Weighting and Extrapolation Schemes for Stacking

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Abstract

Among the things that influence the suppression of multiples on a stack, the finite width of the CDP gather, and the corresponding truncation effect, is significant. Truncation of strong multiples at the edge of a gather widens the sidelobes of the multiples in the velocity spectral domain to the point where they may overlap the desired primary components. Analagous to taper and transform methods of spectral estimation, the truncation sidelobes may be suppressed by a suitable weight applied to the gather before stack. The calculation of weights is complicated by the fact that a multiple to be suppressed has a nonlinear moveout. The most effective weights are likely proportional to a power of the local difference in dip between the multiple and the primary. Of course, all this holds for any weak coherent event to be stacked in the presence of a strong event of different dip. Another way to reduce any truncation error present is to extrapolate, weight and stack the data, which is analogous to the role of prediction error filtering in spectral estimation. Problems with spatial aliasing on the gather are not dealt with in this paper.

Introduction -- Problems with Separating Events by Stacking

The operation of stacking is meant to extract a desired event from the gather in the presence of noise and other unwanted coherent events on the gather. Since the stacking operator on NMO-corrected data is a low-pass dip filter, it will give good results when no event other than the primary event to be stacked (or noise for that matter) has any significant zero-dip component. Intuitively, the components of the desired primary lie apart from those of the undesired multiple or noise in a particular transform space. For example, random noise is assumed to be uncorrelated from trace to trace, but the primary is highly correlated across the traces of the raw gather. A second example is one which attention will be focused upon for the remainder of this paper: the case of undesired multiple events in the presence of a weak primary. If all coherent events on a two-dimensional data set display linear moveout, a natural domain is the two-dimensional Fourier transform domain, since the Fourier components of all events line up on discrete lines passing through the origin. As long as energy from one event does not overlap that of another, stacking an event involves picking its spectral components off the Fourier transform plane. On a common-midpoint gather, however, events do not have linear moveout, and selecting a natural domain in which primaries separate from multiples of different moveout is not as simple as that of a Fourier transform. But in principle these transform spaces can be defined. One specific way is to linearize the moveout of all hyperbolic events on a gather by a t^2-x^2 stretch, and then transform into the Fourier domain.

The notion of a "distinct" event on a gather is not so distinct. The human eye seems to easily perceive "distinct" primaries and multiples on a noise-free gather, so the mind must be perceiving these events in the proper transform space, only we don't know what that space is. A good estimate of it may be made, and the additional operations of extrapolation, interpolation and weighting of the data should make the distinct events actually separable.

With the idea that a space exists in which distinct events are separate (or orthogonal), and that stacking is the selection (or projection) of one of these events, there exists an estimate on the goodness of the stack: the less energy from undesired events in the projection window, the better the stack. Consider a data set consisting of coherent events with linear moveouts of different dips, as in figure 1. If it is transformed into Fourier space, these events will interfere with one another because of various artifacts. The spatial bound on the data set introduces side lobes into the spectrum of each event, so that the side lobes from one event will interfere with another when they are separated by only a small dip. Because it is discretized, aliased data will overlap in the Fourier domain. If the traces are unequalized, so that events vary in magnitude from trace to trace, the spectra of these events will have large side lobes attached to them. All these effects contribute to interfering energy in the Fourier domain. In particular, energy from unwanted events will overlap into the low-pass window defined by the stacking operation.

For now, we shall try to compress the unwanted events via extrapolation, weighting or trace equalization so that they "separate" from the primary. The next section illustrates the effects of truncation in the special case of stacking linear-moveout data sets (figure 1). Of course the same types of effects arise in the problem of stacking common midpoint gathers which have events with hyperbolic moveout. In the following sections the data sets will be assumed to be continuous, so that the problem of aliasing will not be addressed. These methods may seem somewhat ad-hoc. Very little physics is involved in the arguments to justify the multiple-suppression techniques described in this paper. What is involved is the fact that multiple events, when they are a problem, are clearly coherent and visible across the common midpoint gather. The coherency of an event from trace to trace simply means that the event as a whole is identifiable and can be discriminated against by suitably designed methods.

