5.4 Multiple Reflection, Prospects

To improve our ability to suppress multiples, we try to better charactorize them. The trouble is that there are so many ingredients to a realistic model. Few of the theories which abound in the literature have had much influence upon routine industrial practice. I would put the unsuccessful literature into two catagories:

(1) Those that try to achieve everything with statistics, oversimplifying the complexity of the spatial relations.

(2) Those that try to achieve everything with mathematical physics, oversimplifying the noisy and incomplete nature of the data.

Multiple reflection is a good subject for nuclear physicists, astro-physicists, and mathematicians who enter our field. Those who are willing to take up the challange of trying to carry theory through to industrial practice, are rewarded by learning some humility. I'll caution you now that I haven't gotten it all together in this chapter either!

Here two approaches will be proposed, both of which attend to geometry ***and*** statistics. Both approaches are fairly new and little tested. Regardless how well they may or may not work, they illuminate the task.

The first approach, called ***CMP slant stack***, is a simple one. It transforms data into a form where all offsets mimic the simple one-dimensional, zero-offset model. That model has a rich literature in both statistics and mathematical physics. From there the choice is yours.

The second approach is based on a ***replacement impedance*** concept. It is designed to accomodate rapid lateral variations in the near surface. It is easiest to explain for a hypothetical marine environment where the sole difficulty arises from lateral variation in the seafloor reflectivity. The basic idea is downward continuation of directional shots and directional geophones to just beneath the sea floor, but no further. Then they are upward continued through a replacement medium which has a zero sea floor reflection coefficient. This doesn't eliminate all the multiple reflections, but it should eliminate the most troublesome ones. Statistical considerations are required to find the laterally variable reflectivity. The shot waveform may be spectrally incomplete or unknown.

The replacement impedance concept can be implemented with varying degrees of accuracy. In simplest form it is just two filters, one for the shot location, and another for the geophone location. In a more general form it could incorporate offset and dip angles. Implementations are described from the doctoral dissertation of Larry Morley.

Transformation to One Dimension by Slant Stack

There is a rich literature on the one dimensional model of multiple reflections. Some authors develop many facets of wave propagation theory. Others begin from a simplified propagation model and develop many facets of information theory. These one dimensional theories are often regarded as applicable only at zero offset. However, we will see that all other offsets can be brought into the domain of one dimensional theory by application of slant stacking.

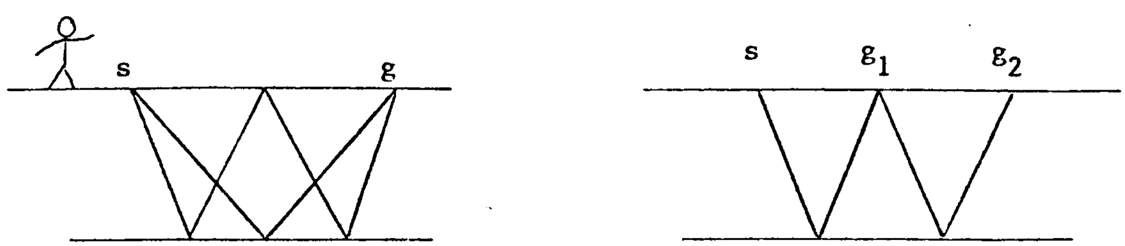

FIG. 1. Rays at constant offset (left) arrive with various angles, hence various Snell parameters. Rays with constant Snell parameter (right) arrive with various offsets. At constant p all paths have identical travel times.

The way to get the timing and amplitudes of multiples to work out like vertical incidence is to stop thinking of seismograms as time functions at constant offset, and start thinking of constant Snell parameter. In a layered earth the complete ray path is constructed by summing the path in each layer. At vertical incidence $p = 0$ it is obvious that when a ray is in layer i its travel time t_i for that layer is independent of whatever other layers may also be traversed on other legs of the total journey. This is also true for any other fixed p. But as shown in figure 1 it is not true for a ray whose total offset $\sum f_i$ is fixed instead of p being fixed.

Likewise, for fixed p, the horizontal distance f_i which a ray travels while in layer i is also independent of other legs of the journey. Thus $t_i + const\ f_i$ for any layer i is also independent of other legs of the journey. So $t_i' = t_i - pf_i$ is a property of the i-th layer and has nothing to do with what other layers may be in the total path. Given the layers that a ray crosses, you add up the t_i and the f_i for each layer, just as you would in the vertical incidence case. Some paths are shown in figure 2.

