

### 1.1. Conclusions.

In this chapter we have defined a linear velocity spectrum as the image of a CMP gather in Snell midpoint coordinates for a non-vertical Snell reference wave. This definition fulfills most requirements of a velocity spectrum. 1) It is obtained through linear transformations in the data. 2) Energy distribution is a local function of velocity. 3) Velocity estimation can be done with any slanted reference Snell wave. 4) Interval and RMS velocities are measured directly in the data. 5) Cable truncations and refractions do not severely impair resolution. Also, unlike semblance techniques, it is easy to identify the events used to get the velocity function. The spectrum is flexible, the choice of the reference Snell wave could be done to suit the particular data set of interest. When several reference Snell waves are used, the LMO method can be used to quantify the effect of velocity variations between reflectors in the interval velocity estimates. For computation, Stolt's method is fast and can be combined with a hyperbolic deformation to sharpen the image. The fifteen degree wave equation in offset space is particularly useful since  $v(h, \tau)$  can be used to resolve multi-valued velocity functions. We do not need *a priori* knowledge of velocity. The method is sensitive to aliasing, but the LMO correction partially solves the problem. All these properties make the spectrum attractive, not only for velocity estimation but in applications demanding a data space satisfying linearity and locality requirements.

### Appendix A: Wave Equation In Snell Midpoint Coordinates

The most important wave-field extrapolation equation in reflection seismology is the *double square root equation*. In this appendix we transform it into Snell midpoint coordinates.

The double square root equation is defined by (Clærbout, 1982)

$$Z = \left[ 1 - S^2 \right]^{1/2} + \left[ 1 - G^2 \right]^{1/2} \quad (\text{A.1})$$

where

$$\begin{aligned} S &= \frac{k_s v}{\omega} = v \frac{dt}{ds} = \sin \vartheta_s \\ G &= \frac{k_g v}{\omega} = v \frac{dt}{dg} = \sin \vartheta_g \\ Z &= \frac{k_z v}{\omega} \end{aligned} \quad (\text{A.2})$$

$\vartheta_s$  is the takeoff angle and  $\vartheta_g$  is the emergent angle to the vertical.

A representation of the wave-field at any depth is given by

$$\begin{aligned} F(s, g, z, t) = & \quad (\text{A.3}) \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_s, k_g, z=0, \omega) \exp \left\{ i \left[ \int_0^z \frac{\omega}{v(\xi)} Z d\xi + k_s s + k_g g - \omega t \right] \right\} dk_s dk_g d\omega \end{aligned}$$

where we assumed depth dependence in  $Z$  through the velocity.

Retarded Snell midpoint coordinates are defined as:

$$\begin{aligned}
 t' &= t - p_0(g - s) + 2 \int_0^z \frac{\cos \vartheta}{v(\xi)} d\xi \\
 y &= \frac{g + s}{2} \\
 h &= \frac{g - s}{2} + \int_0^z \tan \vartheta d\xi \\
 \tau &= 2 \int_0^z \frac{\cos \vartheta}{v(\xi)} d\xi
 \end{aligned}
 \tag{A.4}$$

Using Snell's law,  $p_0 v = \sin \vartheta$ , we can rewrite these equations as function of  $p_0$  only

$$\begin{aligned}
 t' &= t - p_0(g - s) + 2 \int_0^z \frac{[1 - p_0^2 v(\xi)^2]^{1/2}}{v(\xi)} d\xi \\
 y &= \frac{g + s}{2} \\
 h &= \frac{g - s}{2} + \int_0^z \frac{p_0 v(\xi)}{[1 - p_0^2 v(\xi)^2]^{1/2}} d\xi \\
 \tau &= 2 \int_0^z \frac{[1 - p_0^2 v(\xi)^2]^{1/2}}{v(\xi)} d\xi
 \end{aligned}
 \tag{A.5}$$

