To Understand Diffractions

Boris Zavalishin

Diffracted wave theory in seismic prospecting gradually draws more attention as it becomes clear, that diffracted waves contain useful information about both geometrical construction and physical properties of seismic boundaries. Yet, from the point of view of a conventional data processing scheme, where stacking of CMP gathers is an essential element, diffractions are considered as something confusing. We start to think about diffractions only when the stacking procedure fails to suppress them completely. The stacking process suppresses diffractions in many cases rather effectively because of R.M.S. velocities differences. In such cases we lose useful information.

There are several reasons for diffractions to be neglected during data processing and interpretation. One of them is the complicated physical nature of diffracted waves and as a consequence of this the complicated mathematical description of the phenomena or time consuming programs for its computer simulation. That is why, though there are plenty of books and papers on the subject, one of them (Trorey, 1970) is to be considered very important because it makes you think, that diffraction is not as complicated as is commonly believed. Only simple ideas and simple theories can actually work in spheres, where big group of people are involved, and seismic interpretation is such a sphere. So simplification of diffraction theory is an important aim of applied seismology.

On the present level of its development, applied seismology studies diffractions as some additional term to the ray- tracing theory of wave propagation. This term is extracted from a solution of the scalar wave equation. But as the scalar wave equation presents rather rough approximation to the real case of elasticity it is unreasonable to use its most sophisticated solutions, when they are complicated or expensive. Reasonably approximated solutions may help a lot.

Modeling of seismic events is a subject, where consideration of diffractions is desirable in the fist place. Now modeling is a part of interpretation and it's becoming more and more popular and useful. But in most cases modeling is based on a ray-tracing technique and can not implement diffractions even when they are badly needed, in the case of a fault for example, to say nothing about cases when physical properties are changed or layers are bent. It is not because of absence of proper modeling programs but because of their high costs.

Modeling of diffractions is expensive because numerical evaluation of surface integrals is involved. Trorey's approach enables one to replace expensive integration by much cheaper convolution, in cases when under-integral expression can be presented analytically in an explicit form. It opens a way for inexpensive modeling of diffractions in some special cases (Trorey, 1977; Berryhill, 1977; Hilterman, 1975, 1982). But the number of cases, where this approach is valid, is limited: straight diffracting edge perpendicular or parallel to a shot-receiver line. Twelve years since the original publication (1970) have not seen any other solution. Obtaining an explicit expression for other cases is a problem.

So, all known analytical solutions of the diffraction problem are limited to a few special cases and this restrains study and use of diffracted waves in seismic interpretation. In this situation it may be useful to recollect that the Fresnel approach has been enabling solution of some diffraction problems on the physical level. Furthermore, these solutions proved to be rather good approximations to sophisticated mathematical solutions found later. In this paper we shall try to explain some results, which can be achieved on mathematical level, from the physical point of view. It may lead to further simplifications in understanding of diffractions and expansion of their use for modeling and interpretation.

Trorey's (1970) approach permits no simple physical explanation of the phenomena because his solution contains a singularity at the front of the diffracted pulse. To overcome this problem we suggest using in the solution the step-function

$$s(t) = \int \delta(t)\, dt \tag{1}$$

instead of delta-function $\delta(t)$.

To recall the whole the problem, figure 1 was adapted from Trorey's paper.
An absolutely reflecting half-plane σ is to the right from AC, the source and the receiver are at P. Kirchhoff's solution of this diffraction problem is represented by the integral

$$\varphi_{P(t)} = \frac{z}{\pi} \int\limits_{\sqrt{z^2+x^2/\cos^2\theta}}^{\infty} \int\limits_0^{\pi/2} \left[\frac{1}{\zeta^3} f\left(t - \frac{2\zeta}{v}\right) + \frac{1}{v\zeta^2} \frac{\partial f\left(t - \frac{2\zeta}{v}\right)}{\partial t} \right] d\zeta\, d\theta \tag{2}$$