To see how to relate field data to slant stacks, begin by searching on a common-midpoint gather for all those patches of energy (tangency zones) where the hyperboloidal arrivals attain some particular numerical value of slope $p = dt/df$. These patches of energy seen on our surface observations each tell us where and when some ray of Snell's parameter p has hit the surface. Typical geometries and synthetic data are shown in figures 2 and 3.

Both the t_i and the t'_i behave like the times of normal-incident multiple reflections. Unfortunately, the lateral location of any patch depends upon the velocity model $v(z)$. But slant stacking makes the lateral location irrelevant. In principle, slant stacking could be done for many separate values of p so that the (f,t)-space gets mapped into a (p,t)-space. The nice thing about (p,t)-space is that the multiple-suppression problem decouples into many separate one-dimensional problems, one for each p-value. Not only that, but you do not need to know the material velocity to solve these problems. It is up to you to select one of the many published methods. After suppressing the multiples you inverse slant stack. Once back in (f,t)-space you could estimate velocity and further suppress multiples by your favorite stacking method.

To the my knowledge, the above method hasn't been seriously tried. Its strength is in correctly handling the angle dependences which arise from the source/receiver geometry as well as the intrinsic angle dependence of reflection coefficient. A weakness is the assumption of lateral homogeneity in the reverberating layer. Water is extremely homogeneous, but the sediments at the water bottom can be quite inhomogeneous.

