WAVE EQUATION VELOCITY ANALYSIS

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Wave equation velocity analysis

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Abstract

Velocity sensitive (wide propagation angle) seismic data does not comply with the root–mean–square small propagation angle approximation. A hyperbolic velocity spectrum and Dix's equation cannot use wide angle arrivals to estimate interval velocity accurately. The wave equation velocity spectrum gives a better alternative.

Snell midpoint coordinates are a convenient frame of reference to study slant wave propagation in the earth. A consequence of the properties of Snell midpoint coordinates at image locations is the linear moveout method of velocity estimation. Energy in Snell midpoint coordinates focuses at the arrival coordinates of a fixed reference Snell wavefront. A function of these coordinates estimates interval velocities accurately.

In this thesis the image of a common midpoint seismic gather in Snell midpoint coordinates, for a non–vertical reference Snell wave, defines the wave equation velocity spectrum. Two important properties of the velocity spectrum are locality, energy is a local function of velocity; and linearity, it is invertible using linear transformations.

Velocity sensitivity of wave equation extrapolation operators increases with angle of propagation. In Snell midpoint coordinates angles are measured relative to an arbitrary slant reference Snell wave. At this particular angle wave equation operators are exact, independent of velocity. Group velocity equations prove

that even approximations of the wave equation in Snell midpoint coordinates satisfactorily image wide angle energy.

The phase shift method is used to illustrate downward continuation of common midpoint gathers in Snell midpoint coordinates. To compute the velocity spectrum, a process to deform CMP gathers, using an estimate of the depth velocity function, to constant velocity hyperbolas enhances resolution of Stolt's imaging method. A more velocity independent formulation is based on the fifteen degree finite difference wave equation in the frequency domain. This formulation resolves multivalued, wide velocity spectrum data using inhomogeneous, offset and depth dependent, downward continuation velocity. Stepout filtering concurrent with downward continuation eliminates wide angle propagation energy not modeled by the fifteen degree wave equation.

Introduction

Multiple coverage 2-D seismic reflection data constitute a 3-D data base in midpoint, offset and time coordinates. A standard way of reducing the dimensionality of the data is to apply a process that will make the data offset independent followed by integration over offset[1]. When collapsing the offset space a velocity–like function, usually referred as *stacking* velocity, is determined. This velocity is assumed to equal the RMS velocity of a vertically propagating reference wavefront. Interval velocities, which describe the material properties of the medium, are then estimated using Dix's equation. (Dix, 1955).

Ray methods have traditionally been used to stack seismic data. Taner and Koehler (1969) defined the velocity spectrum based on a quadratic approximation of traveltime as function of offset. In this method the data is stacked for a range of trial velocities. The stacking velocity is defined as the velocity that optimizes some semblance measure of the stacked energy.

In stratified media the stacking velocity is close to the vertical RMS velocity in the near zero offset region. However, when wide offsets are available the RMS velocity, which is a function of angle, may vary considerably with offset. Residual NMO will exceed half a wavelength at the far offsets, degrading the quality of the stacked section. Quadratic NMO with only two degrees of freedom is not enough to approximate traveltimes for all offsets. High order NMO corrections with more degrees of freedom may be used to improve the quality of the stack. (May and Straley, 1979). Unfortunately improved stacking velocities do not imply improved interval velocities.

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[1]It is important to recognize that any such process can never be strictly defined. Offset space is not a formal physically realizable space.

We need a velocity spectrum specifically designed to estimate interval velocity. It should exploit the enhanced sensitivity of seismic data to velocity for large offsets. (Wide propagation angles). Accurate interval velocities give a more realistic description of the material properties of the subsurface. Also accurate estimates of interval velocity can be used in high–order migration algorithms.

The wave equation provides a deterministic physical model of the reflection seismology experiment. For velocity estimation Doherty and Clærbout (1976) used the wave equation to downward continue the data and remove structure effects from the offset coordinate. Yilmaz and Chambers (1981) proposed to use the wave equation to stack the data as an alternative to ray methods. Thorson and Yedlin (1980) used the wave equation to do NMO without stack. All these methods are designed to measure RMS velocities. Schultz (1981) defined a method using slant stacks to measure interval velocity directly. His method requires to chose events in the data before velocity estimation.

In this thesis we define the *wave equation velocity spectrum* of a CMP gather as its image in Snell midpoint coordinates for a non–vertical reference wavefront. Snell midpoint coordinates provide an optimal frame of reference for slant wave propagation and velocity estimation problems. Our velocity spectrum is invertible with linear transformations. Energy is a local function of velocity. Interval velocity can be estimated accurately using the LMO method. (Clærbout, 1982). There is no need of geometric approximations that deteriorate with angle. (Figure 0.1). Our velocity space is also a convenient model space with linearity and locality properties.

