Enhancing Velocity Analysis by Prestack Partial Migration of Radial Trace Sections

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Abstract

Prestack partial migration increases the resolution and accuracy of conventional velocity analysis by correcting for the effects of dip. In radial trace coordinates, prestack partial migration is insensitive to vertical velocity variations and vertical grid sampling.

Introduction

Judson, et. al. (1978) demonstrated that a prestack partial migration can enhance velocity analysis. This process makes the offset moveout of dipping events resemble that of flat events. The apparent velocity increase caused by dip is removed, increasing the *accuracy* of velocity estimation. Also, the moveout function becomes more hyperbolic, increasing the *resolution* of velocity estimation. The example in this paper illustrates these properties.

Yilmaz (SEP-18) discovered a method of implementing prestack partial migration on constant offset sections. However, mathematical approximations in the derivation decreased the utility of this method at wide offsets. Both unmoved out and moved out finite difference algorithms were proposed. The unmoved out version had a mathematical pole in it requiring infinitesmal z steps for shallow reflectors. The other version was more numerically robust, but required knowledge of velocity in order to perform the moveout.

Ottolini (SEP-28) proposed a method of implementing prestack partial migration using Snell trace sections. A *Snell trace section* selects the portion of the wavefield propagating at approximately the same offset angle. Such a prestack migration scheme is accurate for steep dips, wide offests, and vertical velocity variations. However, the formation and partial migration of Snell traces require knowledge of the vertical velocity function.

This paper uses radial trace sections in prestack partial migration. *Radial traces* are the seismic gather at constant ratio of offset to time. They are the constant velocity special case of Snell traces. The transformation into radial traces is simpler than Snell traces Both the transformation and migration are velocity independent. Mathematical accuracy is lost in assuming constant velocity. But as the example shows, the loss is not significant in practice.

Migration Equation

The derivation of the prestack partial migration equation in radial trace coordinates follows the paradigm set forth by Yilmaz. That is, this operation is the difference between the operations for full migration before stack and conventional processing. Conventional processing is defined to be zero dip stacking and the migration of a zero offset section.

First, we recall the migration before stack equation for radial trace sections derived by Ottolini (SEP-28).

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\left(\frac{1 - Y^2}{1 - \frac{4r^2}{v^2}}\right)^{1/2} P \tag{1}$$

where Y is the ratio of lateral wavenumber to frequency wavenumber $vk_y / 2\omega$. r is the ratio of offset to time, equal to h/t, also called the radial parameter. Equation 1 is derived by determining the point reflector travel time function on a radial trace section. It happens to be a squashed hyperbola. The numerator of the square root in equation 1 migrates hyperbolic travel time functions, while the denominator corrects for the 'squashing'.

Various portions of equation 1 can be associated with conventional processing. If the square root were unity, then equation 1 converts time to depth. The portion of the square root which migrates zero offset sections is

$$\sqrt{1 - Y^2} - 1 \tag{2}$$

(The -1 removes the time to depth conversion.) The portion which corrects for offset dependent moveout when dip is zero (Y=0) is

$$\frac{1}{\sqrt{1 - \frac{4r^2}{v^2}}} - 1 \tag{3}$$

These latter two processes comprise conventional processing. The partial migration operator is obtained by subtracting equations 2 and 3 from 1. Making some low order approximations in the square root arithmetic leaves

$$\frac{dP}{dz} = -i2\frac{\omega}{v}\left(1 - \frac{r^2k_x^2}{2\omega^2}\right)^{1/2} P \tag{4}$$

If depth in equation 4 is converted into migrated travel time by $z = v\tau$, then equation 4 is velocity independent.

Processing Sequence and Example

The following processing sequence was used on the synthetic example shown in figure 1. Straight reflectors dip from zero to 80 degrees. The background velocity increases linearly from .5 to 3. CDP gathers were generated by analytic ray tracing as described in the appendix of the other paper by Ottolini in this volume. The processing sequence is:

(1) Extract radial trace sections for several radial parameters. A radial trace follows a diagonal line from the origin on a CDP gather. About as many radial parameters as geophone offsets were used, evenly ranging across existing events. Radial traces of the same radial parameter from each CDP gather are assembled into a radial trace section.