Using the chain rule for partial differentiation

$$\begin{vmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial g} \\ \frac{\partial}{\partial s} \\ \frac{\partial}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{\partial t'}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial h}{\partial t} & \frac{\partial \tau}{\partial t} \\ \frac{\partial t'}{\partial g} & \frac{\partial y}{\partial g} & \frac{\partial h}{\partial g} & \frac{\partial \tau}{\partial g} \\ \frac{\partial t'}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial h}{\partial s} & \frac{\partial \tau}{\partial s} \\ \frac{\partial t'}{\partial z} & \frac{\partial y}{\partial z} & \frac{\partial h}{\partial z} & \frac{\partial \tau}{\partial z} \end{vmatrix} \begin{vmatrix} \frac{\partial}{\partial t'} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial h} \\ \frac{\partial}{\partial \tau} \end{vmatrix}$$

we get

$$\begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial g} \\ \frac{\partial}{\partial s} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -p_0 & 1/2 & 1/2 & 0 \\ p_0 & 1/2 & -1/2 & 0 \\ \frac{2[1-p_0^2 v^2]^{1/2}}{v} & 0 & \frac{p_0 v}{[1-p_0^2 v^2]^{1/2}} & \frac{2[1-p_0^2 v^2]^{1/2}}{v} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial t'} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial h} \\ \frac{\partial}{\partial \tau} \end{pmatrix}$$

assuming constant velocity, we can Fourier transform all the variables to get

$$\begin{pmatrix} -\omega \\ k_g \\ k_s \\ k_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -p_0 & 1/2 & 1/2 & 0 \\ p_0 & 1/2 & -1/2 & 0 \\ \frac{2[1-p_0^2 v^2]^{1/2}}{v} & 0 & \frac{p_0 v}{[1-p_0^2 v^2]^{1/2}} & \frac{2[1-p_0^2 v^2]^{1/2}}{v} \end{pmatrix} \begin{pmatrix} -\omega' \\ k_y \\ k_h \\ k_\tau \end{pmatrix}$$

(A.6)

This matrix defines the transformation between field coordinates and Snell midpoint coordinates in the frequency-wavenumber ( $\omega-k$ ) domain.

With Eq. (A.6) we can transform the double square root equation (A.1) into Snell midpoint coordinates. It is convenient to define normalized variables for the offset  $h$ , midpoint  $y$  and time-depth  $\tau$  variables in the frequency-wavenumber ( $\omega-k$ ) domain as follows

$$H = \frac{k_h v}{2\omega'} \quad (\text{A.7a})$$

$$Y = \frac{k_y v}{2\omega'} \quad (\text{A.7b})$$

$$T = \frac{k_\tau}{\omega'} \quad (\text{A.7c})$$

When there is no dip  $H$  equals the sine of the stepout angle. Near zero offset  $Y$  equals the sine of the geologic dip.

Since  $\omega = \omega'$ , from now on we will drop the prime from  $\omega$ . Rewrite the transformation (A.6) as

$$\begin{array}{c} S \\ G \\ Z \end{array} = \begin{array}{cccc} -p_0 v & 1 & 1 & 0 \\ p_0 v & 1 & -1 & 0 \\ 2[1 - p_0^2 v^2]^{1/2} & 0 & 2 \frac{p_0 v}{[1 - p_0^2 v^2]^{1/2}} & 2[1 - p_0^2 v^2]^{1/2} \end{array} \begin{array}{c} -1 \\ Y \\ H \\ T \end{array} \quad (\text{A.8})$$

making the appropriate substitutions in Eq. (A.1) we get the result

$$T = 1 - \frac{p_0 v}{1 - p_0^2 v^2} H - \frac{1}{2} \left\{ \left[ 1 - \frac{2p_0 v(H - Y) + (H - Y)^2}{1 - p_0^2 v^2} \right]^{1/2} + \left[ 1 - \frac{2p_0 v(H + Y) + (H + Y)^2}{1 - p_0^2 v^2} \right]^{1/2} \right\} \quad (\text{A.9})$$

This equation describes wave propagation with a wavefront that leaves the surface of the earth with angle  $\vartheta_0 = \sin^{-1}(p_0 v_z = 0)$ . When the value of the ray parameter  $p_0$  is zero we obtain the double square root in *midpoint offset* coordinates:

$$T = 1 - \frac{1}{2} \left\{ \left[ 1 - (H - Y)^2 \right]^{1/2} + \left[ 1 - (H + Y)^2 \right]^{1/2} \right\} \quad (\text{A.10})$$

### Appendix B: Phase shift method

The phase shift method (Gazdag, 1981. Dubrulle and Gazdag, 1979) implements directly Eq. (3.1). This method has the advantages of being exact, up to Nyquist frequency, and to accept stratified velocity  $v(z)$ . However it is slow in its computer implementation. We derive here the discrete version of it.