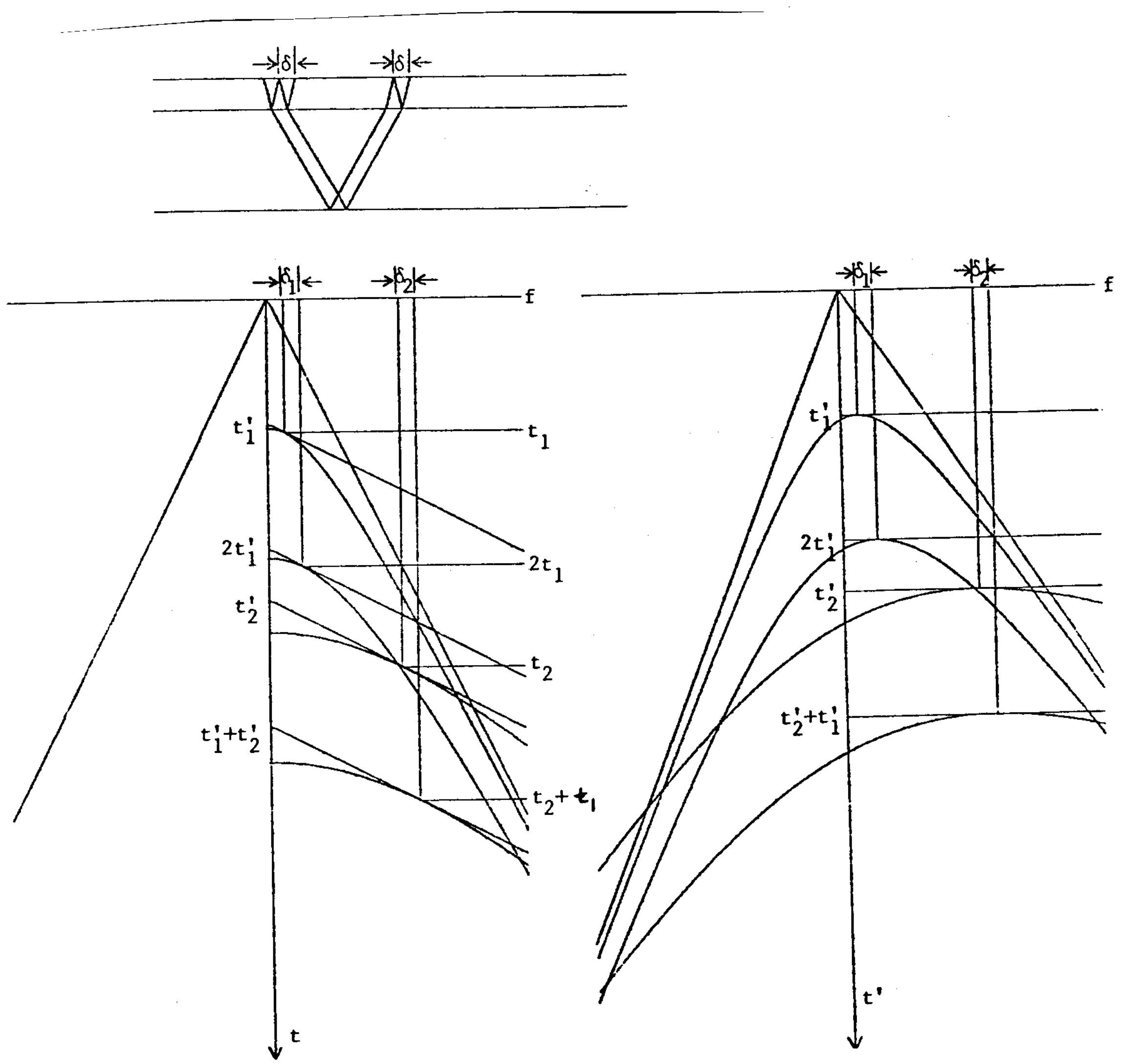

FIG. 2. (Gonzalez) A two-layer model showing the events $(t_1, 2t_1, t_2, t_2+t_1)$. Top is a ray trace. On the left is the usual data gather. On the right it is replotted with linear moveout $t' = t - pf$. Plots were calculated with $(v_1, v_2, 1/p)$ in the proportion (1,2,3). Fixing attention on the patches where data is tangent to lines of slope p, we see that arrival times are in the vertical-incidence relationships. That is, the reverberation period is fixed, and it is the same for simple multiples as it is for peglegs. This must be so because the ray trace at the top of the figure applies precisely to those patches of the data where $dt/dx = p$. Furthermore, since $\delta_1 = \delta_2$, the times $(t_1', 2t_1', t_2'+t_2')$ also follow the familiar vertical-incidence pattern.

