5.5 Moveout-Corrected, Radial Coordinates

If the earth were truly inhomogeneous in all three dimensions, we could hardly expect a single seismic line to make any sense at all. It says something that reflection seismology usually seems to work, even when restricted to a single line. It says that the layered model of the earth is a pretty good starting point. Normal moveout correction is usually a good idea. Mathematically, we can say that NMO is an excellent tool to deal with depth variation in velocity, but its utility drops in the presence of steep dip or a wide dip spectrum.

In my first efforts to migrate reflection seismic data with the wave equation, I began by transforming the data to a moveout corrected coordinate system. This approach to migration is well suited to data which is sparsely sampled on the geophone axis. It is even better suited to data which is sparsely sampled on the shot axis. When steepness of dip becomes the grounds on which migration is evaluated, then moveout correction offers little advantage, indeed, it suffers by its comparative complexity. Improving technology is leading to much greater sampling density on the geophone axis. However, there is little likelihood that we will see significantly increasing densities in shot space, particularly as we work more in three dimensions. With three dimensional data undersampling is far worse than it was in two dimensions. In this section the older moveout correction approach will be brought up-to-date. It could easily see a revival in one form or another.

The U:D Imaging Concept

The U:D imaging concept says that *reflectors exist in the earth at places where the onset of the downgoing wave is time coincident with an upcoming wave*. Figure 1 illustrates the concept. It is easy to confuse the *experiment sinking* concept and the *U:D* concept because of the similarity of the phrase "downward continue the shots" to the phrase "downward continue the downgoing wave". The first situation refers to computations involving only an upcoming wavefield $U(s,g,z,t)$ whereas the second situation refers to computations involving both upcoming $U(x,z,t)$ and downgoing $D(x,z,t)$ waves.

The downgoing wave is usually handled as an impulse whose travel time is theoretically known in a layered medium. The upcoming wave could be treated by any of the methods of previous chapters. Alternately, the upcoming wave could be effectively

FIG. 1. (Riley) Up and downgoing waves observed with buried receivers. A disturbance leaves the surface at $t=0$ and is observed passing the buried receivers $G_1 \ldots G_5$ at progressively later times. At the depth of a reflector, z_3, the G_3 receiver records both the upcoming and downgoing waves in time coincidence. Shallower receivers also record both waves. Deeper receivers record only D. The fundamental principle of reflector mapping states that reflectors exist where U and D are time coincident. An alternate principle is that the upcoming waves must vanish for all time prior to the first arrival of the downgoing wave.

treated by the moveout coordinate system to be described. The time coincidence can be quantified in a number of ways. The most straight-forward seems to be to look at the zero lag of the cross-correlation of the up and down going waves. The image is creating by displaying the zero-lagged cross-correlation everywhere in (x,z)-space.

The sinking concept seems to demand complete coverage in shot-geophone space whereas the U:D concept works for any downgoing wave. It could be the spherical wave from a point source but it could also be a plane wave or Snell wave. An example of a data base which could be handled with U:D but not sinking would be that of a sonobuoy. A sonobouy is a hydrophone with a radio transmitter. It is thrown overboard and the ship sails away, repeatedly firing a source until it is out of range. The reciprocal principle says that the data is equivalent to a single source with a ***very*** long line of geophones.

The $U{:}D$ concept is used extensively in FGDP for the three problems of migration, velocity analysis, and multiple suppression. Later, for the single application of zero offset migration, $U{:}D$ was superceded by Sherwood's approach of using the exploding reflector concept. (I don't know how the exploding reflector concept of seismic migration began, but John Sherwood was the first to use it to explain migration with finite differences.) Still later, the experiment sinking concept emerged from work with Doherty, Muir, and Clayton.

Moveout/Radial Coordinates in Geophone Space

NMO reduces the problem of spatial alias on the offset axis because the earth usually has modest dips. But NMO has reduced utility for data with severe dip or a wide dip spectrum. On the other hand NMO is especially suited to large depth variations in velocity. Whatever its merits or drawbacks, NMO commands our attention by its near universal use in the industrial world.

Our theoretical analysis will abandon the geophone axis g in favor of a radial-like axis charactorized by a Snell parameter p. This really says nothing about the implied data processing itself, since it is simple enough to transform final equations back to offset. The coordinate system we will be defining will be called a Snell *trace* coordinate system. It is similar to the Snell *wave* coordinate system in that downward continuation by a fixed distance is mainly just a fixed time shift of the entire (p,t')-space. The difference is that the Snell wave coordinates measure times by vertical and horizontal *phase* velocities, whereas Snell trace coordinates measure all times along rays.

p	$(\sin\vartheta)/v$, the Snell ray parameter
t_p	any one way time from the surface along a ray with parameter p
g	the surface separation of the shot from the geophone
t'	one way time, surface to reflector along a ray
τ	travel time depth of buried geophones, one way time along a ray
t	travel time seen by buried geophones.
$v(p,t_p)$	a stratified velocity function, $v'(z)$, in the new coordinates