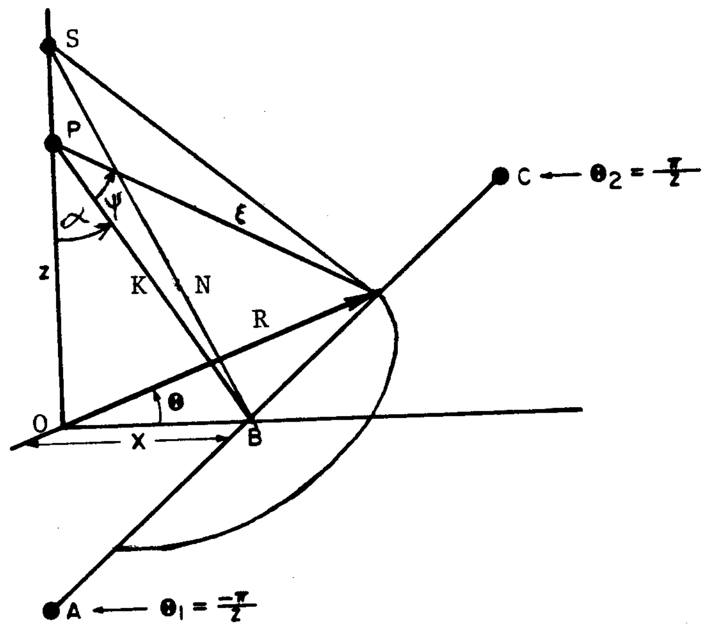

FIG. 1. Geometry of the diffraction problem.

where v is velocity, $f(t)$ is a wave-form in the source. To integrate over ζ we substitute

$$\frac{\partial f\left(t - \frac{2\zeta}{v}\right)}{\partial t} = -\frac{v}{2}\frac{\partial f\left(t - \frac{2\zeta}{v}\right)}{\partial \zeta}$$

and

$$\frac{1}{\zeta^3} f\left(t - \frac{2\zeta}{v}\right) - \frac{1}{2\zeta^2}\frac{\partial f\left(t - \frac{2\zeta}{v}\right)}{\partial \zeta} = \frac{d}{d\zeta}\left[-\frac{1}{2\zeta^2} f\left(t - \frac{2\zeta}{v}\right)\right] .$$

Then

$$\varphi_p(t) = \frac{z}{\pi}\int_0^{\pi/2}\left[-\frac{1}{2\zeta^2} f\left(t - \frac{2\zeta}{v}\right)\Bigg|_{\sqrt{z^2+x^2/\cos^2\theta}}^{\infty}\right] d\theta \qquad (3)$$

Choosing $f(t) = s(t) = 1$ and substituting limits inside brackets in Equation (3), we get

$$\varphi_p(t) = \frac{z}{2\pi}\int_0^{\pi/2}\frac{d\theta}{z^2 + \frac{x^2}{\cos^2\theta}} \qquad (4)$$

$$= \frac{1}{2\pi z}\left[\theta - \frac{x}{\sqrt{x^2 + z^2}} arctg\left[\frac{x}{\sqrt{x^2 + z^2}} tg\ \theta\right]\right]$$

Expression (4) is the diffraction response to the unit step-function. As it must be expected, derivative of (4) over t gives Trorey's (1970) result. Using the same definitions: τ_x is the two-way time from the origin to point B (Figure 1), τ is the two-way time from P to B, $\theta = arctg(\sqrt{t^2 - \tau^2}/\tau_x)$, we get

$$\frac{d\varphi_p(t)}{dt} = \frac{2z\tau_x}{\pi t v^2(t^2 + \tau_x^2 - \tau^2)\sqrt{t^2 - \tau^2}} \tag{5}$$

(Reflection coefficient R in our consideration is equal 1.)