(2) Partially migrate each radial trace section using equation 4. I used the phase shift method to implement the solution to equation 4 (see Ottolini, SEP-28).

(3) Map radial trace sections back into CDP gathers. Figure 2 shows the same CDP gather before and after prestack partial migration. The shallow moveouts of the steep dipping reflectors have been increased to approximately what flat reflectors would be.

(4) Estimate velocities using conventional procedures. Figure 3 shows v-t graphs computed from unmigrated and migrated CDP gathers. The migrated result is has a higher resolution for steep events and more accurate rms velocities.

(5) Perform conventional stacking and migration on the processed gathers to obtain a reflector image.

Discussion

Several recently developed migration-before-stack schemes are closely related to the results presented in this paper. First, Biondi et. al. (1981) in what they call offset continuation implement the equation

$$P_{ht} = -\frac{h}{t}k_x^2 P \tag{5}$$

This equation is essentially the retarded 15 degree expansion of equation 4. It is velocity independent, computationally robust, and produces good seismic images. There are some sampling/stability problems when h > vt. However, this region of the CDP gather is near the direct arrival where refraction energy dominates and often is muted out.

Second, the phase shift term from the abstract of Hale's paper on constant offset section migration (this SEP volume) closely resembles equation 4. It is (slightly rewritten)

$$\omega t \left[1 - \frac{4h^2}{v^2 t^2} + \frac{h^2 k_x^2}{t^2 \omega^2} \right]^{1/2} \tag{6}$$

The second term, which performs normal moveout, is dropped to obtain the prestack partial migration phase shift. (The sign difference between equations 6 and 4 is due to that the former is implemented as a diffraction algorithm while the latter as a migration algorithm.)

The main difference between the three schemes- Biondi's, Hale's, and mine- is whether to implement the ratio h/t on radial trace sections or constant offset sections. In the case of radial trace sections, much of the computational work is done during transforming to radial traces, while the migration phase shift is very simple to calculate. In the case of constant offset sections, the phase shift must be recomputed for every time step because it is a function of time. Or if it is implemented in the time domain, then the h/t ratio sometimes poses numeric problems. However, the extraction of radial traces is sensitive to the offset spacing on CDP gathers, while h/t methods can work on a single constant offset section.

REFERENCES

Biondi, G., Loinger, E., and Rocca, F., 1981, Offset continuation of seismic sections, Preprint from the 43rd EAEG meeting

Judson, D.R., Schultz, P.S., and Sherwood, J., 1978, Equalizing the stacking velocities of dipping events via Devilish, Preprint from the 48th SEG meeting

