1.1 Exploding Reflectors[1]

The most basic reflection seismic prospecting equipment is a source for impulsive sound waves, a geophone (something like a microphone), and a multichannel waveform display system. A survey line is defined along the earth's surface. It could be the path for a ship, in which case the receiver is called a hydrophone. About every 25 meters or so the source is activated and the echoes are recorded nearby. The sound receiver will have almost no directional tuning capability, owing to the fact that the frequencies which have deep-earth penetrating ability are those with wavelengths longer than the ship. Consequently, echoes can arrive from several directions at the same time. It is the joint task of geophysicists and geologists to interpret the results. Geophysicists assume the quantitative, physical and statistical tasks. Their main goals, and the goal to which this book is mainly directed, is to make good pictures of the earth's interior from the echoes.

A Powerful Analogy

Figure 1 depicts two wave-propagation situations which are apparently quite different. The first is our situation with field recording. The second is a thought experiment in which all of the reflectors in the earth suddenly explode. Waves from the hypothetical explosion propagate up to the earth's surface where they are observed by a hypothetical string of geophones along the earth's surface. Even if the earth had exploding reflectors, we would have difficulty recording the waves because of the need for so many geophones. It is surely much easier to tow one geophone past a thousand locations than to operate a one-thousand-channel recording system.

Notice in the figure that the raypaths in the field recording situation seem to be the same as those in the exploding reflector situation. It is a great conceptual advantage to imagine that the two wave fields, the observed and the hypothetical, are indeed the same. If they are the same, then we can ignore the many thousands of experiments which have actually been done and think only of the one hypothetical experiment. The one major, obvious difference between the two situations is that in the field geometry waves must first go down and then return upward along the same path, whereas in the hypothetical experiment they just go up. This difference could be accounted for in either of two

1 SEP-25, pp 191-201.

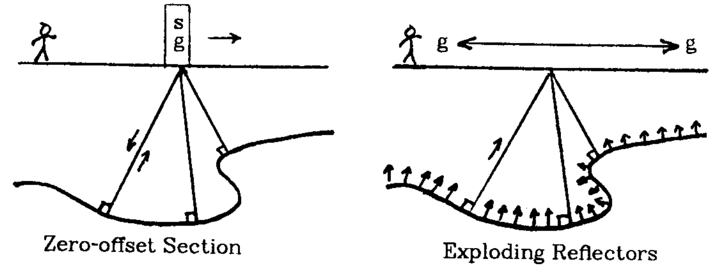


FIG. 1. The field geometry of echoes collected with a source-receiver pair at all places on the earth's surface (left) and the "exploding reflectors" conceptual model (right).

ways. We could take the traveltime in field experiments and divide by two. In practice, the data of the field experiments (two-way time) is analyzed assuming the sound velocity to be half its true value.

Huygens Secondary Point Source

Waves on the ocean have wavelengths comparable to those of waves in seismic prospecting (15-500 meters), but they are conveniently different in that they move slowly enough to be easily observed. Imagine a long harbor barrier parallel to the beach with a small entrance in the barrier for the passage of ships. This is depicted in figure 2. A plane wave incident on the barrier from the open ocean will send a wave through the gap in the barrier. It is an observed fact that in the harbor the wavefront becomes a circle with the gap as its center. The difference between this beam of water waves and a light beam through a window is in the ratio of wavelength to hole size.

A Cartesian coordinate system has been superimposed upon the ocean surface with x going along the beach and z measuring the distance from shore. To draw the analogy to reflection seismology we must say that we are confined to the beach (the earth's surface) where we can make only measurements of wave height as a function of x and t. From this data we can make inferences about the existence of a gap in the barrier out in the (x,z)-plane. Figure 3a depicts the arrival time at the beach of a wave from the ocean. The earliest arrivals occur nearest

