Stolt migration; interpolation artifacts

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Introduction

The formulation of the "Stolt method" (Stolt, 1978) is:

$$model(x,z) = 2DFT^{-1}\left\{ data\ [\ k_x, \omega(k_x,k_z)\ ]\ \frac{d\omega}{dk_z} \right\}$$

This method requires interpolation in the $\omega - k$ domain, and discrete Fourier transforms (DFT's) in x,t and also z directions. Each transform assumes periodicity and homogeneity in its direction. The DFT's in x and in t are inherited from the "Phase Shift Method" (Claerbout ,1977, 1980; Gazdag, 1978). The DFT in the z direction is added by the "Stolt Method".

If the sampled data are not aliased we can reconstruct the continuos data fully by multiplying the time domain data by a rectangle function (to get rid of the replicates), which corresponds to convolving with a sinc in the frequency domain (Bracewell, 1978, p.189). The problem with sincs is that they decay slowly and superposing all of them is expensive.

Impulse responses

Data consisting of one impulse, in a homogeneous velocity medium, should migrate to a semicircle, (Claerbout, 1980, p.198). Migrating an impulse data in the Stolt method, using linear interpolation (Figure 1), produces spurious events in addition to the expected one (Lynn, 1978). Similar events appear when cubic spline interpolation is used (Figure 2). Using geometric interpolation, these events are absent when we have one impulse data (Figure 3). However, they show up when the input has more than one impulse. (Figure 4).

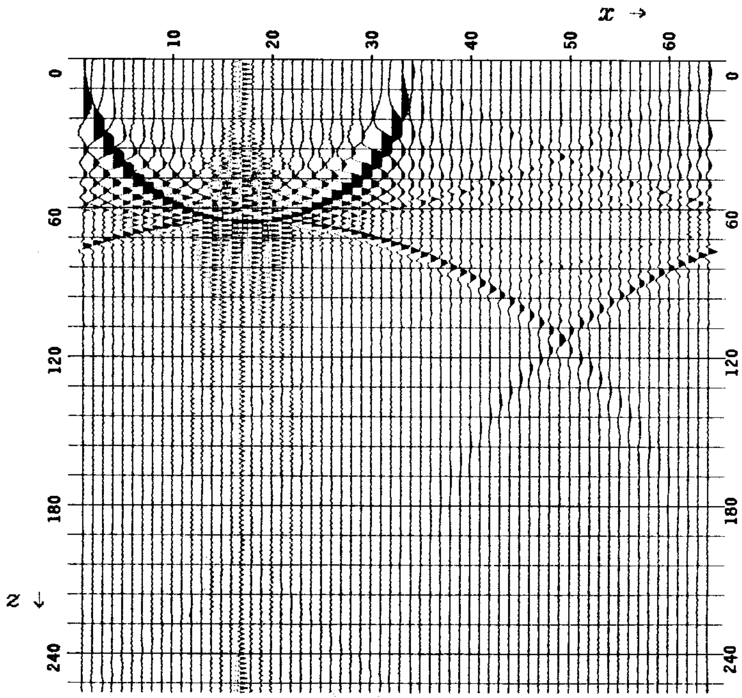

FIG. 1. Migration of an impulse by "Stolt method", using linear interpolation. The weaker and bigger semicircle is an artifact.

To get an idea why this happens (Bill Harlan, this report) we can plot the error in the interpolation of a cosine which is the FT of a symmetric two impulse data.

$$DFT \left\{ \delta(x - x_0) \left[\delta(t - t_0) + \delta(t + t_0) \right] \right\} = 2cos(\omega t_0)\, e^{ik_x x_0}$$

where:

$$\omega = 2\pi j / N_t \qquad j = 0,1,\ldots,N_t - 1 \qquad (N_t = \textit{number of samples in } t \textit{ direction})$$

$$k_x = 2\pi j / N_x \qquad j = 0,1,\ldots,N_x - 1 \qquad (N_x = \textit{number of samples in } x \textit{ direction})$$

In the interpolation k_x is fixed, and we interpolate only the cosine.

In all these examples, (Figures 5,6,7), the error has high frequency components, corresponding to spurious impulses in long and negative times, resulting from replicating the data in the DFT.