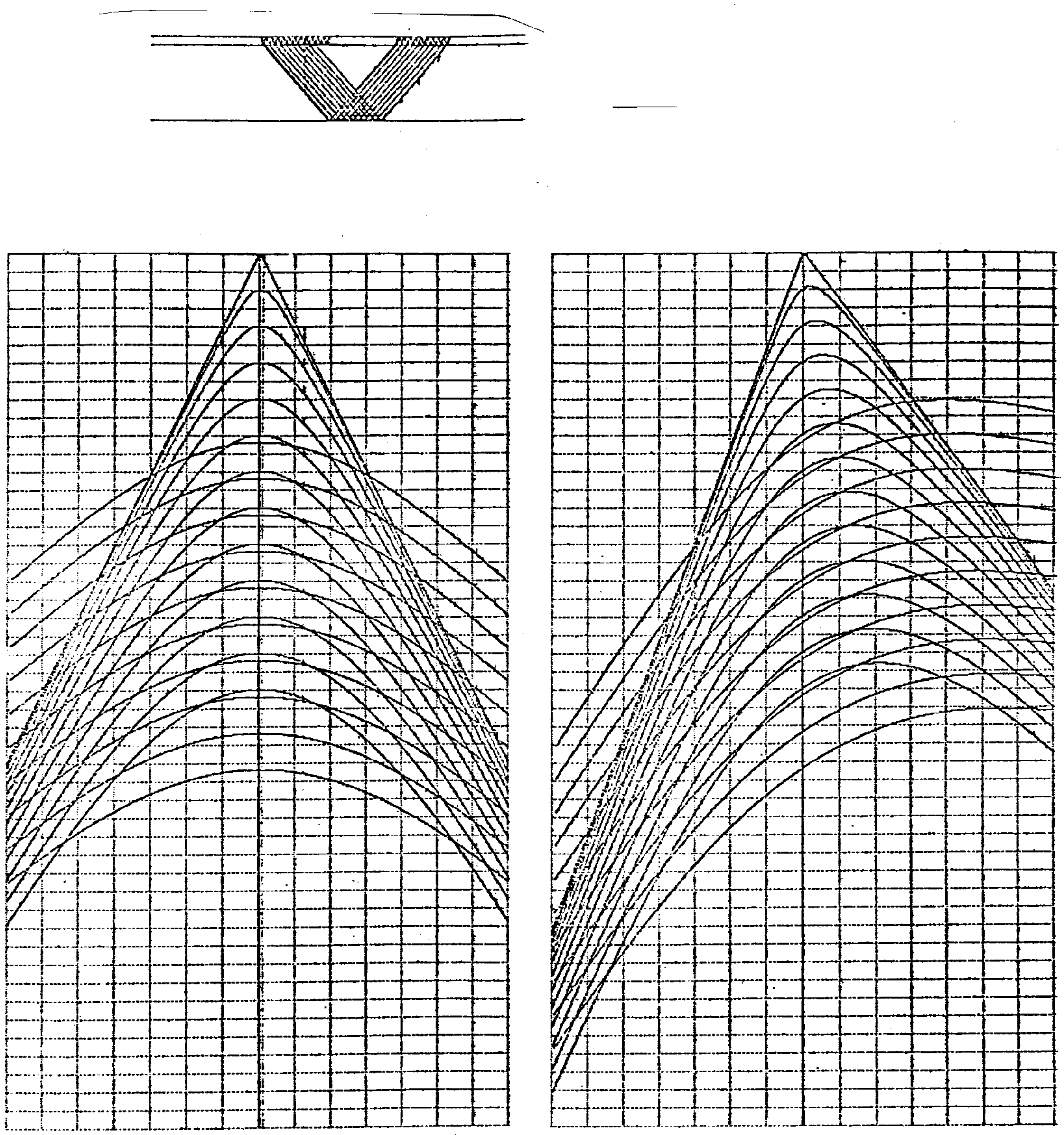

FIG. 3. (Gonzalez) This figure is the same as figure 2 but more multiple reflections are shown. This simulates much marine data. By picking the tops of all events on the right-hand frame and then connecting the picks with dashed lines, the reader will be able to verify that sea-bottom peg-legs have the same interval velocity as the simple bottom multiples. The interval velocity of the sediment may be measured from the primaries. The sediment velocity can also be measured by connecting the n-th simple multiple with the n-th peg-leg multiple.

Near Surface Inhomogeneity

Soils have strange acoustic behavior. Their seismic velocities are usually less than or equal to the speed of sound in water (1500 meters/sec). It is not uncommon for the velocity to be five times slower, namely, as slow as the speed of sound in air (300 meters/sec). Where practical, seismic sources are buried under this weathered zone. But for all but the most abnormal environments, the receivers are stuck above the weathered zone.

A source of much difficulty is the fact that soils are severely laterally inhomogeneous. There is probably no such thing as a "typical" situation, so instead I will describe the local California situation. The California central valley is a flat plain, near sea level, about 150 by 1000 kilometers in extent. A theoretical seismologist would have to be forgiven for making the erroneous assumption that the extreme flatness at the surface implies a flatness in the immediate subsurface. Anywhere on this plain it is not unusual for two geophones separated by 10 meters to see quite different seismograms. In particular, the uphole transit time (seismic traveltime from the bottom of a shot hole to the surface near the top of the hole) can easily exhibit time anomalies of a full wavelength.

How can we understand such severe, unpredictable, traveltime anomalies in the weathered zone? It is important to realize that in recent geological time the rivers in such a valley have been meandering all over the valley floor.

The Stanford University Geophysics Department field trips have always been able to get good geophysical measurements of gravity, magnetism, and electrical conductivity, but reflection seismology has never been successful. It is because a training operation with modest funds can't afford the deep shot holes, large charges and multichannel surface receiver arrays necessary to penetrate the weathered zone. Maybe this explains why so few academic seismologists specialize in reflection seismology!

The shallow marine situation is somewhat better. There are still ample opportunities for lateral variations. There are buried submarine channels as well as buried fossil river channels. But for shallow marine data the dominant aspect of the problem becomes the resonance in the water layer. The power spectrum of the observed data will be controlled by this resonance.

Likewise, with land data it is commonly observed that the power spectrum varies rapidly from one recording station to the next. These changes in spectrum may be interpreted as changes in the multiple reflections due to changes in the effective depth or character of the weathered zone.

Modeling Regimes

Downward continuation equations contain four main ingredients, the slowness of the medium at the geophone $v(g)^{-1}$, likewise at the shot $v(s)^{-1}$, the stepout in offset space k_h / ω, and dip in midpoint space k_y / ω. These all have the same physical dimensions, and modeling procedures can be catagorized by what numerical inequalities among the four are presumed to exist. One dimensional work ignores three of the four, namely dip, stepout, and the the difference $v(g)^{-1} - v(s)^{-1}$. We have just seen how CMP slant stack includes the stepout k_h / ω. Next we get our choice to include either the dip or the lateral velocity variation. The lateral velocity variation is often severe near the earth surface where the peglegs live. Recall the simple idea that typical rays in the deep subsurface, emerge steeply at a low velocity surface. If we use continuation equations only in the near surface, we are particularly justified in neglecting dip, that is $v^{-1} \gg k_y / \omega$. It is nice to find an excuse to neglect dip because our field experiments are so poorly controlled with regard to dip out of the plane of the experiment. Offset stepout, on the other hand, is probably always much larger in the plane of the survey line than out of it.

Another important ingredient for modeling or processing multiple reflections is the coupling of up and down going waves. This introduces the reflectivity beneath the shot $c(s)$ and receiver $c(g)$. An important possibility, to which we will return, is that $c(s)$ may be different from $c(g)$ even though all of the angles may be neglected.

Subtractive Removal of Multiple Reflections

As stacking may be thought of as a multiplicative process, modeling leads to subtractive processes. The subtractive processes are a supplement to stacking, not an alternative. After subtracting, you can stack.