Let  $q$  be the discrete version of  $f$

$$q(l, m, n) = f(l\Delta h, m\Delta\tau, n\Delta t') \quad (\text{B.1})$$

where  $\Delta h$ ,  $\Delta\tau$ , and  $\Delta t'$  are the sampling intervals,  $l = 0, 1, 2, \dots, L-1$ ,  $m = 0, 1, 2, \dots, M-1$ ,  $n = 0, 1, 2, \dots, N-1$ .

The discrete Fourier Transform is defined by

$$Q(\alpha, \beta, \gamma) = \sum_{i=0}^{L-1} \sum_{j=0}^{M-1} \sum_{k=0}^{N-1} q(i, j, k) W_L^{-i\alpha} W_M^{-j\beta} W_N^{k\gamma} \quad (\text{B.2})$$

$$q(l, m, n) = \frac{1}{LMN} \sum_{i=0}^{L-1} \sum_{j=0}^{M-1} \sum_{k=0}^{N-1} Q(i, j, k) W_L^{il} W_M^{jm} W_N^{-kn} \quad (\text{B.3})$$

where  $\alpha = 0, 1, 2, \dots, L-1$ ,  $\beta = 0, 1, 2, \dots, M-1$ ,  $\gamma = 0, 1, 2, \dots, N-1$ , and

$$\begin{aligned} W_L &= e^{2\pi\sqrt{-1}/L} \\ W_M &= e^{2\pi\sqrt{-1}/M} \\ W_N &= e^{2\pi\sqrt{-1}/N} \end{aligned} \quad (\text{B.4})$$

in these equations  $\sqrt{-1}$  implies the positive branch.

In the frequency domain the discrete data is given by

$$Q(\alpha, \beta, \gamma) = F(\alpha\Delta k_h, \beta\Delta k_\tau, \gamma\Delta\omega) \quad (\text{B.5})$$

where

$$\begin{aligned}\Delta k_h &= \frac{2\pi}{\Delta h L} & \alpha &= 0, 1, 2, \dots, L-1 \\ \Delta k_\tau &= \frac{2\pi}{\Delta \tau M} & \beta &= 0, 1, 2, \dots, M-1 \\ \Delta \omega &= \frac{2\pi}{\Delta t' N} & \gamma &= 0, 1, 2, \dots, N-1\end{aligned}\quad (\text{B.6})$$

The observed data at the surface is given by  $q(l, m=0, n)$ . Its Fourier transform is  $Q(\alpha, \beta=0, \gamma)$ . Downward continuation is achieved using the discrete version of Eq. (3.3),

$$Q(\alpha, m, \gamma) = Q(\alpha, m=0, \gamma) \exp \left\{ \sqrt{-1} \sum_{j=1}^m T_{\alpha, j, \gamma} \gamma \Delta \omega \Delta \tau_j \right\} \quad (\text{B.7})$$

$$T_{\alpha, j, \gamma} = 1 - \frac{p_0 v_j}{1 - p_0^2 v_j^2} H_{\alpha, j, \gamma} - \left[ 1 - \frac{2p_0 v_j H_{\alpha, j, \gamma} + H_{\alpha, j, \gamma}^2}{1 - p_0^2 v_j^2} \right]^{1/2} \quad (\text{B.8})$$

$$H_{\alpha, j, \gamma} = \frac{\alpha \Delta k_h v_j}{2\gamma \Delta \omega'} \quad (\text{B.9})$$

where the velocity is given at intervals  $\Delta \tau_j$ ,  $j = 1, 2, \dots, M-1$ . In practice  $\Delta \tau$  is kept constant.