For the sampling points (all integers), the error vanishes, and there is usually a discontinuity in high order derivatives (for the linear and geometric interpolations in the second derivative, for the spline in the third)

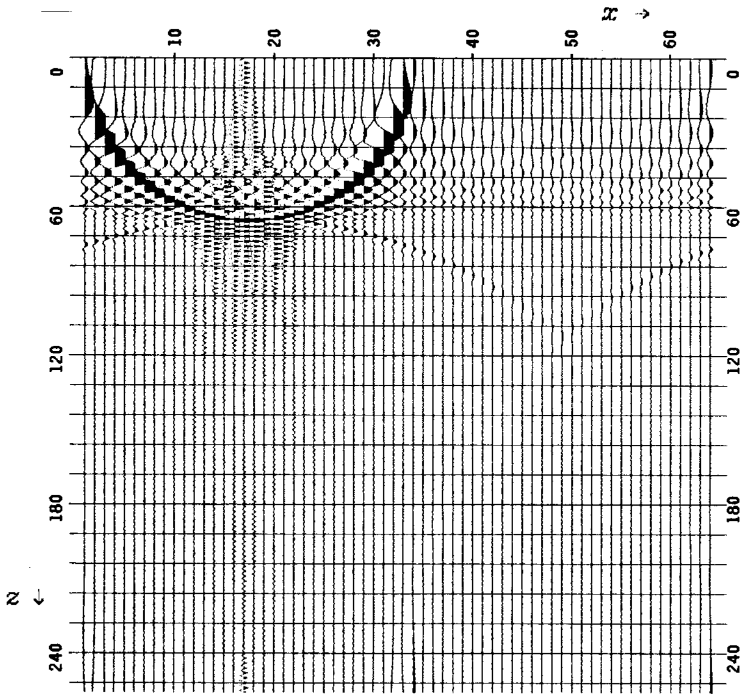

FIG. 2. Migration of an impulse, using cubic spline interpolation. Comparing to linear interpolation (Figure 1), the artifact is weaker, however in the same place.

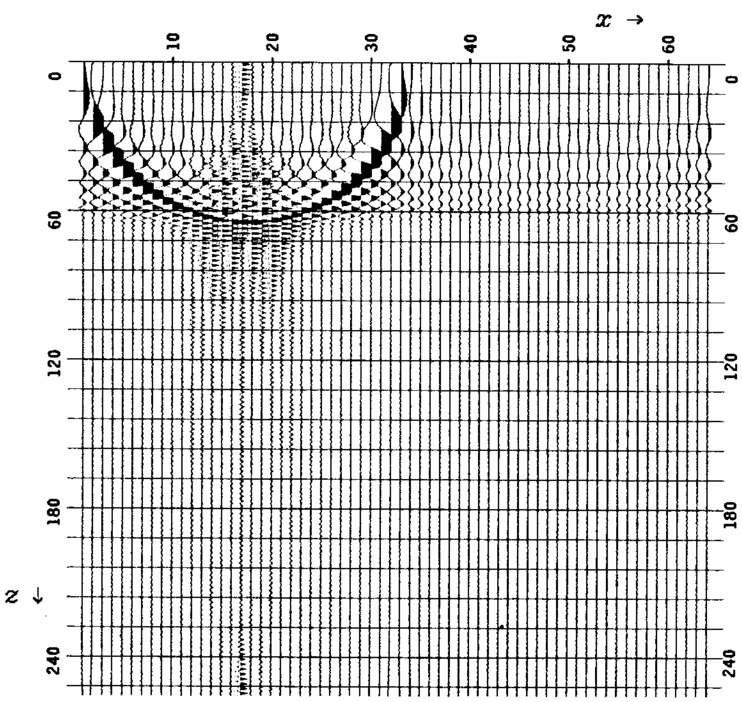

FIG. 3. Migration of an impulse, using geometric interpolation. The spurious events are gone.