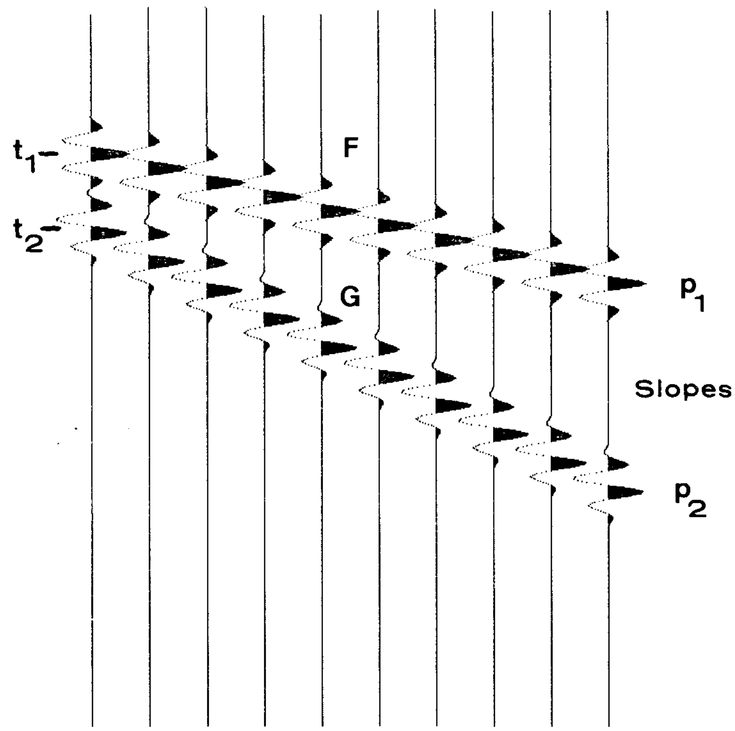


FIG. 1. Data set u comprising two events: f with slope p_1 and intercept t_1, g with slope p_2 and intercept t_2. Each has a distinct waveform and different linear moveout.

Sidelobe Effects in Stacking

This section demonstrates how sidelobe energy arising from truncation influences the stack of a linear moveout gather. The term "linear-moveout" refers to a data set like that of figure 1. In this model two reflections at constant dip are present, one with a signature $f(t)$ and the other with signature $g(t)$. Since aliasing effects are not going to be discussed here, consider the data set $u(x,t)$ to be continuous and bounded in time t and space x. The synthetic gather u in the (t,x) domain and (ω,x) domain is, respectively,

$$u(t,x) = f(t - t_1 - p_1 x) + g(t - t_2 - p_2 x),$$

$$u(\omega,x) = f(\omega)e^{i\omega(t_1 + p_1 x)} + g(\omega)e^{i\omega(t_2 + p_2 x)}.$$

Assume that f is the desired wavelet, and g is meant to be rejected. The stack at slope p_1 for the continuous case is then

$$s(t) = \int_{x_1}^{x_2} W(x)u(t + p_1 x, x)\, dx$$

where x_1 and x_2 are spatial limits of the data set and $W(x)$ is a weighting function normalized to unity:

$$\int_{x_1}^{x_2} W(x)dx = 1$$

Assume the weight $W(x)$ is now defined over all x, but is zero outside the range (x_1,x_2). In the temporal frequency (ω) domain, the stack is

$$s(\omega) = f(\omega)e^{i\omega t_1} + \int_{x_1}^{x_2} W(x)g(\omega)e^{i\omega(t_2+(p_2-p_1)x)}dx$$

The integral in the second term represents the unwanted signal E in the stack $s(\omega)$:

$$E = g(\omega)e^{i\omega t_2}\int_{-\infty}^{+\infty} W(x)e^{i\omega(p_2-p_1)x}\,dx$$

$$= g(\omega)e^{i\omega t_2} W(k_x = \omega p) \qquad \text{where } p \equiv p_2 - p_1$$