For seismic applications both diffraction responses (4) and (5) are to be convolved with certain pulses. According to the common rule

$$\frac{df(t)}{dt} * g(t) = f(t) * \frac{dg(t)}{dt}$$

we get the same result convolving Equation (5) with an arbitrary function $f(t)$ or equation (4) with its derivative $df(t)/dt$. Theoretically both solutions are identical, but computationally equation (4) is preferable. Being free from singularities it provides very cheap calculations in time domain without special precautions (Hilterman, 1982).

Another advantage of the solution (Equation (4)) is that it enables a sort of physical interpretation. When $t < \tau$ the wavefront has not yet reached the diffractor's edge and the diffraction's response is equal to zero. At the moment $t = \tau$ the wavefront is tangent to the diffractor's edge and the solution is still equal to zero because the radiating area is zero. From this moment on a radiating area within the reflector increases proportionally to t, all points of it radiate step-function-like waves, which are summed up at P. So diffraction response must increase with time. This is exactly what the solution (4) shows.

Now we try to solve the same problem for nonzero separation of source and receiver. We'll show how to do it without any analytical derivations (Fig.2). Let the source be at S and the receiver at A . The horizontal reflecting plane P terminates at D . $S'DA$ is the straight line between the image source and the receiver A . We know, that the effective area of reflection for an infinite plane reflector is an ellipse (Zavalishin,1975).
In the case of the receiver A exactly a half of this ellipse is within the reflecting part of P to the right from D . On this ground we understand, that the wave field at A is exactly a half of the field, which infinite plane P would reflect into this point. It is also known (Trorey,1970), that we get the same result, if we place the screen P_A,which terminates at

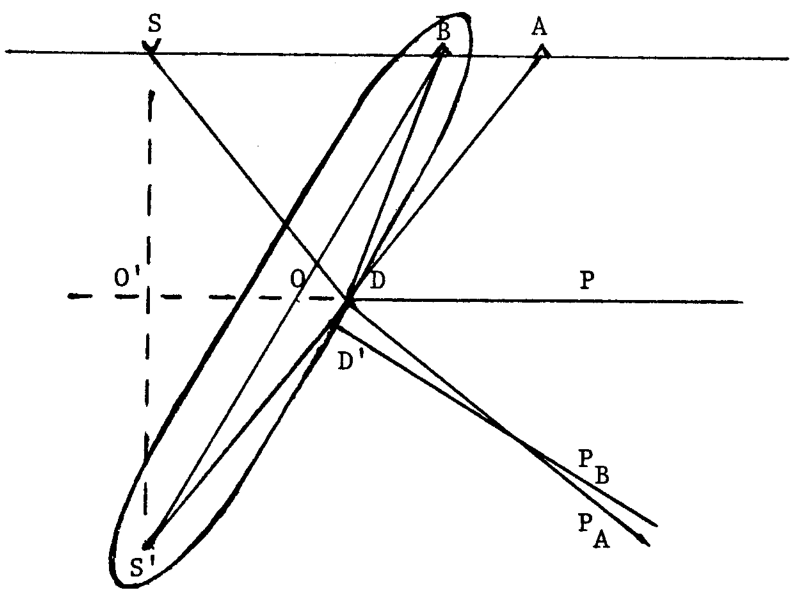

FIG. 2. Geometry of diffraction problem for nonzero source-receiver separation.

D, between S' and A perpendicularly to $S'A$. It can be proved by the solution (4). By this example we state, that there is no necessity to solve the new diffraction problem, which is much more complicated (Trorey,1977; Berryhill,1977), because this solution must be identical to that of of the simpler and already known case. Mathematically this proposition may be described as follows. If we try to solve the problem in P-plane we shall come to elliptical integrals, which are not expressed in elementary functions. We transform coordinates so that P becomes P_A. For this geometry we already have the solution (4). It is logical to suppose that the same similarity must exist and for other receivers.