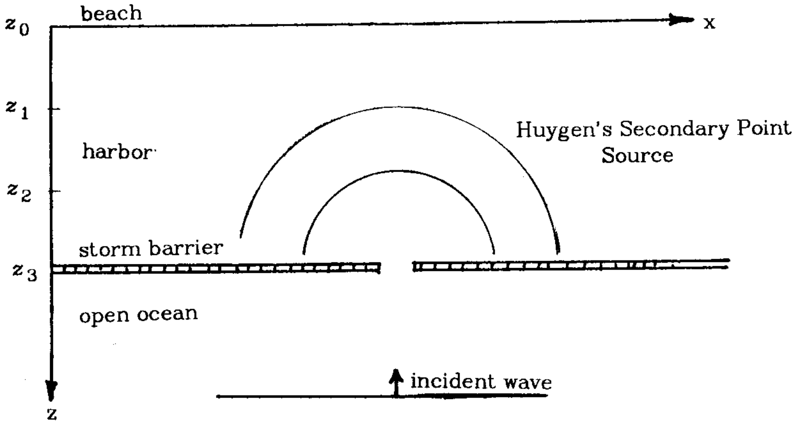


FIG. 2. Waves going through a gap in a barrier have semi-circular wavefronts (provided that the wavelength is long compared to the gap size).

the gap. What mathematical expression determines the shape of the arrival curve seen in the (x,t)-plane?

The waves of interest are expanding circles. An equation for a circle expanding with velocity v about a point (x_3,z_3) is

$$(x-x_3)^2 + (z-z_3)^2 = v^2t^2 \qquad (1)$$

Considering t to be a constant, i.e. taking a snapshot, (1) is the equation of a circle. Considering z to be a constant, (1) is an equation in the (x,t)-plane for a hyperbola. Considered in the (t,x,z)-volume, (1) is the equation of a cone. Slices at various values of t show circles of various sizes. Slices of various values of z show various hyperbolas. Figure 3 shows four hyperbolas. The first is our observation on the beach $z_0 = 0$. The second is a hypothetical set of observations at some distance z_1 out in the water. The third, at z_2, is an even greater distance from the beach. The fourth, z_3, is nearly all the way out to the barrier where the hyperbola has degenerated to a point. All these hyperbolas are from a family of hyperbolas, each with the same asymptote. The asymptote refers to a wave which turns nearly 90° at the gap and is found moving nearly parallel to the shore at the speed dx/dt of a water wave. [For this water wave analogy we presume (incorrectly) that the speed of

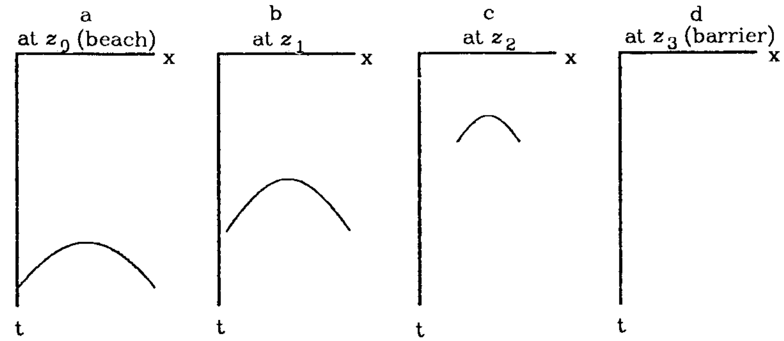


FIG. 3. The left frame shows the hyperbolic wave arrival time seen at the beach. Frames to the right show arrivals at increasing distances out in the water. (The x-axis is compressed from figure 2.)

water waves is a constant independent of water depth.]

Linearity is a property of all low-amplitude waves (not those foamy, breaking waves you see near the shore). This means that if we have two gaps in the harbor barrier we will have two semi-circular wavefronts. Where the circles cross, the wave heights combine by simple linear addition. It is interesting to think of a barrier with very many holes such as that shown in figure 4. The many semi-circles and hyperbolas combine, tending to give the wave which would have been seen if there were no barrier. Indeed, in the limiting case where the barrier disappears, being nothing but one gap alongside another, the semi-circles and the hyperbolas should all combine to make only the incident plane wave. All those waves at non-vertical angles must somehow combine with one another to extinguish all evidence of anything but the plane wave. If the original incident wave was a positive pulse, then the Huygens secondary source must consist of both positive and negative polarities in order to enable the destructive interference of all but the plane wave. So the Huygens waveform has a phase shift. Eventually we will find mathematical expressions for the Huygens secondary source. Another property we will discover, well known to boaters, is that the Huygens semi-circle has its largest amplitude pointing straight towards shore. The amplitude drops to zero for waves moving parallel to the beach. In optics this amplitude dropoff with angle is called the ***obliquity factor***.