Noting that the horizontal distance traveled by a ray in a certain time is the time integral of $v\sin\vartheta = pv^2$, we have the coordinate transform:

$$t(t',p,\tau) \;=\; 2t' - \tau \tag{1}$$

$$g(t',p,\tau) \;=\; 2p\int_0^{t'} v(p,t_p)^2\,dt_p \;-\; p\int_0^{\tau} v(p,t_p)^2\,dt_p \tag{2}$$

$$z(t',p,\tau) \;=\; \int_0^{\tau} \frac{dt_p}{v(p,t_p)} \tag{3}$$

Were the above system to be inverted to get (t',p,τ) as a function of (t, g, z), we would say that surfaces of constant t' are reflections, surfaces of constant p are rays, and surfaces of constant τ are datum levels. Unfortunately, the inversion of the coordinate system cannot be done explicitly. We can still proceed analytically with the differentials. Form the Jacobian, that is

$$\begin{bmatrix} \partial_{t'} \\ \partial_p \\ \partial_\tau \end{bmatrix} = \begin{bmatrix} t_{t'} & g_{t'} & z_{t'} \\ t_p & g_p & z_p \\ t_\tau & g_\tau & z_\tau \end{bmatrix} \begin{bmatrix} \partial_t \\ \partial_g \\ \partial_z \end{bmatrix} \tag{4}$$

Performing differentiations only where they lead to obvious simplifications, we get the transformation equation for Fourier variables:

$$\begin{bmatrix} -\omega' \\ k_p \\ k_\tau \end{bmatrix} = \begin{bmatrix} 2 & g_{t'} & 0 \\ 0 & g_p & z_p \\ -1 & g_\tau & v(\tau)^{-1} \end{bmatrix} \begin{bmatrix} -\omega \\ k_g \\ k_z \end{bmatrix} \tag{5}$$

It should be noted that (5) is a linear relation involving the Fourier variables, but the coefficients involve the original time and space variables. So it is in both domains at once. This is useful and valid so long as when we compute second derivatives we do wish to neglect the derivatives of the coordinate frame itself. This is often a rather benign assumption, amounting to something like spherical divergence correction.

I'd like to elucidate the properties of the coordinate frame without getting bogged down in details. The most obvious approximation is to set z_p to zero. Then the system is essentially triangularized. To get further, faster, just set $v = 1$ in the definitions and obtain

$$k_g = \frac{k_p}{2t' - \tau} \tag{6}$$

$$\omega = \frac{\omega'}{2} + p \frac{k_p}{2t' - \tau} \tag{7}$$

$$k_z = k_\tau - \frac{\omega'}{2} \tag{8}$$

Now substitute these into the familiar square root equation

$$k_z = -\sqrt{\omega^2 - k_g^2} \tag{9}$$

To see what is new, we simply drop second order terms in k_p and get

$$k_\tau \;=\; -\,\frac{p\,k_p}{2t' - \tau} \tag{10}$$

The full implications for data processing have not yet been worked out.

CMP, Radial, Moveout Corrected Coordinates

Nothing could be more central to industrial data processing than NMO at common *midpoint*. Our first goal is to transform the DSR equation to CMP, moveout corrected coordinates. We start with $DSR(\omega, k_y, k_h)$ and do moveout correction which takes ω to ω'. The h-axis is then transformed to a Snell p-axis by ray tracing, thus taking k_h to k_p. Setting k_p to zero implies integration over p at constant (t', y) which is really just a CDP stack. So we expect to see how *CDP stacks* should be downward continued. Up until now we have been pretending that a CDP stack was the same as a zero offset section or as a CMP vertical stack. Definitions are

p	$(sin\,\vartheta)/v$, the Snell ray parameter
t_p	any two way time from the surface along a ray with parameter p
$2h$	the surface separation of the shot and geophone
t'	two way time, surface to reflector and back
τ	travel time depth of buried experiment, two way time along a ray
t	two way time in buried experiment
$v(p, t_p)$	a stratified velocity function, $v'(z)$, in the new coordinates

Noting that the horizontal distance traveled by a ray in a certain time is the time integral of $v \sin\vartheta = pv^2$ we have the coordinate transform:

$$t(t', p, \tau) \;=\; t' - \tau \tag{11}$$

$$h(t', p, \tau) \;=\; \frac{p}{2}\int_0^{t'} v(p, t_p)^2\, dt_p \;-\; \frac{p}{2}\int_0^{\tau} v(p, t_p)^2\, dt_p \tag{12}$$

$$z(t', p, \tau) \;=\; \frac{1}{2}\int_0^{\tau} \frac{dt_p}{v(p, t_p)} \tag{13}$$