To get the solution for the receiver B the plane P_B will serve the purpose. P_B is perpendicular to $S'B$ at the point O. To terminate P_B at D' we draw the ellipse passing through D with focuses at S' and B. Then we are sure that $S'D+DB=S'D'+D'B$ and $x=OD'=\sqrt{(S'D+DB)^2-S'B^2}/2$, so we can evaluate (4) with $z=S'B/2$, $x=OD'$. In example shown on Fig.3 : $z=522m$, $x=48m$, $SS'=1000m$, $O'D=200m$, $SB=300m$. To make sure that the result is correct we calculated the field at B also by Kirchhoff algorithm (2) with the grid on P.

The diffraction response, calculated by (4) was convolved with the time derivative of the wave-pulse, used in Kirchhoff's algorithm calculations. Dimension of the grid was 2.5 x 2.5

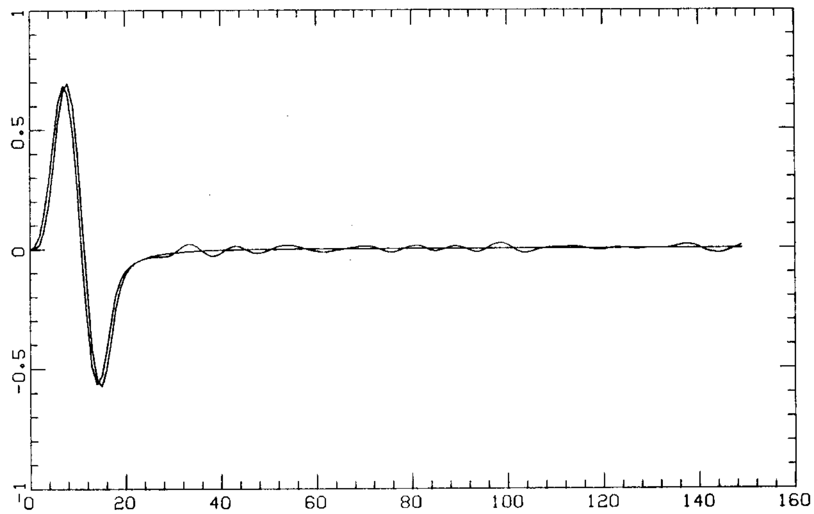

FIG. 3. Diffractions on P and P_B - planes are the same.

m, so the noisy tail of the pulse is the result of spatial sampling of the data. To show pulses calculated by (4) and Kirchhoff algorithm on the same graph (Fig.3) we shifted one of them by one millisecond, otherwise they are undistinguishable. That allows us to say, that a simple solution of the diffraction problem for a horizontal half- plane is found for a nonzero source-receiver separation.

To be particular, it is advisable to mention that instead of straight-forward Kirchhoff algorithm we first tried to use Trorey's algorithm in the time domain, using centroid sampling (Hilterman,1982) and other techniques to override the problem of singularity. Nothing worked properly. Accuracy in this case depends on the sampling technique.

The property of similarity of diffraction we have shown here may help in solving more complicated diffraction problems in the time domain (it means - very cheaply) by using simpler solutions. We illustrate this approach by another example - dipping absolutely reflecting terminated plane. To solve this problem by using the similarity property of diffraction we need to know the solution for the case, when the source and receiver do not coincide but are on the same axes perpendicular to the reflecting plane. It's the case, for example, when the source is in S (Fig.1) and the receiver is in P. It is easy to show, that the solution in elementary functions, similar to (4), is not attainable as it is a case of an elliptical integral. So we try to get an appropriate approximate solution by discussing the solution (4). The following part of the paper is devoted to this purpose.

