THE PRE-STACK MIGRATION OF PROFILES

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Abstract

Conventional processing of seismic data, consisting of the sequence of normal moveout correction, stacking, and migration, often yields a good image of the earth. The quality of this image will be degraded by lateral variations of acoustic velocity and by sparse sampling of the shot axis. The degradation appears as lower signal-to-noise, misplacement of seismic reflectors in space, and blurriness.

The answer to the problem posed by sparse sampling of the data along the shot axis is to work in physical coordinates and form separate, independent images of the earth from common shot profiles. The two shot profile migration techniques that are discussed here are denoted the Cartesian and the NMO-coordinate methods. The Cartesian method is a solution to the problem of lateral velocity variations as well. The algorithms migrate shot profiles and then stack the images.

The Cartesian algorithm downward continues both an upgoing wave recorded at the geophones of a profile and a downgoing wave originating at the shot point. The downward continuation is done in a Cartesian coordinate system. An image is obtained from the two downward continued waves by taking the sum of the frequency domain quotients of the upgoing wave by the downgoing wave.

The Cartesian algorithm for the downward continuation of profiles must be able to handle large propagation angles about the vertical. Therefore, a high sampling rate along the geophone axis and high order migration operators are a necessity. These are developed using a continued fraction approximation to the square root that appears in the one-way wave equation.

Spatial aliasing can also be a problem along the geophone axis. The Cartesian migration algorithm, for instance, requires dense sampling of the data along the geophone axis because it uses g-derivatives. In the absence of steep dips and strong lateral velocity variations, one way to ease sampling requirements along the geophone axis is to migrate in an NMO- based coordinate system. The algorithm that downward continues shot profiles in an NMO-coordinate frame is closely related to the commonly applied steps of normal moveout correction and time-to-depth conversion.

Introduction

A reflection seismic experiment consists of surface recordings of the response of the earth to a known source of sound waves. Reflection interpreters use the recordings to build models of the subsurface that are consistent with the data. Among the tools that interpreters bring to this task are a set of algorithms that process seismic data sets. Two conditions that degrade the performance of these algorithms are insufficient sampling of the data along the shot axis and lateral velocity variations.

Lateral velocity variations occur in a number of situations that are of economic or scientific significance. A dipping sea floor is a particularly common example. This instance of lateral velocity variation is characterized by its geometric simplicity and by the fact that the acoustic velocity of the material above the contact (sea water) is known. The overthrust faults in the Rocky Mountains have another geological structure that juxtaposes high and low velocity material; this time low velocity sediments are placed below high velocity igneous rocks and the velocity function on both sides of the contact is unknown. Detection of lateral velocity variation is important in the identification of petroleum traps. For example, carbonate reefs and stratigraphic traps can be found through the observation of travel time anomalies of finite lateral extent. This thesis provides an accurate means of getting images of acoustic waves when material properties vary laterally, but is only suggestive of solutions to the problem of estimating the magnitude and location of these variations.

Insufficient sampling of the data along the shot axis is, and probably will be, more common than sparse sampling of the data along the geophone axis. The number of geophones per experiment and their density in the field have increased ever since the first seismic surveys, and the pace of the trend is increasing today. On the other hand, the number of shots one can set off in a given time period is not likely to decrease, because an irreducible amount of time must be allotted for the earth to quiet down between each of the shotpoints. The basic point is that if a fixed area must be surveyed in a fixed number of months, then the average shot spacing has a minimum that is not reducible by technological improvements.

A typical sequence of processing steps for seismic data, in the order of their application, often includes the application of a normal moveout correction to a suite of common midpoint (CMP) gathers, summation over offset to get a stacked section, and migration. In figure 0.1, normal moveout correction and stacking work on data sets of constant midpoint y, where s indexes the lateral position of the shots, g is the lateral coordinate of the geophones, and $y = (s + g)/2$. Similarly, migration works on data sets indexed by y, and for which the half-offset h is a constant.