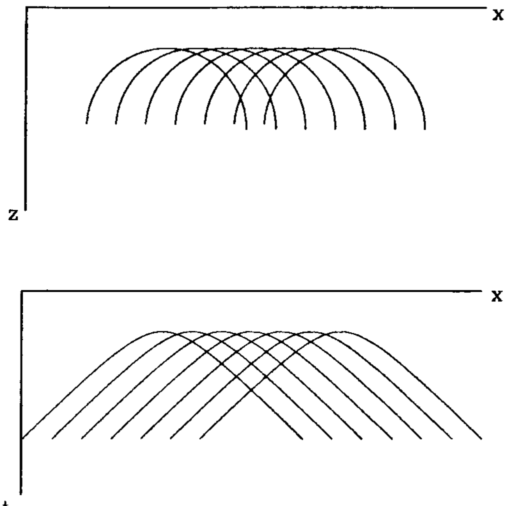


FIG. 4. (Gonzalez) A barrier with many holes. Top shows a snapshot, that is, the (x,z)-plane at some t_0. It is a superposition of many Huygens semi-circular wavefronts which nearly create a plane wave. Bottom shows the superposition of the hyperbolas in the (x,t)-space of geophysical observations.

Migration Defined

Looking in the dictionary at the word "run" you find many definitions. They are related, but they are distinct. The word "migration" in geophysical prospecting likewise has about four related but distinctive meanings. The simplest is like the meaning of the word "move." When an object at some location in the (x,z)-plane is found at a different location at a later time t, then we say it ***moves***. Analogously when a wave arrival (often called "an event") at some location in the (x,t)-space of geophysical observations is found at a different position for a different survey line at a greater depth z, then we say it ***migrates***.

To see this more clearly we imagine the four frames of figure 3 being taken from a movie. During the movie, the depth z changes beginning from the beach (earth's surface) going out to the storm barrier. The frames are superimposed in figure 5a. Mainly what happens in the movie is that the event migrates upward toward $t=0$. To remove this dominating effect of vertical translation we make another superimposition, keeping the hyperbola tops all in the same place. This is done by replacing the time t-axis by a so-called ***retarded*** time axis $t'=t+z/v$, as shown in figure 5b. Our second, more precise, definition of ***migration*** is the motion of an event in (x,t')-space as z changes. Having removed the vertical shift, we are seeing mainly a shape change.

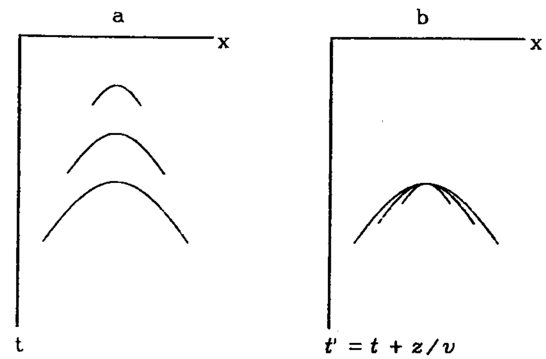


FIG. 5. (Gonzalez) Left shows a superposition of the hyperbolas of figure 3. At the right the superposition incorporates a shift, called retardation $t'=t+z/v$, to keep the hyperbola tops together.