The wave equation velocity spectrum requires a stratified model of the earth without geologic dip. When dipping events are present in the gather, the data must be pre–processed with a pre–stack partial migration algorithm before veloc-

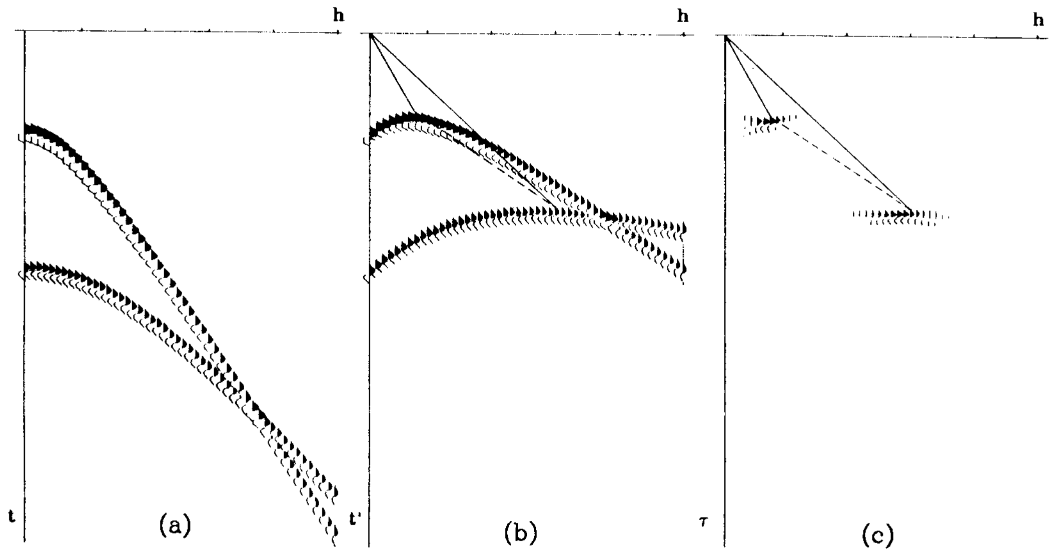

FIGURE 0.1. LMO velocity estimation. (a) Synthetic CMP data. (b) Linear moveout (LMO) corrected data. A reference Snell wave is fixed when applying this correction to the data. The reference Snell wave has the minimum traveltime of an event. RMS velocity is a function of the slope between the origin and the reference Snell wave arrival coordinates. (Continuous line). Interval velocity is a function of the slope joining the reference Snell wave arrivals between two consecutive primary events. (Dashed line). (c) Imaged data in Snell midpoint coordinates. In this coordinate system the energy focuses at the reference Snell wave arrival position. Velocity estimation is possible identifying focused energy.

ity estimation. Pre–stack partial migration is a general mathematical approach to make seismic data offset independent. It consists of an operator that applies NMO plus an operator that corrects for dipping events. (Sherwood *et al*, 1976; Yilmaz and Clærbout, 1980; Deregowski and Rocca, 1981; Ottolini, 1982; Hale, 1982).

We start the thesis defining Snell midpoint coordinates in chapter (I). At the surface of the earth CMP gathers can be put in Snell coordinates by simply applying a Linear Moveout (LMO) correction. The LMO method of velocity estimation can be derived from the properties of the coordinate frame at the image coordinates. LMO velocity estimation is independent of depth. The method can be applied to the image of the data itself. Linearity ensures the timing relationships of multiple reflections are preserved. Multiple reflection energy becomes easy to discriminate from primary energy by their velocity.

In chapter (II) group velocity equations for wave equation operators in retarded Snell midpoint coordinates are derived. These group velocity equations are used to analyze the sensitivity of several wavefield extrapolation operators to the background velocity. We will learn that velocity accuracy imposes strong limitations to the use of exact algorithms at wide propagation angles. The advantage of Snell midpoint coordinates is that operators are referenced at an arbitrary propagation angle. Wide angle energy can still be focused with small–angle low–order operators.

Chapter (III) defines a process to deform CMP gathers with some velocity estimate $\hat{v}(z)$ to hyperbolas. The motivation is given by seismic data processes, such as hyperbolic velocity estimation and Stolt's imaging, that expect a constant velocity seismic wavefield. The transformation defines Snell traces according to the estimate velocity $\bar{v}(z)$ and maps them to constant velocity $\hat{v}$ radial traces.

In chapter (IV) we define the wave equation velocity spectrum. The most general physical description of the reflection seismic model is given by the *double square–root* equation. (Stolt, 1968; Claerbout, 1982). The double square root equation is formulated in Snell midpoint coordinates. Imaging of CMP gathers with the phase shift, Stolt's and finite difference methods are analyzed. Stolt's method can be combined with the hyperbolic deformation of chapter (III). The fifteen degree wave equation in (h, t', τ) is particularly useful because its insensitivity to the background velocity. $\bar{v}(h, \tau)$ can be used to resolve multiple–valued velocity functions. This velocity can be defined with the LMO method. No *a priori* knowledge of velocity is necessary. Also, using the finite–difference formulation of the fifteen degree equation, it is possible to do stepout filtering concurrent with wave extrapolation.

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