Migration is a process for obtaining images from stacked sections by computationally sinking into the ground all the shots and geophones of a seismic line. For this purpose, a stacked section is treated as a zero offset section recording of an upgoing wave U. Such a wavefield can be downward continued with a single square root equation, and an image obtained by stripping off the zero travel time plane (Loewenthal, Lu, Roberson, and Sherwood, 1976).

In regions where subsurface dips are too large and variable a modification of the conventional process, called pre-stack migration, may be performed. Pre-stack migration is usually described as a sequence of operations, resembling the post-stack migration operation, on constant offset sections. In practice, pre-stack migration may be applied as a small correction to the conventional sequence of processing operations.

The conventional processing sequence and its pre-stack variants all assume that the seismic experiment records only an upgoing wave U, and that an image can be formed from this upgoing wave by downward continuing the shots and geophones of a seismic line into the ground (Yilmaz, 1979). These processes obtain images from that part of the downward continued data set that lies at zero offset and zero travel time. For theoretical purposes, the downward continuation can be thought of as a solution of the partial differential equation called the double square root equation.

The techniques explored here are migration algorithms that differ from the conventional sequence in four ways. First, the schemes introduced here work in shot-geophone coordinates whereas the conventional sequence works in midpoint-offset space. Second, both an upgoing wave recorded at the geophones, U, and a downgoing wave caused by the shot, D, are dealt with. These two waves are downward continued independently on separate computational meshes. The conventional process uses only an upgoing wave. Third, the new procedures downward continue with single square root equations. No use is made of the double square root equation that conventional processing is based on. Fourth, profile downward continuation uses the time coincidence of the upgoing wave, U, and the downgoing wave, D, as an imaging condition (Claerbout, 1970, and Claerbout, 1971). The conventional process strips off the zero travel-time plane from a downward continued upgoing wave.

Velocity analysis after profile migration is different from conventional velocity analysis in that it does not make any geometrical assumptions. Though not pursued here, images from correctly migrated common shot profiles should be

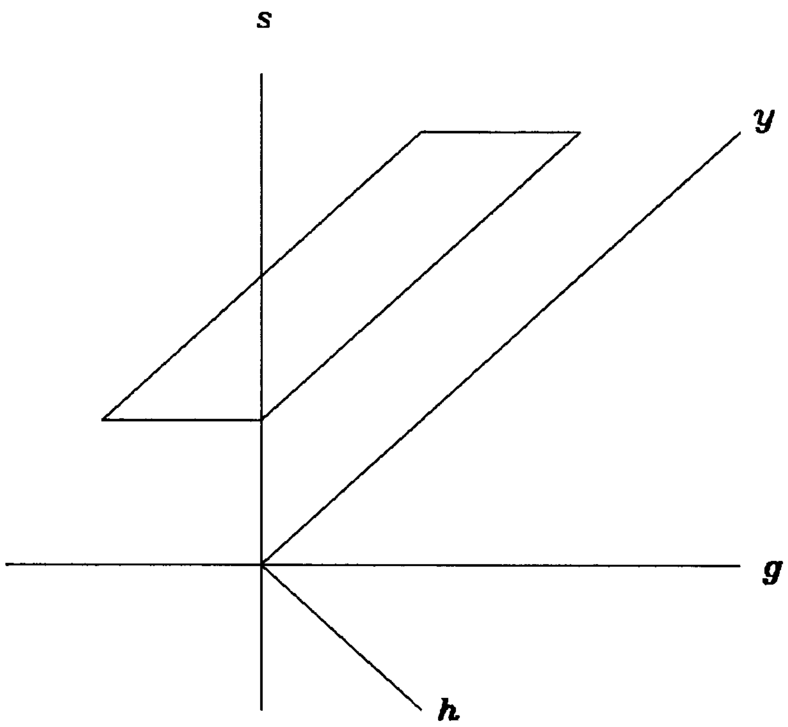

FIGURE 0.1. Shot-geophone coordinates. The s and g-axes index the spatial locations of the sources and receivers, respectively, of the traces of a seismic line. The finite dimensions of the seismic line and cable imply that all of the recorded data occurs in the outlined parallelepiped. The data in this quadrilateral is usually processed in offset-midpoint coordinates. The variables y and h denote midpoint and half-offset, respectively. This study concerns itself with processes that work on common shot gathers, slices of data parallel to the g-axis.

independent of illumination angle. Consider a suite of correctly migrated and imaged common shot profiles. Reorder this processed data as a suite of migrated common geophone gathers. Each such common geophone gather will contain events that are linear and horizontal and no others. Deviations from zero-dip linearity can be used to detect and (possibly) estimate velocity anomalies.