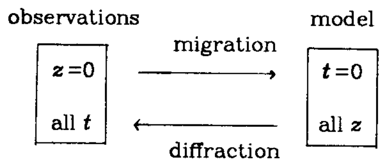
It is of interest to see how the shape actually changes. Think of a pebble thrown into the water and the ensuing circular wave. At the end of any ray from the center to the circle is a wavefront whose slope is given by some $dx/dz = \tan\vartheta$. This angle is constant as the circle grows with t. Likewise, in (x,t)-space, the wavefront, called an event, has a slope $dt/dx = \sin\vartheta/v$ which remains constant as z increases. Figure 5b was drawn so that the hyperbolas end at $\vartheta = 45°$. These endpoints migrate along a straight line in the (x,t')-plane toward the center, which they hit at depth z_3.

In this case the exploding reflector is like a short line segment across the barrier gap. At depth z_3 all the energy in the (x,t')-space of migrated data is located in the position of the gap. In other words, it is focused. The third definition of migration is that it is the process which somehow pushes observational data — wave height as a function of x and t —from the beach to the barrier.

To go farther we need a more general example than the storm barrier example. The barrier example is confined to making Huygens sources only at some particular z, and we need sources at other depths as well. Then, given a wave extrapolation process to move data to increasing z values, we can construct our exploding reflector images with

$$Image\,(x,z) \quad = \quad Wave\,(t=0,x,z) \tag{2}$$

Our fourth definition of migration also incorporates the definition of "diffraction" as the opposite of migration.



Diffraction is sometimes regarded as the natural process which creates and enlarges hyperboloids. *Migration* is the computer process which does the reverse.

Another aspect of the use of the word "migration" arises in Chapter 3 where the horizontal coordinate can be either midpoint y or shot to geophone offset h. Hyperboloids can be downward continued in both the (y,t) and the (h,t)-plane. In the (y,t)-plane this is called *migration* or *imaging* and in the (h,t)-plane it is called *focusing* or *velocity analysis*

An Impulse in the Data

We have seen that Huygens diffraction takes an isolated pulse function (delta function) in (x,z)-space and makes it into a hyperbola in (x,t)-space at $z=0$. The converse is to start from a delta function in (x,t)-space at $z=0$. This converse refers to a seismic survey in which you record no echoes except at one particular location and there you record only one echo. What earth model is consistent with such observations? As shown in figure 6 this earth must contain a spherical mirror whose center is at the anomalous recording position.

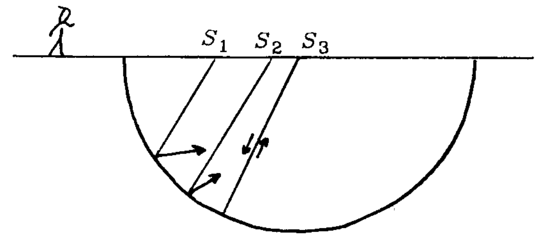


FIG. 6. When the seismic source S is at the exact center of a semi-circular mirror, then, and only then, will an echo return to the geophone at the source. This semi-circular reflector is the logical consequence of a data-set where one echo is found at only one place on the earth.

It is very unlikely that the processes of nature have created many spherical mirrors inside the earth. But when you look at processed geophysical data, you will often see spherical mirrors. Obviously, such input data contains impulses which are not consistent with the wave propagation theory being explained here. This illustrates why petroleum prospectors study reflection seismic data processing even though they personally plan to write no processing programs. The raw data is too complex to comprehend. The processed data gives an earth model, but its reliability is difficult to know. You may never plan to build an automobile, but when you drive far out into the desert, it is prudent to know as much as you can about automobiles.