First we try to model the multiple reflections, then we try to subtract them from the data. In general, removal by subtraction is more hazardous than removal by multiplication. To be successful, subtraction requires a correct amplitude as well as timing error less than a half wavelength.

Statistically determined empirical constants may be introduced in order to account for discrepancies between reality and the modeling. In statistics this is known as *regression*. For example, knowing that a collection of data points should fit a straight line, the method of least-sum-squared-residuals can be used to determine the best parameters for the line. A careful study of the data points might begin by removal of the straight line, much as we intend to remove multiple reflections. Naturally we will want an adjustable parameter to account for the difficulty we expect in calculating the precise amplitude for the multiples. An unknown timing error is much harder to model. Because of the non-linearity of the mathematics, a slightly different, more tractable approach is to take as adjustable parameters the coefficients in a convolution filter. Such a filter could represent any scale factor and time shift. It is tempting to use a time variable filter to account for time variable modeling errors. An inescapable difficulty is that a filter can also represent a lot more than scaling and amplitude. And the more adjustable parameters, the more the model will be able to fit the data, whether or not the model is actually related to the data.

The difficulty of subtracting multiple reflections is really just this: If you do an inadequate job of modeling the multiples, say for example that you do a poor job of modeling the geometry or velocity, then you will wish to compensate by using many adjustable parameters in the regression. With many adjustable parameters, you find that you are subtracting primary reflections as well as multiples. Out goes the baby along with the wash water.

Slanted Deconvolution and Inversion

Because of the wide offsets used in practice, it became clear that we must pay attention to differences in the sea floor from bounce to bounce. A straight-forward and appealing method of doing so was introduced by Taner (1980) in his *radial trace* method. A radial trace is a line cutting through a common shot profile along some line of constant $r = h/t$. Instead of doing deconvolution on a seismogram of constant offset, one does deconvolution on a radial trace. The deconvolution can be generalized to a downward continuation process. Downward continuation of a radial trace may be approximated by time shifting. Unfortunately, there is a problem when the data on the line consists of both sea-floor multiples and peglegs, because both require different trajectories. This problem is resolved, at least in principle, by means of Snell waves. Estevez, in his dissertation showed theoretically how Snell waves could also be used to resolve a number of other difficulties such as diffraction and lateral velocity variation (if known). An example of Estevez, illustrating the relevance of the depth different depth of the sea floor on different bounces is shown in figure 4.

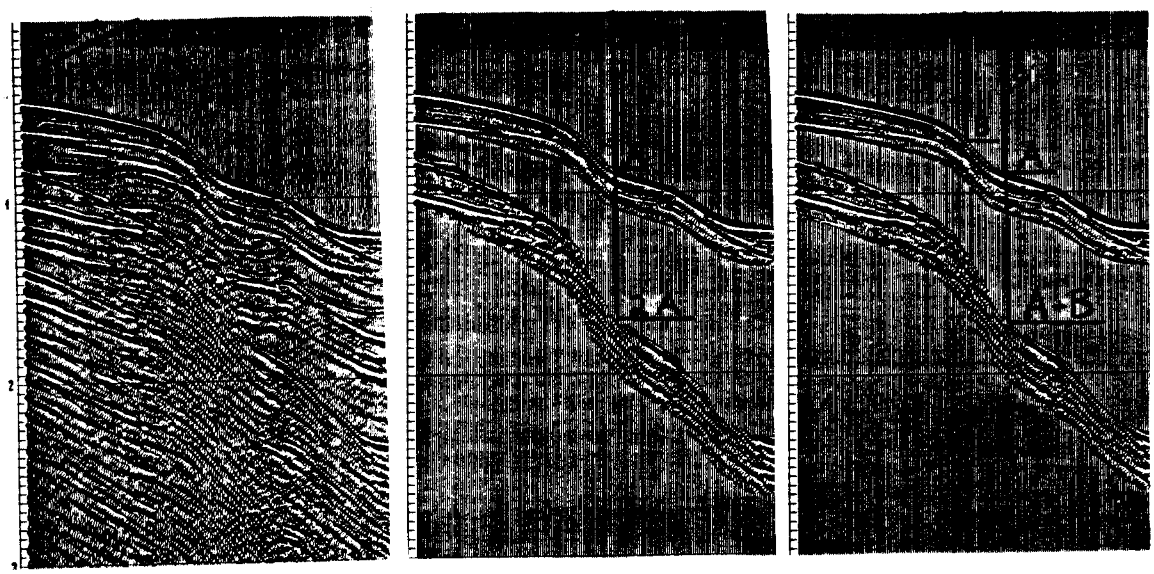

FIG. 4. SEP12p77-80 (Estevez) Depth of multiple depends on sum of all depths.