Look at the modulus of E in the ω domain:

$$|E(\omega)| = |g(\omega)| \cdot |W(\omega p)|$$

For example, when $W(x)$ is a uniform weighting from x_1 to x_2 (i.e. no weighting at all), the transform $W(k_x)$ is a *sinc* function sampled at $k_x = \omega p$. The shaded area in figure 2 represents the significant pass band of $g(\omega)$, say from ω_1 to ω_2. So a measure of the total unwanted energy in the stack is

$$E \equiv \int |E(\omega)|^2 d\omega = \int_{\omega_1}^{\omega_2} |g(\omega)|^2 |W(\omega p)|^2 d\omega \tag{1}$$

This is to be compared with the energy in the desired signal $\int |f(\omega)|^2 d\omega$. Suppose that no other a priori assumptions about $g(\omega)$ are made other than it is pass-limited between the frequencies (ω_1, ω_2). The weights $W(x)$ can be selected by a minimization of the energy

$$E_w = \int_{\omega_1}^{\omega_2} |W(\omega p)|^2 d\omega$$

which depend on the data only through the parameters ω_1, ω_2, and p. If we define the uniform measure of energy on g as $E_g = \max|g(\omega)|^2$ the unwanted energy E is bounded by $E_g E_w$. Of course one can minimize (1) to determine weights if g has some significant spectral holes that do not change from trace to trace.

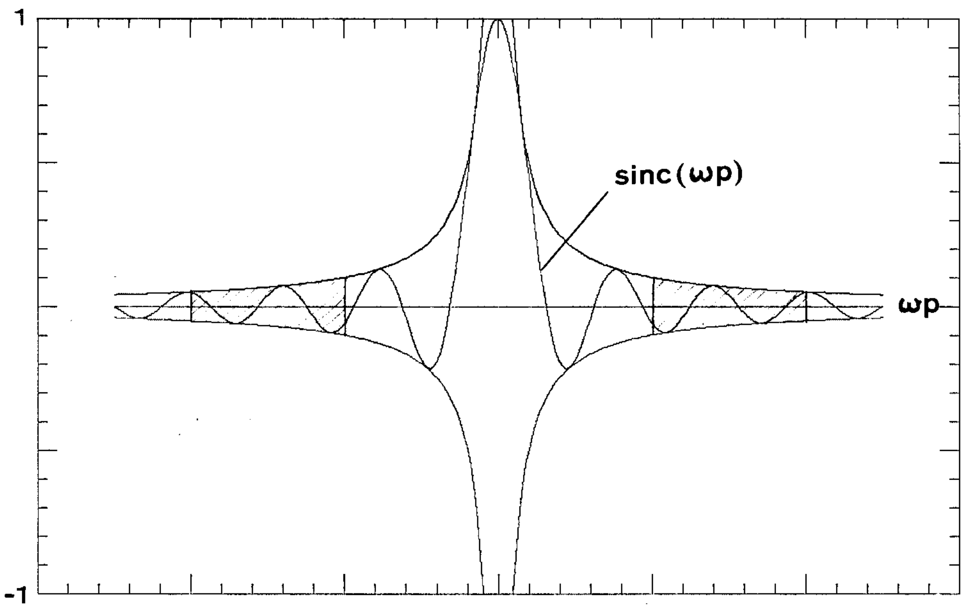


FIG. 2. The weight function $W(\omega p)$ is a sinc function with envelope $1/|x|$. ω_1 and ω_2 are the limits to the frequency components in waveform $g(t)$. As the dip of the g event increases, the farther out on the sidelobes the shaded window shifts. The shaded area is a measure of how much of g is included in the stack.