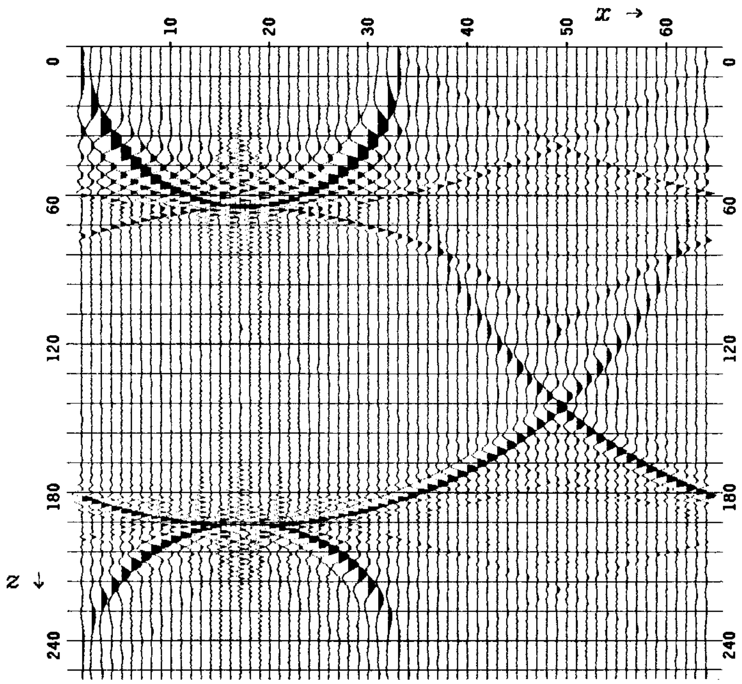

FIG. 4. Migration of two impulses, using geometric interpolation. Not like the single impulse, the transform has changing amplitude for constant k_x. In this case the geometric interpolation is not exact. Comparing to Figure 3, we can see the non linearity of the process.

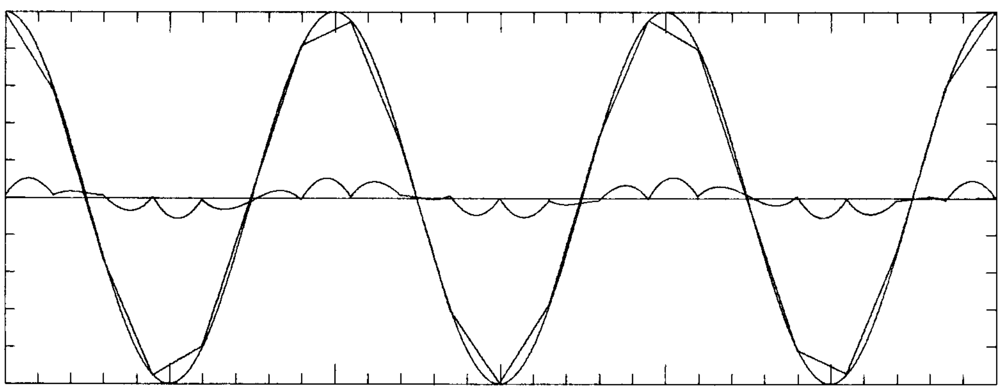

FIG. 5. Linear interpolation: a cosine function, its interpolation, and the error.

Migration of an impulse, using geometric interpolation, does not have the meaning of impulse response, since geometric interpolation is not a linear process. In Figure 4, the migration of the sum of two impulses is not equal to the sum of the migrations of each impulse. Linear interpolation is a linear process, however, cubic spline