The imaged data is found Fourier Transforming Eq. (B.7). Assuming constant  $\Delta \tau$

$$q(l, m, n) = \sum_{i=0}^{L-1} \sum_{k=0}^{N-1} Q(i, m=0, k) \exp \left\{ \sqrt{-1} \sum_{j=1}^m T_{i, j, k} k \Delta \omega \Delta \tau \right\} W_L^i W_N^{-kn} \quad (\text{B.10})$$

It is convenient to assume the sampling intervals are the same in time and time-depth coordinates,  $\Delta \tau = \Delta t'$ ,

$$q(l, m, n) = \sum_{i=0}^{L-1} \sum_{k=0}^{N-1} Q(i, m=0, k) \exp \left\{ 2\pi \sqrt{-1} \left( \sum_{j=1}^m T_{i,j,k} k \right) / N \right\} W_L^{il} W_N^{-kn} \quad (\text{B.11})$$

substituting the imaging condition  $n \Delta t' = m \Delta \tau$ , or  $n = m$  in our case gives

$$q(l, m, n = m) = \sum_{i=0}^{L-1} \sum_{k=0}^{N-1} Q(i, m=0, k) W_N^{k \sum_{j=1}^m (T_{i,j,k} - 1)} W_L^{il} \quad (\text{B.12})$$

To use this equation in a computer algorithm it is more efficient to find the wave-field  $q(l, m, n = m)$  not from  $Q(i, m=0, k)$  but from  $Q(i, m-1, k)$ . This way only one phase shift is needed and previous computations are saved. Rewriting Eq. (B.12) as

$$q(l, m, n = m) = \sum_{i=0}^{L-1} \sum_{k=0}^{N-1} Q(i, m-1, k) W_N^{k (T_{i,m,k} - 1)} W_L^{il} \quad (\text{B.13})$$

This equation can be implemented in the computer with the aid of a Fast Fourier Transform algorithm.

### Appendix C: Stolt method

Stolt's method (Stolt, 1978) has the attributes of being exact in the propagating region. Since fast Fourier transform algorithms are used, it is fast. However, it expects constant velocity fields. The deformation introduced in Chap. (III) may be useful as a preprocessor when the velocity function has sharp discontinuities.

Snell midpoint coordinates were introduced as a *retarded* frame (Eq. A.13). To derive a Stolt algorithm, coordinates need to be redefined as *non-retarded*. They are:



$$\begin{aligned}
 t' &= t - p_0(g - s) \\
 y &= \frac{g + s}{2} \\
 h &= \frac{g - s}{2} + \int_0^z \tan \vartheta \, d\xi \\
 \tau &= 2 \int_0^z \frac{\cos \vartheta}{v(\xi)} \, d\xi
 \end{aligned} \tag{C.1}$$

In this system the imaging conditions become  $s = g$ , and  $t' = 0$ .

The dispersion relation in this non-retarded Snell midpoint frame is:

$$T = -p_0 \frac{v}{1 - p_0^2 v^2} H - \left[ 1 - \frac{2p_0 v H + H^2}{1 - p_0^2 v^2} \right]^{1/2} \tag{C.2}$$

Stolt imaging is a transformation from  $(h, t')$  space to  $(h, \tau)$  space in the  $\omega$ - $k$  domain. This transformation is achieved changing variables in the following integral:

$$f(h, \tau, t' = 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_h, \tau = 0, \omega) e^{i \int_0^{\tau} T(\xi) \omega \, d\xi} e^{ik_h h} \, dk_h \, d\omega \tag{C.3}$$

in a way that it resembles a 2D-fft of the form

$$f(h, \tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_h, k_\tau) e^{-i(k_h h + k_\tau \tau)} \, dk_h \, dk_\tau \tag{C.4}$$