Two techniques for the downward continuation of profiles are discussed in this thesis. These algorithms will be denoted the Cartesian method and the NMO-coordinate method. An advantage that both the Cartesian and NMO-coordinate downward continuation algorithms have over conventional processing is that they do not require dense sampling along the shot axis. Profile migration avoids artifacts caused by subsampling along the shot coordinate by working with traces sharing a common shotpoint and therefore does not take derivatives with respect to s. For example, insufficient sampling becomes a problem in areas where the subsurface dips are so non-uniform that two hyperboloidal events on a CMP gather cross one another. In this case, simple dip correction of the rms velocity is not possible and a pre-stack migration algorithm, such as a profile imaging scheme, is appropriate. Most pre-stack algorithms work in midpoint-offset coordinates and apply a dip correction directly to the data instead of a moveout velocity function (Yilmaz, 1979). These schemes generally form a dip estimate for this correction by differentiating with respect to the midpoint coordinate y. On a constant offset section from a data set in which the shot spacing, Δs, is greater than the geophone sampling interval, the sampling rate for the y axis is proportional to Δs. Thus, if s is too sparsely sampled, then unaliased derivatives with respect to y cannot be taken. Aliasing artifacts can be expected in the output of a pre-stack migration algorithm that works in midpoint-offset coordinates.

The Cartesian method works in the rectilinear coordinate system defined by the lateral position, depth, and time axes. The method is the conceptually simplest way to downward continue a profile. The algorithm images profiles by separately downward continuing an upgoing wave recorded at the geophones and a downgoing wave caused by the shot. These two downward continuations are based on implementations of the single square root equation. There are many well-known techniques for implementing this differential equation in the special case where the acoustic velocity function is a function of z alone, so the problem of implementation in a laterally varying medium is taken up in this thesis. One problem that was encountered was the generation of migration artifacts caused by sparse sampling of the data along the geophone axis.

While retaining the ability to image data sparsely sampled along the shot axis, the NMO-coordinate method has an advantage over the Cartesian method of profile imaging in that it admits sparse sampling along the geophone axis, too. The NMO-coordinate scheme for obtaining images from profiles works in a curvilinear coordinate system. The seismic response over a zero-dip, planar interface is a horizontal line in the new coordinate system. Like the conventional processing sequence, the NMO-coordinate method relies on a normal moveout correction step to do most of the work involved in forming an image. The application of NMO to a profile also reduces its bandwidth along the geophone wavenumber axis, provided dips in the subsurface are small. The size of the temporal and spatial shifts performed by the migration algorithm increase with increasing dip. Thus, the NMO-coordinate method is appropriate in regions where dips are gentle. The coordinate system differs from those used earlier (Claerbout and Doherty, 1972) because one axis is indexed by a radial coordinate. THe wave equation for a constant or z-variable medium in a radial coordinate system has to do less work shifting acoustic waves laterally than it does in other coordinate systems. Radial coordinate systems lead to natural side boundary conditions, too.

Chapters 1 and 2 contain descriptions of the Cartesian and NMO-coordinate algorithms, respectively. Both of these chapters contain applications of profile migration to a marine seismic line from the Gulf of Alaska. Later chapters pick up the theoretical development of profile migration schemes. In particular, the Cartesian method of profile migration requires an operator that remains valid over a wide range of stepouts in a medium with laterally varying velocity. Thus, the derivation of the single square root equation and its approximation by a continued fraction expansion are taken up in chapters 3 and 4. In the process, a number of fitting parameters are introduced. A discussion of methods for picking these parameters is found in chapter 5. Chapter 6 extends the single square root theory to cover the propagation of pseudo-compressional and pseudo-shear waves in a hexagonally anisotropic earth.

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