There is a basic problem with most inversion methods. It is that they are recursive. An error made at shallow depths will compound as you descend. This is especially a problem because of the incomplete nature of seismic data. First of all there is some uncertainty in the shot waveform and it is spectrally incomplete. Second, at the p-values for which pegleg multiples are a problem, we often find that the first seafloor bounce occurred too close to the ship to be properly recorded. Taner built a special auxiliary recording system.

It was an advantage for the Snell wave methods that the stacking created some signal to noise enhancement from the raw field data, but it was a disadvantage that the downward continuation had to continue to all depths. The methods to be discussed next, are before-stack methods, but they do not require downward continuation much below the sea floor.

The Split Backus Filter

We are leading up to a general strategy, *impedance replacement*, for dealing with surface multiple reflections. This strategy will require heavy artillery from both regression theory and wave extrapolation theory. So as not to lose sight to the goals, we will begin with an example drawn from a highly idealized geometry. The fact that reality is not too far from this idealization was demonstrated by Larry Morley whose doctoral dissertation illustrates a successful test of this method and describes the impedance replacement strategy in more detail.

Imagine that the sea floor is flat. Near the shot the seafloor reflection coefficient will be taken as c_s. Near the geophone it will be taken to be c_g. In the vicinity of the geophone we may expect to see a reverberation pattern denoted by

$$\frac{1}{1 + c_g Z} = 1 - c_g Z + c_g^2 Z^2 - c_g^3 Z^3 + c_g^4 Z^4 + \cdots \qquad (1)$$

Where Z is the two way delay operator for travel to the water bottom. Near the shot we expect to see a similar reverberation sequence.

$$\frac{1}{1 + c_s Z} = 1 - c_s Z + c_s^2 Z^2 - c_s^3 Z^3 + c_s^4 Z^4 + \cdots \qquad (2)$$

Ignoring the difference between c_s and c_g leads to the Backus reverberation sequence which is the product of (1) and (2).

$$\frac{1}{1 + c\,Z}\;\frac{1}{1 + c\,Z} = 1 - 2cZ + 3c^2 Z^2 - 4c^3 Z^3 + 5c^4 Z^4 + \cdots \qquad (3)$$

The denominator in (3) is the Backus filter. Applying this filter should remove the reverberation sequence. Morley called the filter resulting from explicitly including the difference at the shot and geophone a *split Backus* filter. If you are willing to ignore the effect of dip, you may allow the depth as well as the reflection coefficient to vary laterally. Thus the split Backus operator can be taken to be

$$\left[1 + c_s\, e^{i\omega\tau(s)}\right]\;\left[1 + c_g\, e^{i\omega\tau(g)}\right] \qquad (4)$$

Inverting (4) into an expression like (3) you will find that the n-th term splits into n terms. It just means that paths with sea floor bounces near the shot will have a somewhat different travel times than those with bounces near the geophone.

The next two figures from Morley's dissertation show that split pegleg multiples are an observable phenomenon. His interpretation of the figures follows:

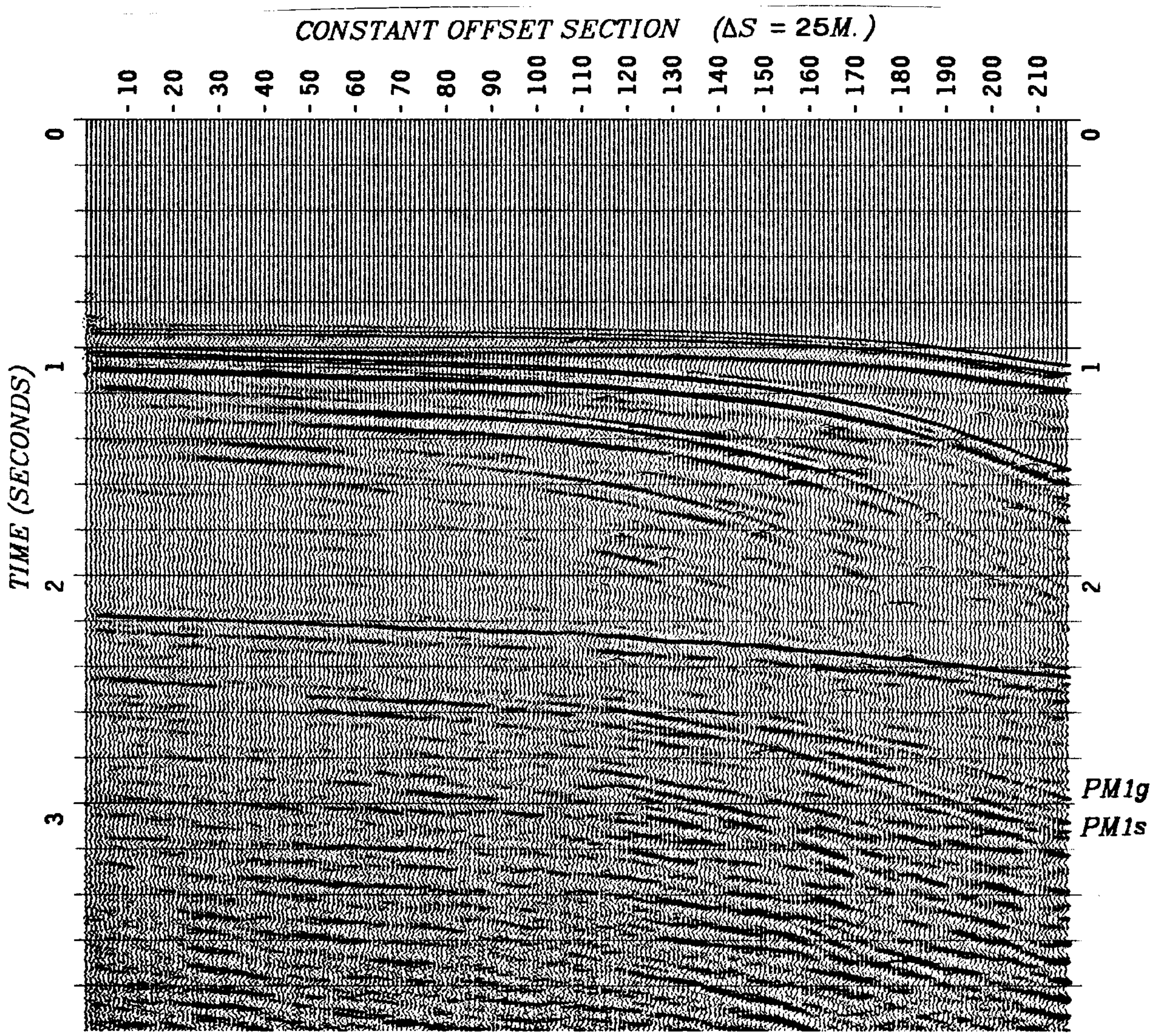

FIG. 5. Constant Offset Section (COS) from the same line as [figure 3 in section 5.3] Offset distance is about 46 shotpoints. Notice that the first order pegleg multiple is now split into two distinct arrivals, $PM1s$ and $PM1g$.

"[The figure] is a constant offset section (COS) from the same line for an offset half-way down the cable (a separation of 45 shot points with this geometry). The first order pegleg multiple starting at 2.5 seconds on the left and running across to 3 seconds on the right is "degenerate" (unsplit) on the near-trace section but is split on the COS due to the sea floor topography. The maximum split is some 200 mils around shot points 180-200. This occurs, as one might expect, where the seafloor has maximum dip; i.e., where the difference between seafloor depths at the shot and geophone positions is greatest."

Most present processing ignores the Backus filter altogether and solves for an independent deconvolution filter for each seismic trace. This introduces a great number of free parameters. So by comparison, a split-Backus approach should do a better job of preserving primaries.

In practice it is expected that any method based on the split-Backus concept would need to consider the effect of moveout. Luckily, velocity contrast would reduce the emerging angle for peglegs. Of course, residual moveout problems would be much more troublesome with water bottom multiples. Presumably the process should be applied after normal moveout. Let us take a look at the task of actually estimating a split-Backus operator.

Sea Floor Consistent Multiple Suppression

Erratic time shifts from trace to trace have long been dealt with by means of the so-called *surface consistent statics model.* By this model, the observed time shifts, say $t(s,g)$ are fit to a regression model $t(s,g) \approx t_s(s) + t_g(g)$. The statistically determined functions $t_s(s)$ and $t_g(g)$ may be interpreted as being derived from altitude or velocity variations directly under the shot and geophone. Recently, Taner and Coburn (1980) introduced the closely related idea of a surface-consistent frequency response model as part of the statics problem. We will be interpreting and generalizing that approach. Our intuitive model for the data $P(s,g,\omega)$ is basically

$$P(s,g,\omega) \approx \frac{1}{1 + c_s\, e^{i\omega\tau(s)}} \; \frac{1}{1 + c_g\, e^{i\omega\tau(g)}} \times \qquad (5)$$

$$e^{i\omega\sqrt{z^2+h^2/v^2}}\, H(h,\omega)\, Y(y,\omega)\, F(\omega)$$

The first two factors represent the split Backus filter. The next factor is the normal moveout. The factor $H(h,\omega)$ is the residual moveout. The factor $Y(y,\omega)$ is the depth dependent earth model beneath the midpoint y. The last factor $F(\omega)$ is some average filter which results from both the earth and the recording system.