I wish I could invert the above system to get (t', p, τ) as a function of (t, h, z), but I can't. So we will form the Jacobian, that is

$$\begin{bmatrix} \partial_{t'} \\ \partial_p \\ \partial_\tau \end{bmatrix} = \begin{bmatrix} t_{t'} & h_{t'} & z_{t'} \\ t_p & h_p & z_p \\ t_\tau & h_\tau & z_\tau \end{bmatrix} \begin{bmatrix} \partial_t \\ \partial_h \\ \partial_z \end{bmatrix} \tag{14}$$

Performing differentiations only where they lead to obvious simplifications, we get the transformation equation for Fourier variables:

$$\begin{bmatrix} -\omega' \\ k_p \\ k_\tau \end{bmatrix} = \begin{bmatrix} 1 & h_{t'} & 0 \\ 0 & h_p & z_p \\ -1 & h_\tau & .5v(\tau)^{-1} \end{bmatrix} \begin{bmatrix} -\omega \\ k_h \\ k_z \end{bmatrix} \tag{15}$$

The real hazard is that the coordinate system may have a pole located just where we are planning to extrapolate.

At this stage, a bit of courage is required to invert equation (15). The results can be substituted into the *DSR* expressing it in terms of the new coordinates. The new *DSR* will not be simple, nor will it offer special insights until simplifications are made.

In the hopes of elucidating the frame itself without getting into any of the complexities, I set $v = 1$ in the definitions and obtained

$$\frac{k_h}{2} = \frac{k_p}{t' - \tau} \tag{16}$$

$$\omega = \omega' + p \frac{k_p}{t' - \tau} \tag{17}$$

$$k_z = 2k_\tau - 2\omega' \tag{18}$$

which when substituted into the DSR gives

$$k_\tau = \omega' - \frac{1}{2}\left[\left(\omega' + \frac{pk_p}{t'-\tau}\right)^2 - \left(\frac{k_y}{2} \pm \frac{k_p}{t'-\tau}\right)^2\right]^{1/2} - ditto\,(\mp) \tag{19}$$

The new coordinate system has not altered the fact that the DSR is still not separable in k_y and k_p. Setting k_p equal zero achieves separability while keeping full accuracy in k_y. But setting k_p equal zero implies CDP stacking. Thus we get an equation which shows how CDP stacks really should be downward continued! The only approximation made is the very reasonable one that derivatives of the coordinate frame itself can be neglected.

To have practical separability we need to neglect terms like $k_y k_p$. The way to neglect this product is to get the individual factors as small as possible. There is really nothing we can do about k_y which is given to us by the earth. What we can do is to choose a good moveout correction function which should help by reducing k_p. This says to moveout at the *stacking* velocity, not the *earth* velocity. We should work out the DSR for this case.

A novel approach is to decompose the CDP gathers by stacking velocity, say

$$D \quad = \quad D(v_1) + D(v_2) + D(v_3) + D(v_4) + \cdots \tag{20}$$

The best technique for such a decomposition has yet to be described. But let us presume that this can be done.

After the decomposition each $D(v_i)$ could be stacked over its appropriate stacking velocity. Then each could be downward continued with a migration equation according to the earth velocity. Finally the decomposed data could be recombined.

Alternately, express each individual panel $D(v_j)$ in its own moveout corrected space. Thus as far as each panel is concerned, $k_y k_p$ is small thereby justifying separability. Because of the smoothness in the lateral direction, each panel could be subsampled laterally. You could do full before stack migration of each panel. All these possibilities have yet to be studied, especially in regard to their relation to velocity estimation.

There is a focus in the coordinate system itself, and it is located just where we are headed. Looking back at the coordinate system near the focus $t' = \tau$, we see that $h = 0 \times p$. This means that the coordinate system is collapsing, the coefficient of the ∂_{pp} derivative will have a pole. As we approach the focused image, the coordinate system is unsuitable for energy which has the wrong velocity. This is not a computational disaster, it just means that the whole conceptual basis for velocity estimation in conjunction with this frame has yet to be worked out. On the other hand, perhaps we will be seeking implementations which are immune to this kind of difficulty.

Anyway, whenever you are ready for it, the separable approximation is always available

$$SEP(k_y,k_p) \quad = \quad DSR(k_y,0) \; + \; DSR(0,k_p) \; - \; DSR(0,0) \tag{21}$$

To look at prestack partial migration, convert k_p back to k_h. I always think of prestack partial migration operators as $PSPM = DSR(h) - DSR(h{=}0)$, but this point of view does not seem to incorporate the dip bandwidth as well as Rocca's approach because it doesn't fully overcome the separability problem. The reason for a detour through k_p-space is to enhance the validity of any square root approximations. (If you omit the detour, your square root expansions get overwhelmed by the main thing that the *DSR* wants to do which is to push energy to zero offset.)

Applying (21) to (19) and subsequently dropping quadratics and higher in k_p we get

$$k_\tau \;=\; \omega' \;-\; \sqrt{\omega'^2 - k_y^2/4} \;-\; \frac{p\ k_p}{t' - \tau} \tag{22}$$

Interpretation of the data processing implications will require further work.