Using the dispersion relation (C.2), we can solve for  $\omega$  to get

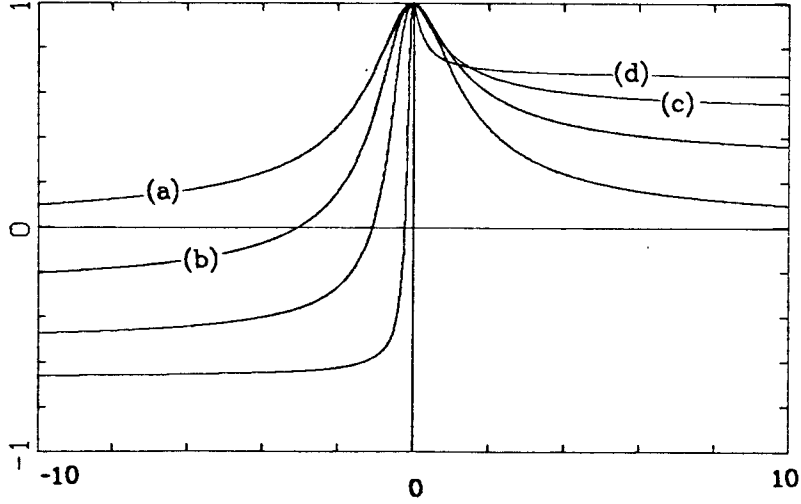


FIG. C.1. Obliquity function. Snell midpoint coordinates. The horizontal axis is  $\frac{k_h v}{2k_\tau}$ ; (a)  $\vartheta_0 = 0^\circ$ ; (b)  $\vartheta_0 = 17^\circ$ ; (c)  $\vartheta_0 = 37^\circ$ ; (d)  $\vartheta_0 = 64^\circ$ .

$$\omega = -\frac{p_0 v^2 k_h}{2(1 - p_0^2 v^2)} + \left[ k_\tau^2 + \frac{k_h^2 v^2}{4(1 - p_0^2 v^2)} + \frac{p_0 v^2 k_h k_\tau}{1 - p_0^2 v^2} + \frac{p_0^2 v^4 k_h^2}{2(1 - p_0^2 v^2)^2} \right]^{1/2} \quad (C.5)$$

Next, assuming constant velocity and changing variables in Eq. (C.3) from  $\omega \rightarrow k_\tau$  we obtain

$$f(h, \tau) = \int_{-\infty}^{\infty} \int F(k_h, k_\tau = k_\tau(k_h, \omega)) e^{-i(k_h h + k_\tau \tau)} \left| \frac{d\omega}{dk_\tau} \right| dk_h dk_\tau \quad (C.6)$$

where the obliquity factor is given by

$$\frac{d\omega}{dk_\tau} = \frac{k_\tau + \frac{p_0 v^2 k_h}{2(1 - p_0^2 v^2)}}{\left[ k_\tau^2 + \frac{k_h^2 v^2}{4(1 - p_0^2 v^2)} + \frac{p_0 v^2 k_h k_\tau}{1 - p_0^2 v^2} + \frac{p_0^2 v^4 k_h^2}{2(1 - p_0^2 v^2)^2} \right]^{1/2}} \quad (C.7)$$

Eq. (C.6) can be implemented into a computer algorithm with a 2-D fast Fourier transform. We take the data  $f(h, \tau=0, t')$  and Fourier transform it to  $F(k_h, \omega)$ . Then use Eq. (C.3) to map this data into  $F(k_h, k_\tau)$  space and inverse Fourier transform to  $F(h, \tau)$ . The obliquity factor  $\left| \frac{d\omega}{dk_\tau} \right|$  weights by a cosine-like factor amplitudes. For arrivals traveling with a fix  $p_0$   $H = 0$ . Here the obliquity factor is unity. At large propagation angles the obliquity factor goes to zero. (Fig. C.1). In computer implementations of Stolt's method the obliquity factor is usually omitted.

#### Appendix D: Finite difference in $(h, \tau, \omega)$ domain.

Finite difference methods (Clærbout, 1976) are appropriate when we need to apply wave extrapolation operators in space domain rather than wavenumber domain. In this section we will derive a finite difference operator in  $(h, \tau, \omega)$  domain.

In  $(h, \tau, \omega)$  space velocity may include variations with angle. This inhomogeneity may be used to partially correct errors in finite difference schemes. Stepout (dip) filters can also be included in the formulation at no extra cost (Clærbout, 1976).