A problem with the split Backus filter is the old one that the time delays $\tau(s)$ and $\tau(g)$ enter in the model in a non-linear way. So to linearize the model we generalize it to

$$P'(s,g,\omega) \approx S(s,\omega)\, G(g,\omega)\, H(h,\omega)\, Y(y,\omega)\, F(\omega) \tag{6}$$

Now S contains all water reverberation effects characteristic of the shot location, including any erratic behavior of the gun itself. Likewise, receiver effects are embedded in G. Moveout correction was done to P thus defining P'.

Theoretically, taking logarithms we get a linear, additive model.

$$\ln P'(s,g,\omega) \approx \ln S(s,\omega) + \ln G(g,\omega) + \ln H(h,\omega) + \ln Y(y,\omega) + \ln F(\omega) \tag{7}$$

The phase of P', which is the imaginary part of the logarithm, contains the ***travel time*** information in the data. This begins to lose meaning as the data consists of more than one arrival. The phase function becomes discontinuous, even through the data are well behaved. In practice, therefore we restrict our attention to the real part of (7) which is essentially a statement about power spectra. The decomposition (7) is a linear problem, perhaps best solved by iteration because of the high dimensionality. In reconstructing S and G from power spectra, Morley used the Wiener-Levinson technique, explicitly forcing zeros in the filters S and G to account for the water path. He omitted the explicit moveout correction in (5) which may account for the fact that he only considered the inner half of the cable.

Replacement Medium Concept of Multiple Suppression

In seismology wavelengths are so long that we tend to forget that it is physically possible to have a directional wave source and a directional receiver. Suppose we had, or were somehow able to simulate, a source which radiated only down and a receiver which received only waves coming up. Then suppose that we were somehow able to downward continue this source and receiver beneath the sea floor. This would eliminate a wide class of multiple reflections. Sea floor multiples and peglegs would be gone. That would be a major achievement. After succeeding, we might have one more minor problem. The data might now lie along a line which is not flat, but follows the sea floor. So there is a final step, which is easy, which is to upward continue through a replacement medium which does not have the strong disruptive sea floor reflection coefficient. The process being described would be called ***impedance replacement***. It is analogus to a replacement medium in gravity data reduction. It is also analogous to time shifting seismograms for some ***replacement velocity***.

The migration operation downward continues an upcoming wave. It is like downward continuing a geophone line in which the geophones can receive only up-coming waves. In reality, buried geophones see both up and down-going waves. The directionality of the source or receiver is built into the choice of sign of the square root equation which is used to extrapolate the wave field. By means of the reciprocal theorem we could also downward continue the shots. So we can deduce the results of four possible experiments at the sea floor, all possibilities of upward and downward directed shots and receivers.

To extrapolate all this information across the sea floor boundary, will require an estimate of the seafloor reflection coefficient. It enters the calculation as a scaling factor in forming linear combinations of the waves above the seafloor. The idea behind the reflection coefficient estimation can be expressed in two ways which are mathematically equivalent.

1. The waves impinging on the boundary from above and below should have a crosscorrelation which vanishes at zero lag.
2. There should be minimum power in the wave which impinges on the boundary from below.

After you have the geophones below, you must start to think about getting the shots below. To invoke reciprocity, it is necessary to invert the directionality of the shots and receivers. This is why we needed to include the auxiliary experiment of upward directed shots and receivers.

EXERCISE

1. Refer to Figure 3.
   a) What graphical measurement shows that the interval velocity for simple sea floor multiples equals the interval velocity for peglegs?

b) What graphical measurements determine the sediment velocity?

c) With respect to the velocity of water, deduce the numerical value of the (inverse) Snell parameter p.

d) Deduce the numerical ratio of the sediment velocity to the water velocity.

2. Consider the up coming wave U observed over a layered medium of layer impedances given by $(I_1, I_2, I_3, \cdots)$ and the up coming wave U' at the surface of the medium $(I_2, I_2, I_3, \cdots)$. Note that the top layer is changed.

   a) Draw ray paths for some multiple reflections which are present in the first medium, but not the second.

   b) Presuming that you can find a mathematical process to convert the wave U to the wave U', what multiples are removed from U' which would not be removed by the Backus operator?

   c) Utilizing techniques in FGDP, chapter 8, derive an equation for U' in terms of U, I_1, and I_2 which does not involve $I_3, I_4 \cdots$.