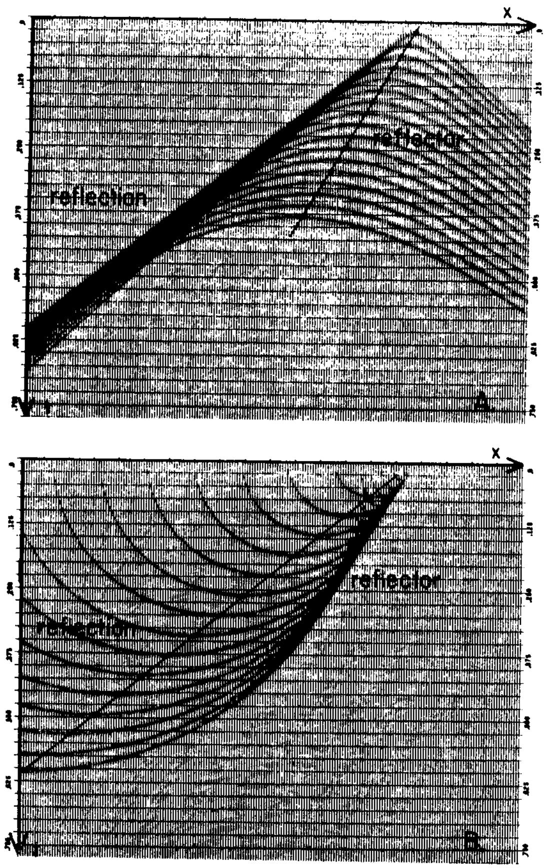


FIG. 7. (Okaya) Top is a superposition of many hyperbolas. The top of each hyperbola lies along a straight line. That line is like a reflector, but instead of a continuous line, we have a sequence of points. Constructive interference gives an apparent reflection off to the side.

Bottom shows a superposition of semi-circles. The bottom of each semi-circle lies along a line which could be the line of an observed plane wave. Instead the plane wave is broken into point arrivals, each of which we interpret as coming from a semi-circular mirror. Adding the mirrors we see a steeply dipping reflector.

Migration Steepens Reflectors

It is true that flanks of hyperbolas migrate without change of slope. But a hyperbola is a special kind of event which comes from a single source at a single depth. Superposing point sources from different depths into a dipping planar reflector we find that migration steepens the reflections. This could be suspected by consideration of the limiting case, a vertical wall. Its reflections, the asymptotes of a hyperbola, have a non-vertical steepness. To see this in a less extreme case, see figure 7, where a dipping bed, of dip of about 60 degrees, is made from a series of points in a line.

Limitations of the Exploding Reflector Concept

The exploding reflector concept is a most powerful and fortunate analogy. For people who spend their time working entirely on data interpretation rather than processing, the exploding reflector concept is more than a vital crutch. It's the only means of transportation! For those of us who work on data processing, the exploding reflector concept has a very serious shortcoming. No one has yet figured out how to extend the concept to apply to data recorded at nonzero offset. Furthermore, most data is recorded at rather large offsets. In a modern marine prospecting survey, the recording cable (a cable containing not one but many hundreds of hydrophones) is typically 2-3 kilometers long. Drilling may be about 3 kilometers deep. So in practice the angles are big. Therein lie both new problems and new opportunities, none of which we will consider until Chapter 3.

Furthermore, even at zero offset, the exploding reflector concept is not quantitatively correct. Later efforts, mainly in Chapter 3 will elucidate, to some degree, the region of validity. For the moment let us just note two obvious failings. First, figure 8 shows a ray which is not predicted by the exploding-reflector model, but which will be present in a zero-offset section. Notice that lateral velocity variation is required for this situation to exist.

Second, consider the situation with multiple reflections. For a flat sea floor with a two-way traveltime t_1, multiple reflections are predicted at times $2t_1$, $3t_1$, $4t_1$, etc. In the exploding-reflector geometry the first multiple has first a path from reflector to surface, then from surface to reflector, then from reflector to surface, for a total time $3t_1$. Subsequent multiples occur at times $5t_1$, $7t_1$, etc. Clearly there is no relationship between the multiple reflections generated on the zero-

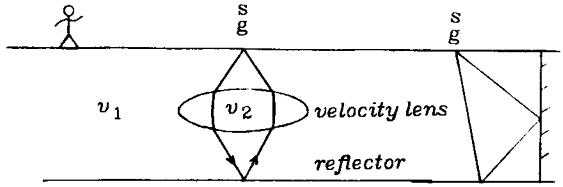


FIG. 8. Two rays, not predicted by the exploding reflector model, which would nevertheless be found on a zero-offset section.

offset section and those of the exploding-reflector model. This explains why Chapter 5 of this book, which has to do with modeling and suppressing multiple reflections, completely abandons the zero-offset approach.

Examples of Migration ....7 pages of half tones in preparation