According to Fresnel, a diffraction response depends not only upon the radiating area but also upon angles between source-diffractor and receiver-diffractor. For the zero separation between the source and the geophone these angles coincide and can be measured by the relation between z and K. This dependence is clearly seen in the solution (4) but it is not simple. Both θ and the second term of the equation (4) depends on the angle between z and K. Trying to simplify it, we rewrite Equation (4) in the form

$$\varphi_p(t) = \frac{1}{2\pi z}\left[\theta - \frac{x}{k}\psi\right] = \frac{\theta}{2\pi z}\left[1 - \frac{x}{k}\frac{\psi}{\theta}\right] \tag{4a}$$

where ψ is the angle between K and ζ

Conditionally we divide this formula into three parts. (1)Multiplier $1/2z$ represents divergence and needs no discussions. (2)Multiplier θ/π is the relation between circumferences of the arc within the diffracting half-plane and the full circle of the R radius (Fig.1). (3)Multiplier inside the brackets is the function of the source - receiver to the diffractor angles. Some previous experience shows that solutions of different diffraction problems consist of these three parts. The first two of them are always the same and create no problems in their physical explanation and estimation. The third one is the angle's function. To demonstrate our understanding of the problem let us consider an example. We change the geometry of our diffraction model in such a way that $\psi=\theta x/K$. It's the case, when AC (Fig.1) is no longer the straight line but the circle arc of radius x. We get from (4a)

$$\varphi'_p(t) \;=\; \frac{\theta}{2\,\pi\,z}\,\frac{z^2}{k^2}. \tag{6}$$

If $\theta=\pi$ this formula represents the known solution (Zavalishin,1981; Hilterman,1982) of the diffraction problem for a circle. θ in this case does not depend upon t and (6) is the amplitude of the step-function. The angle's function is very simple here, it is just $\cos^2\alpha = z^2/K^2$.

It is interesting, that the solution (4),(4a) for the straight diffracting edge (Fig.1) can be approximately represented in as simple form as the solution (6) for a circle. We replace angles inside the brackets in formula (4a) by their tangents and get the formula

$$\varphi_2 \approx \frac{\theta}{2\,\pi\,z}\,\frac{z^2}{K^2}, \tag{7}$$

which looks exactly like (6). The difference is that here $\theta=arctg(\sqrt{t-\tau}/\tau_x)$ depends upon t. As angle's function z^2/K^2 in (7) does not depend upon t we expect, that approximate solution (7) is bigger than the accurate solution (4a), and we suggest to check two more approximations

$$\varphi_3 \approx \frac{\theta}{2\pi z} \frac{z^2}{\zeta^2}, \tag{8}$$

which is smaller than (4a) and

$$\varphi_4 \approx \frac{\theta}{2\pi z} \frac{z^2}{K\zeta}, \tag{9}$$

which is the geometric mean of (7) and (8).

Another approximate solution we get by replacing the angles inside the brackets of (4a) with their sines

$$\varphi_5 \approx \frac{\theta}{2\pi z}\left(1 - \frac{x^2}{K\zeta}\right). \tag{10}$$

We compared these four approximations with the accurate solution (4a) for variety of z and x (Fig.1) and found that they are more sensitive to z than to x. In three examples shown on Fig.4 $x = constant = 100m$ and $z = 1000m$ for a, $z = 500m$ for b and $z = 50m$ for c . The left parts of Fig.4 represent diffraction responses, calculated by formulas: 1 - (4a), 2 - (7), 3 - (8), 4 - (9), 5 - (10). In the right parts of Fig.4 these responses are convolved with the same seismic pulse. This example shows, that difference between the exact and approximate solutions is distinguishable only very near to the diffracting edge and it is small to be taken into consideration in most practical applications.