Now we are at a point where we can make some quantitative statements about how much the event g contributes to the stack. The contribution of g is defined as the truncation error in the stack, and an expression like (1) measures this error. A uniform weight over a data set of width $2l = x_2 - x_1$ has a value of $1/2l$. Its transform $x \to k_x$ is $\sin k_x l / k_x l$ and the measure E_w is

$$E_w = \int_{\omega_1}^{\omega_2} \left[\frac{\sin \omega p l}{\omega p l} \right]^2 d\omega$$

If the interval (ω_1, ω_2) is away from the central lobe of the *sinc* function, a bound on E_w is

$$E_w \leq \int_{\omega_1}^{\omega_2} \frac{d\omega}{(\omega p l)^2} = \frac{1}{(pl)^2} \left[\frac{1}{\omega_1} - \frac{1}{\omega_2} \right]$$

Therefore the truncation error drops off by a power of two as the width l of the data set increases or as the dip p of the event g increases. If the interval (ω_1, ω_2) includes the central lobe of the *sinc* function of figure 2, the envelope $1/p^2l^2$ can be replaced by $1/(1+p^2l^2)$ and approximately,

$$E_w \le \tan^{-1}\omega_2 pl - \tan^{-1}\omega_1 pl$$

which also drops off as an inverse power of two with large p or l.

The advantage to extrapolating an event like g before stacking is that its expected contribution to the stack drops off as $1/l^2$ as the width l of the stack increases. The term "contribution" means the comparison of the energy of g with respect to f, and it is assumed that f as well as g is correctly extrapolated. Assume now that f is not extrapolated. When f is very weak in the presence of g, no information about f may be available for the extrapolation. The energy E_f of f in the stack goes as

$$\int_{x_1}^{x_2} W(x) \int_{\omega_1}^{\omega_2} |f(\omega)|^2 d\omega dx = \left[\int_{\omega_1}^{\omega_2} |f(\omega)|^2 d\omega \right] \int_{x_1}^{x_2} \frac{dx}{2l} \equiv E_f \frac{l_o}{l}$$

where x_1 and x_2 are the bounds of the original gather and $l_o \equiv (x_2 - x_1)/2$ is its width. Therefore as the extrapolation widens, the energy of the waveform $f(t)$ in the stack diminishes as $1/l$, while the "noise" energy from the contribution of g diminishes by $1/l^2$. This is an encouraging result, for it claims that in order to increase the contribution of f to the stack relative to g, one need only deal with eliminating the truncation effects caused by g. A uniform weight, which was implied above, is probably a worst-case weight with respect to resolving f. After extrapolating g an arbitrary distance and applying a smooth weight that favors the original data, the decrease in the energy of f on the stack will be minimal.

The arguments above apply to the model of figure 1, that is, for events with linear moveout. In principle truncation will have the same effect on stacks of CDP gathers, because the contribution to the stack due to a truncation is a local effect, and locally events have linear moveout. More will be said below about stacking gathers whose events exhibit hyperbolic moveout.

Weighting Schemes

Whether or not traces are created by extrapolation and added to the gather, the weight used in stacking is important in the resolution of f. The role of weighting is to minimize as much as possible the sidelobes of g where they overlap the components of f in the transform domain, the sidelobes being due to the finite extent of the gather. Now minimizing the overlap of components of g onto f in the (k_x, ω) domain requires the minimization of $\int |W(\omega p)|^2 d\omega$ (equation 1 above). Since the envelope of the weight $W(\omega p)$ decreases with imcreasing argument, the error in the stack depends on the dip p of g as well as ω.

Let us now raise the question: how to design stacking weights for gathers that have had a hyperbolic moveout correction applied to them? In this case, the meanings of f and g are taken to be the following. Waveform $f(t)$ identifies what is to be stacked: an event that has been moveout corrected to zero dip. Ignore any distortion to the waveform due to moveout stretch. Waveform $g(t)$ identifies one of a series of multiples which exhibit a moveout at water velocity. The residual moveout on g after an RMS moveout correction has been applied to f looks something like that of the multiple at 1.5 to 1.8 seconds in figure 3. Intuitively, when the dip of the undesirable event varies over x and t, we want the stacking weights to be larger in the region of higher dips, and lower elsewhere.