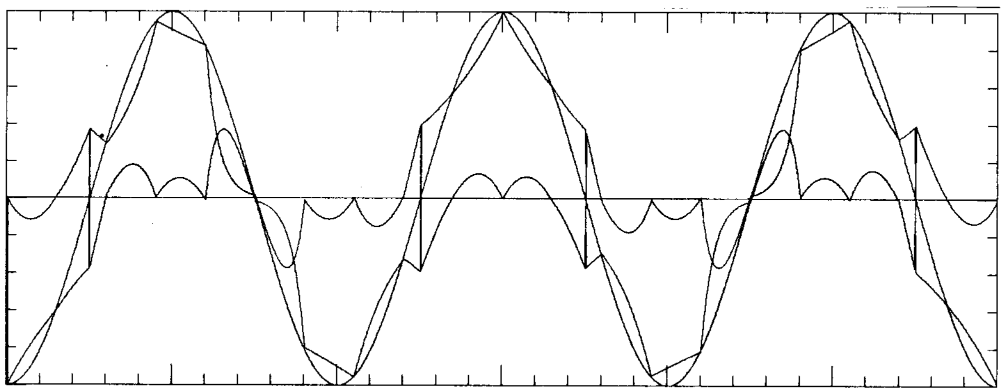

FIG. 6. Geometric interpolation. The interpolation becomes complex when the real interpolated function changes sign. (The phase changes from π to $-\pi$). The absolute value, with the sign of the real part, is plotted.

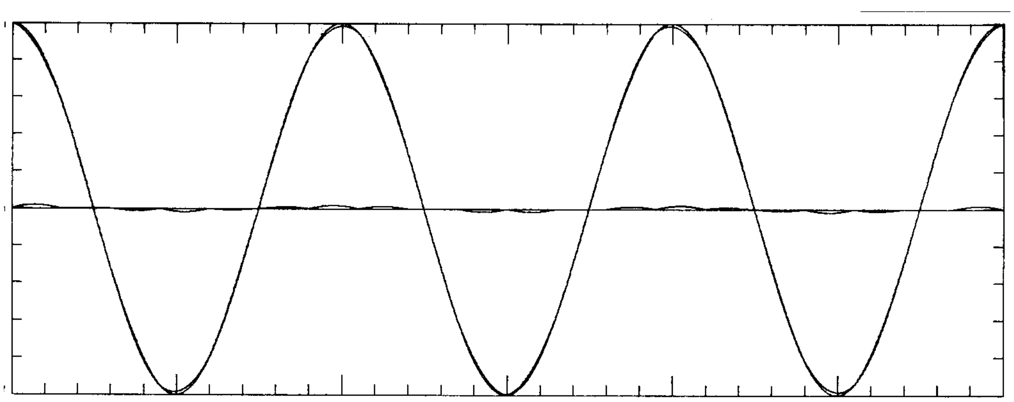

FIG. 7. Cubic spline interpolation: a cosine function, its interpolation, and the error.

interpolation is a linear process only approximately.

One impulse is migrated by geometric interpolation exactly because the Fourier transform has a constant amplitude and a linearly changing phase:

$$DFT\left\{\delta(t-t_0)\right\} = e^{i\omega t_0} \qquad (where\ \omega=2\pi j\,/N_t \quad j = 1,2,\ldots N_t-1)$$

Geometric interpolation of complex numbers interpolates the phase linearly and does not touch the amplitude if it is constant, in this case it is exactly what we need. However, in field data the amplitude is not constant and the phase changes at a variable rate.

Demodulation

The error gets worse when the data includes late events (the phase of the FT oscillates fast). For energy coming at short times, the phase is almost stationary, and it is easy to interpolate. One way to reduce the error is to pad with zeros, thus confining the data to short times.

Another way is to "demodulate" the FT of the data prior to interpolation and modulate it back after, (Claerbout, in class, 1982). By that, we get a slow varying phase of the transform, for data not close to zero time, but close to some later time. Demodulation in the frequency domain, corresponds to shifting backwards in the time domain. It shifts data from a certain late time to zero time. The migration algorithm should look like:

1) Fourier transform:

$$data(x,t) \dashrightarrow data(k_x,\omega)$$

2) Demodulate:

$$data(k_x,\omega) \dashrightarrow data(k_x,\omega)\, e^{-i\omega C}$$

3) Interpolate:

$$data(k_x,\omega)\, e^{-i\omega C} \dashrightarrow data\,[k_x,\omega(k_x,k_z)]\, e^{-i\omega C}$$

4) Modulate:

$$data\,[k_x,\omega(k_x,k_z)]\, e^{-i\omega C} \dashrightarrow data\,[k_x,\omega(k_x,k_z)]$$

5) Cosine correction:

$$data\,[k_x,\omega(k_x,k_z)] \dashrightarrow data\,[k_x,\omega(k_x,k_z)]\,\frac{d\omega}{dk_z}$$

6) Inverse Fourier transform:

$$data\,[k_x,\omega(k_x,k_z)]\,\frac{d\omega}{dk_z} \dashrightarrow model(x,z).$$

This method shifts the time of constant phase from zero to $N_t C/2\pi$. It works (Figure 8), but it does not solve the problem, only reduces it, and it damages the Hermitian symmetry of the transform.