There is no representation of the operator of Eq. (3.1) in the space domain. We need to find approximations for the square root before Fourier transforming back to space. In chapter (II) we have already study the asymptotic behavior of this operator and concluded the first order approximation is sufficient.

The Fresnel approximation (fifteen degree) is found using either Muir's expansion or Taylor expansion. With this first order approximation we get for Eq. (3.1)

$$T \approx \frac{H^2}{2(1 - p_0^2 v^2)} \quad (D.1)$$

Substituting for  $T$  and  $H$  in terms of  $k_\tau$ ,  $\omega$ ,  $k_h$ , and multiplying by the wave-field  $F(k_h, k_\tau, \omega)$ , we get

$$k_\tau F(k_h, k_\tau, \omega) = \frac{v^2}{8\omega(1 - p_0^2 v^2)} k_h^2 F(k_h, k_\tau, \omega) \quad (D.2)$$

Inverse Fourier transforming the  $k_\tau$  and  $k_h$  coordinates gives

$$\frac{\partial}{\partial \tau} f(h, \tau, \omega) = \frac{iv^2}{8\omega(1 - p_0^2 v^2)} \frac{\partial^2}{\partial h^2} f(h, \tau, \omega) \quad (D.3)$$

where  $i = \sqrt{-1}$ . This equation can be directly discretized into a finite difference formulation.

To add a stepout filtering term it is convenient to replace the frequency  $-i\omega$  by  $\omega_0 - i\omega$ . Velocity may be now a function of  $h$  and  $\tau$ ,

$$\frac{\partial}{\partial \tau} f(h, \tau, \omega) = \frac{v(h, \tau)^2}{8(\omega_0 - i\omega) [1 - p_0^2 v(h, \tau)^2]} \frac{\partial^2}{\partial h^2} f(h, \tau, \omega) \quad (D.4)$$

The image is found summing over frequency. This follows from setting  $t = 0$  in the inverse Fourier Transform.

In Eq. (D.4)  $\omega_0$  determines the numerical viscosity added to the extrapolation scheme. After Clærbout (1976) a value for this parameter  $\omega_0$  is found as follows. Rationalize the denominator in Eq. (D.4); Fourier transform the offset  $h$  coordinate; and assume  $\omega_0 \ll \omega$  (this last condition for fix velocity  $v$  and

wavenumber  $k_h$ , assures we are attenuating waves propagating at wide angles).

We obtain

$$\frac{\partial}{\partial \tau} f(k_h, \tau, \omega) = \frac{-v^2}{8(1 - p_0^2 v^2)} \frac{(w_0 + i\omega)}{\omega^2} k_h^2 f(k_h, \tau, \omega) \quad (D.5)$$

The solution of Eq. (D.5) is

$$f(k_h, \tau, \omega) = f(k_h, \tau_0, \omega) \exp \left\{ \frac{-v^2}{8(1 - p_0^2 v^2)} \frac{(w_0 + i\omega)}{\omega^2} k_h^2 (\tau - \tau_0) \right\} \quad (D.6)$$

From this equation the attenuation factor is given by

$$\left| \frac{f(k_h, \tau, \omega)}{f(k_h, \tau_0, \omega)} \right| = \exp \left\{ - \frac{v^2}{8(1 - p_0^2 v^2)} \frac{w_0}{\omega^2} k_h^2 (\tau - \tau_0) \right\} \quad (D.7)$$

Since  $\sin \vartheta' = \frac{k_h v}{2\omega}$  is the sine of the angle measured about the slanted reference wavefront, we can rewrite Eq. (D.7) as

$$\left| \frac{f(k_h, \tau, \omega)}{f(k_h, \tau_0, \omega)} \right| = \exp \left\{ - \frac{w_0}{2(1 - p_0^2 v^2)} \sin^2 \vartheta' (\tau - \tau_0) \right\} \quad (D.8)$$

Numerical values for  $w_0$  can be found using this relation.

In this thesis, the discrete operator used for the second partial derivative was

$$\frac{\partial^2}{\partial h^2} \approx \frac{\delta_{hh} / \Delta h^2}{1 + \delta_{hh} / 6} \quad (D.9)$$

into a finite difference Crank-Nicolson scheme. (Clærbout, 1976).

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