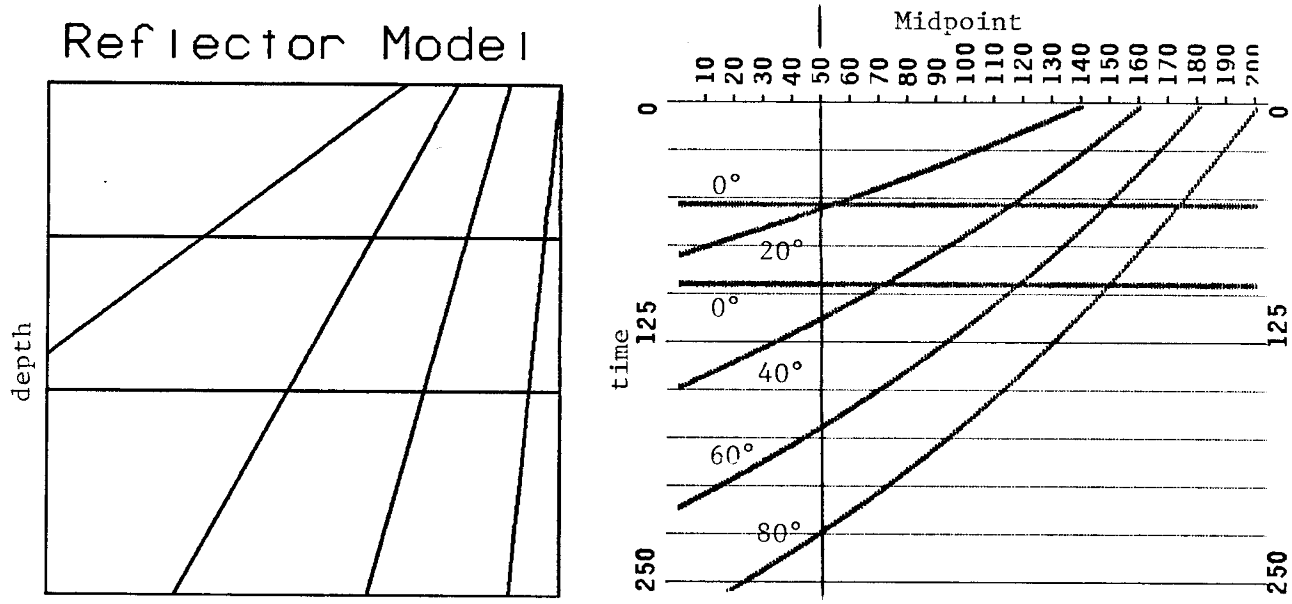

FIG. 1. On the left is the reflector model from which the synthetics were generated. Dip increments are 20 degrees. Velocity increases linearly with depth. Parameters are: nx=200, dx=1, noff=100, doff=1.6, nt=256, dt=1, vmin=.5, dv=.01. Bandpass between 5% and 50% of Nyquist. On the right is the zero offset section. Midpoint 50 is used in figures 2 and 3.

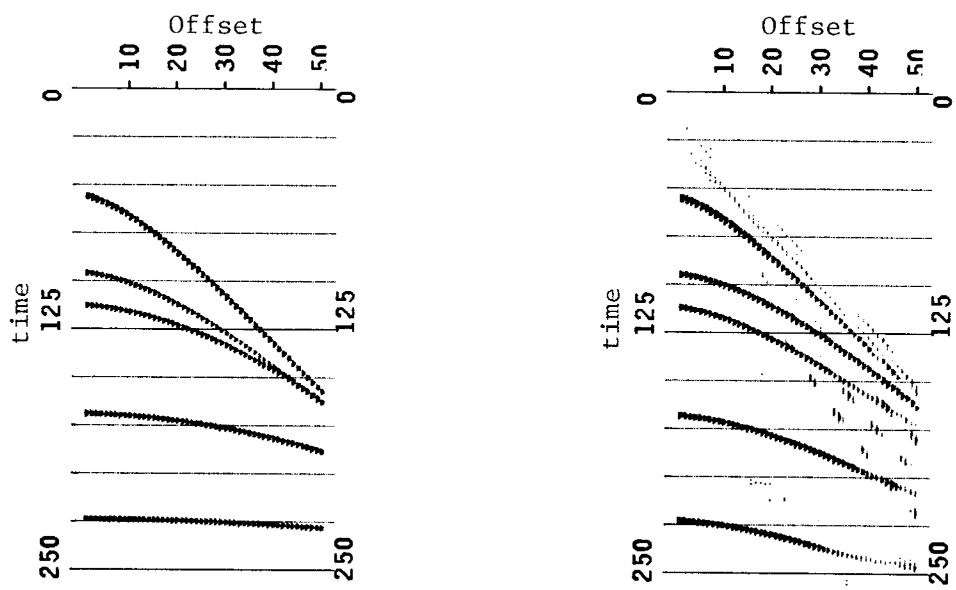

FIG. 2. On the left is an unprocessed CDP gather. On the right is a CDP gather after radial trace prestack partial migration. Prestack partial migration increases the moveout of dipping events to make them resemble flat events.