A way to implement this method can be dividing the data to a few parts, each part is a "window" (can be tapered window) at a certain depth. Then, migrating each part separately, using demodulation. Finly, superposing the migrations. When the "windows" get narrow, it basicly turns into a sort of "Kirchoff summation" method.

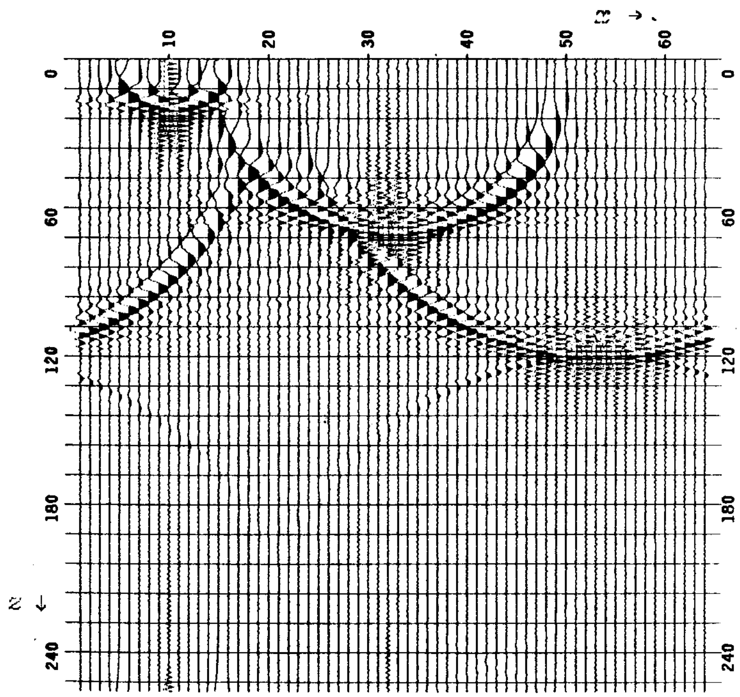

FIG. 8. Demodulation: Migration of three impulses in depth 20,65,120. Demodulating the transform so that an impulse in depth 60 has a constant phase transform. The result is good for the 65 deep event, the spurious events appear in the other reflectors. Linear interpolation was used to get this figure.

REFERENCES

Bracewell R.N., 1978, The Fourier Transform and its application: McGraw-Hill, Inc.

Claerbout J.F., 1977, Migration with Fourier transform: SEP 11, p. 3-5.

------------------, 1980 lecture notes, SEP 25, p. 187-361.

Gazdag J., 1978, Wave equation migration with the phase-shift method: Geophysics, v.43, p.1342-1351.

Lynn W., 1977, Implementing f-k migration and diffraction: SEP 11, p .9-28.

Stolt R.H., 1978, Migration by Fourier transforming: Geophysics v.43 p.23-48.

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34. What number follows logically in this series? 2, 3, 5, 9, 17, ?

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35. Find the number that logically complete this series. 1, 2, 6, 12, 36, ?

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36. What number follows logically in this series? 7, 12, 27, 72, ?

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37. In the square below, a rule applies both from top to bottom and from left to right. Find the rule and figure out the missing number.

Example:	2	7	9
	5	4	9
	7	11	18
	6	2	4
	2	?	0
	4	0	4

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38. Determine the process that was followed in arriving at the prices below and find the price of the last item.

Skirt	$50
Tie	$30
Raincoat	$80
Sweater	$70
Blouse	$?

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39. What is the next number in this series? 21, 20, 18, 15, 11, ?

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40. In a row of four houses, the Whites live next to the Carsons, but not next to the Reeds. If the Reeds do not live next to the Lanes, who are the Lanes' next-door neighbors?

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41. What is a common ten-letter word that can be typed out by using only the top row of letters on a standard typewriter (QWERTYUIOP)?