For further application we chose the simplest approximation, represented by the formula (7). The angle's function here is $\cos^2\alpha$ (Fig.1) and it allows us to think, that in the case of separated source S and receiver P this function should look like a product of two cosines. If $OP = z_1$ and $OS = z_2$, we can easily write the formula

$$\varphi_6 \approx \frac{\theta}{\pi\left(z_1 + z_2\right)} \frac{z_1 z_2}{K N}, \tag{11}$$

which is the approximate solution for separated S and P. Here $\theta = arccos\ x/R$ and R is the root of the equation

$$t = \frac{\sqrt{z_1^2 + R^2} + \sqrt{z_2^2 + R^2}}{v}$$

and equal

$$R = \sqrt{\frac{\left(z_1^2 - z_2^2\right)^2}{4 v^2 t^2} - \frac{\left(z_1^2 + z_2^2\right)}{2} + \frac{v^2 t^2}{4}}$$

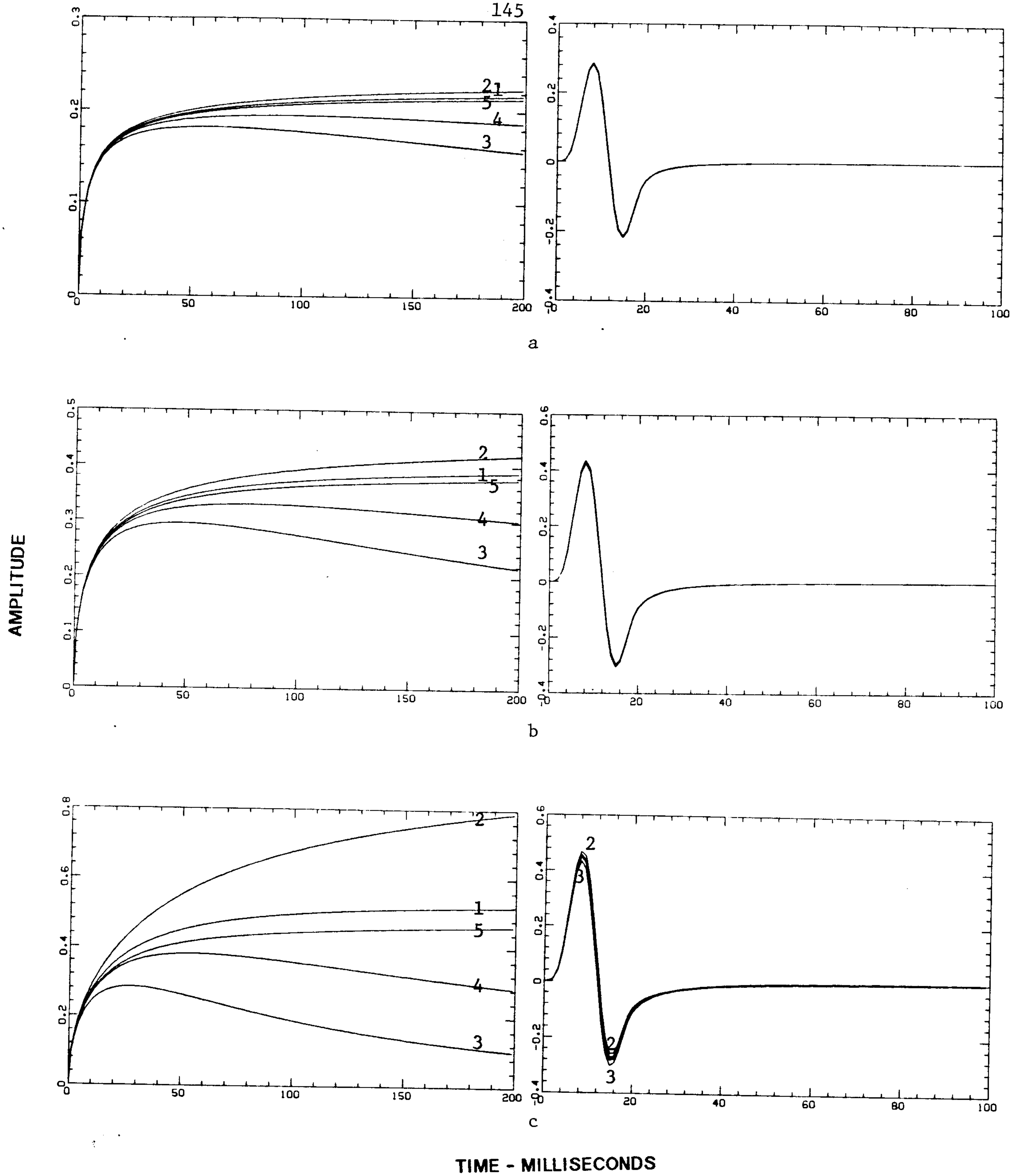

FIG.4. Diffraction responses calculated by formulas:
(4a) - 1, (7) - 2, (8) - 3, (9) - 4, (10) - 5.

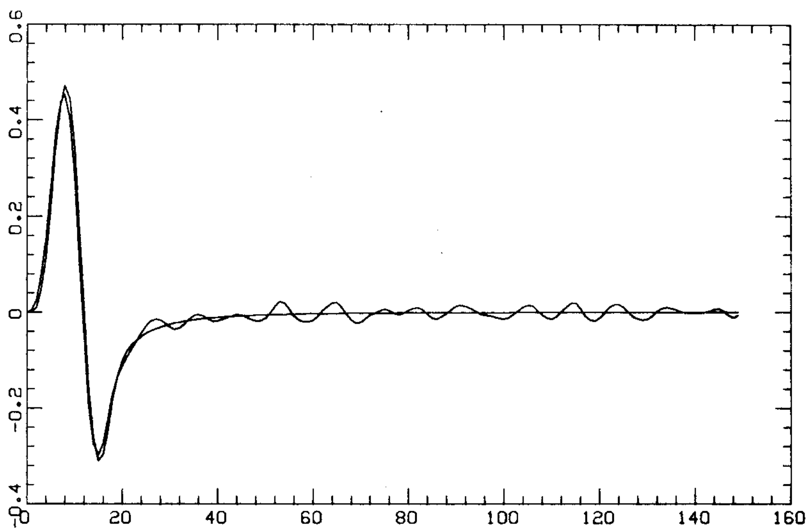

FIG. 5. Accurate (2) and approximate (11) solutions are practically identical.

Fig.5 represents comparison of the solution (11) with Kirchhoff grid solution for: $x=100m$, $z_1=300m$, $z_2=500m$. As they coincide very tightly, we think that one more simple solution is found. Using (11) and the property of diffraction's similarity, which was described in the previous part, we can easily calculate the diffraction from an edge of a dipping terminated reflector, when the edge is perpendicular to the line between the source and the receiver.