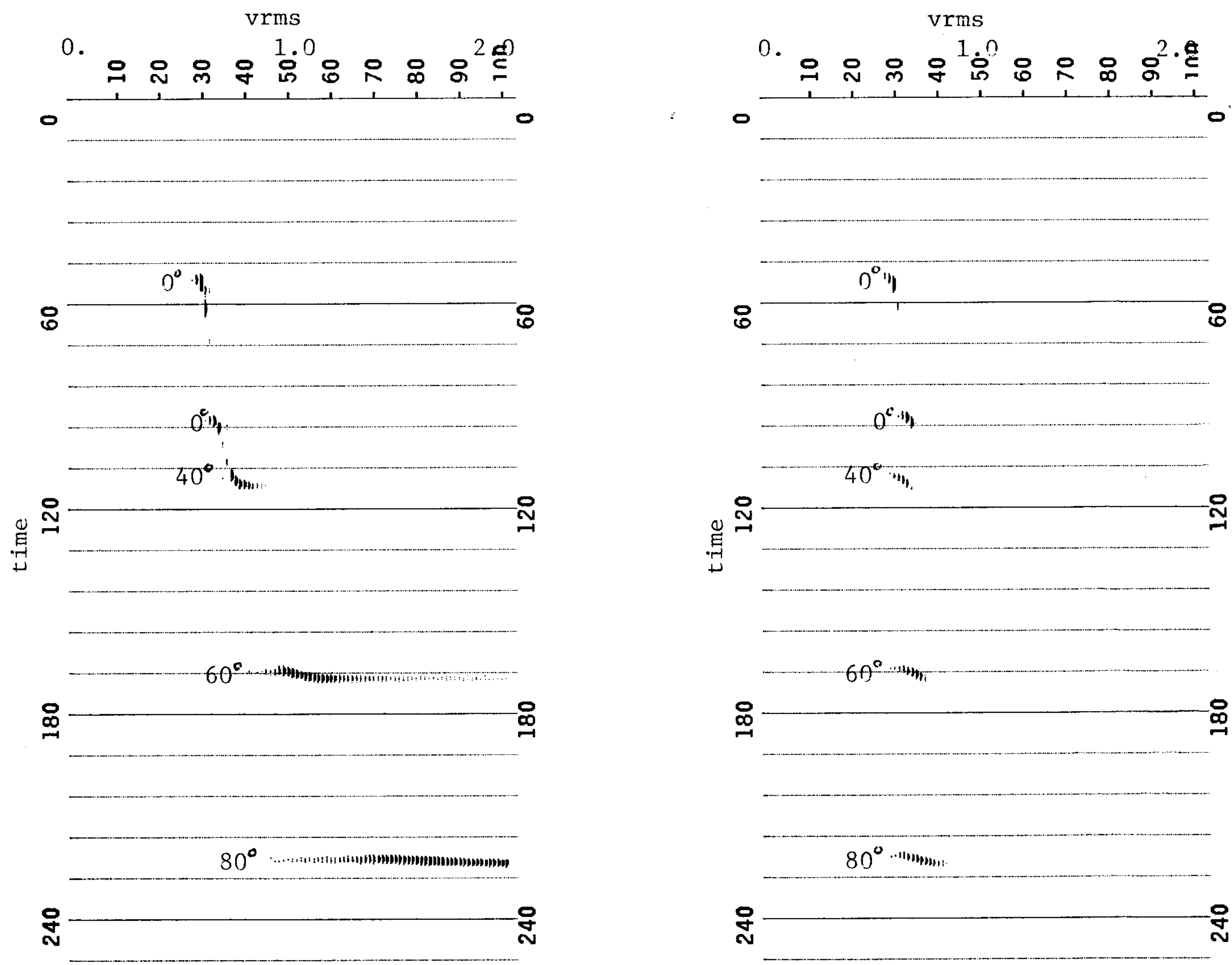

FIG. 3. Velocity-time plots of gathers in figure 2. A mean squared statistic was measured along vrms hyperbolas. On the left is the unprocessed gather. It exhibits the secant dip angle increase in apparent vrms velocity. Velocity errors range from 6% to 400%. As the the dip of an event steepens, the moveout curve becomes less hyperbolic and velocity resolution declines. On the right is the partially migrated gather. Accuracy and resolution have been increased. Velocity errors range from 6% to 10%. Note that though the steep reflections are generally from shallow depths, event though the travel time is large. Therefore the vrms can be low.