for sections. This model is limited to data with a single dominant propagation mode. Concurrent work reported in other papers examines multimode propagation.

Post facto it would be a nice idea to go back and fix up k_0. I guess that half of the spectrum of d should be at frequencies below k_0 and half above. If it isn't so, then k_0 should be moved.

We have not exploited any of the special properties of the tridiagonal matrix. The advantage of a tridiagonal matrix is that the cost of the solution to (7) increases only linearly with the number of missing points. Obviously other banded matrices also provide the same advantage. In many geophysical applications, it is more important to get the "right" tridiagonal matrix than to use more bands. The tridiagonal form of an inverse covariance matrix allows us to represent both time variable envelope and time variable spectra.