Simplicity of physical grounds, which support the approximate solutions shown here, and high accuracy of approximations allow one to believe, that similar approach may help in solving other diffraction problems cheaply and accurately enough for today's interpretation purposes.

References

1. Berryhill, J.F., 1977, "Diffraction response for nonzero separation of source and receiver.": *Geophysics 42, p.1158 -1176.*

2. Born, M. and Wolf, E., 1970, "Principles of optics.": *Oxford, Pergamon Press, 806 p.*

3. Hilterman, F.T., 1975, "Amplitudes of seismic waves - a quick look.": *Geophysics 40, p.745 - 762.*

4. Hilterman, F.T., 1982, "Interpretative lessons from 3-D modeling.": *Geophysics 47, p.784 - 808.*

5. Trorey, A.W., 1970, "A simple theory for seismic diffractions.": *Geophysics 35, p.762 - 784.*

6. Trorey, A.W., 1977, "Diffractions for arbitrary source-receiver locations.": *Geophysics 42, p.1177 - 1182.*

7. Zavalishin, B.R., 1975, "Effective dimensions of reflecting area.": *Applied Geophysics 77, Nedra, Moscow.p.67 - 74. (translation in SEP-28)*

8. Zavalishin, B.R., 1981, "Analysis of ideas on reflecting area dimensions.": *Applied Geophysics 100, Nedra, Moscow. p.36 - 44.*