This can be made more quantitative in the following manner. Divide the CDP gather into small patches, either overlapping or not. It is reasonable to assume that each patch has events on it with near-linear moveout, which may be stacked separately using some linear moveout function and uniform weights. For the uniformly weighted case, we know that the error (the "bad" component g) in the stack is proportional to $1/p^2$, the square of the dip of g. This is approximately true for dips p that cause the stacking operator to sample the weight function $W(\omega p)$ out on its sidelobes. The stacked trace from each small patch mentioned above can be combined into an overall stack, with the contribution from each patch weighted by a "superweight" W_s. We will make the standard assumption that this weight function is normalized to unity. The objective is to minimize the contribution of g to the total stack. It will be treated as if it has all the characteristics of noise: uncorrelated with the signal f or with itself from patch to patch, having a variance proportional to $1/p_i$ since by definition,

$$\sigma_i^2 \equiv \int g_i^2(t)dt$$

where $g_i(t)$ is the contribution of $g(t)$ to the local stack of "patch" i. But this is the stacking error due to g (eq. 1), and with the assumptions made in the discussion of that equation, let us assume that its variance is proportional to inverse dip:

$$\sigma_i = \frac{c_1}{p_i}$$

Thus the overall stack, after assembling the mini-stacks of each patch, is

$$s(t) = \sum_i W_{si}(f(t) + g_i(t))$$

The energy in the stacked trace is

$$\int s^2(t)dt \ = \ \sum_{i,j} W_{si} W_{sj} \left[\int f^2(t)dt \ + \ 2\int f(t)g_i(t)dt \ + \ \int g_i(t)g_j(t)dt \right] \tag{2}$$

$$= \ \int f^2(t)dt \ + \ \sum_{i,j} W_{si} W_{sj} \int g_i(t)g_j(t)dt \tag{3}$$

since the second term on the right in (2) is thrown away by the assumed independence between f and g_i. That which is to be minimized is the second term of (3), which by the assumption of independence reduces to

$$\sum_{i,j} W_{si} W_{sj} \sigma_i^2 \delta_{ij} \ = \ \sum_i W_{si}^2 \sigma_i^2$$

Minimizing this term by variation of W_{si}, with the constraint that $\sum_i W_{si} = 1$, one gets

$$W_{si} \ = \ \frac{c_2}{\sigma_i^2} \ = \ c p_i^2 \tag{4}$$

As the patches shrink in size, the weights W_{si} take on the function of a standard before-stack weighting function $W(x,t)$. So we have arrived at one criterion for selecting weights before stack which depends on the relative dip separation p_i between events f and g.

Another way to derive weights is by transformation. In theory, an unwanted event can be eliminated by stacking only if it exhibits linear moveout. Thus assume a stretch transformation is available that will flatten out g to a constant dip p_0. The Jacobian of this transformation then represents the weights to apply to the original data before a uniformly-weighted stack is made of it. This is of course equivalent to a uniform stack on a gather with linear-moveout events, which theoretically can best suppress linear-moveout events. For example, the x^2-t^2 stretch on a gather obtained from a stratified earth model will make the events -- primaries and multiples -- linear. Specifically, the Jacobian of the transformation $\xi = x^2$, $\tau = t^2$, is equal to $4xt$. At time t that is far enough away from $t=0$, the relative change in the stretch factor $4xt$ in the time direction may be negligible, while the relative change in the x direction is still significant because the data of the gather extends nearly to zero offset. For the events that are near $x=0$, dip p is transformed into dip p_0 where p/p_0 is proportional to x, which in turn is proportional to the desired weight to be applied. So this rule,

$$W \ = \ cp \tag{5}$$

is different from that of equation (4) by a power of two. In both equations (4) and (5), c is a constant independent of coordinates x and t of the gather.

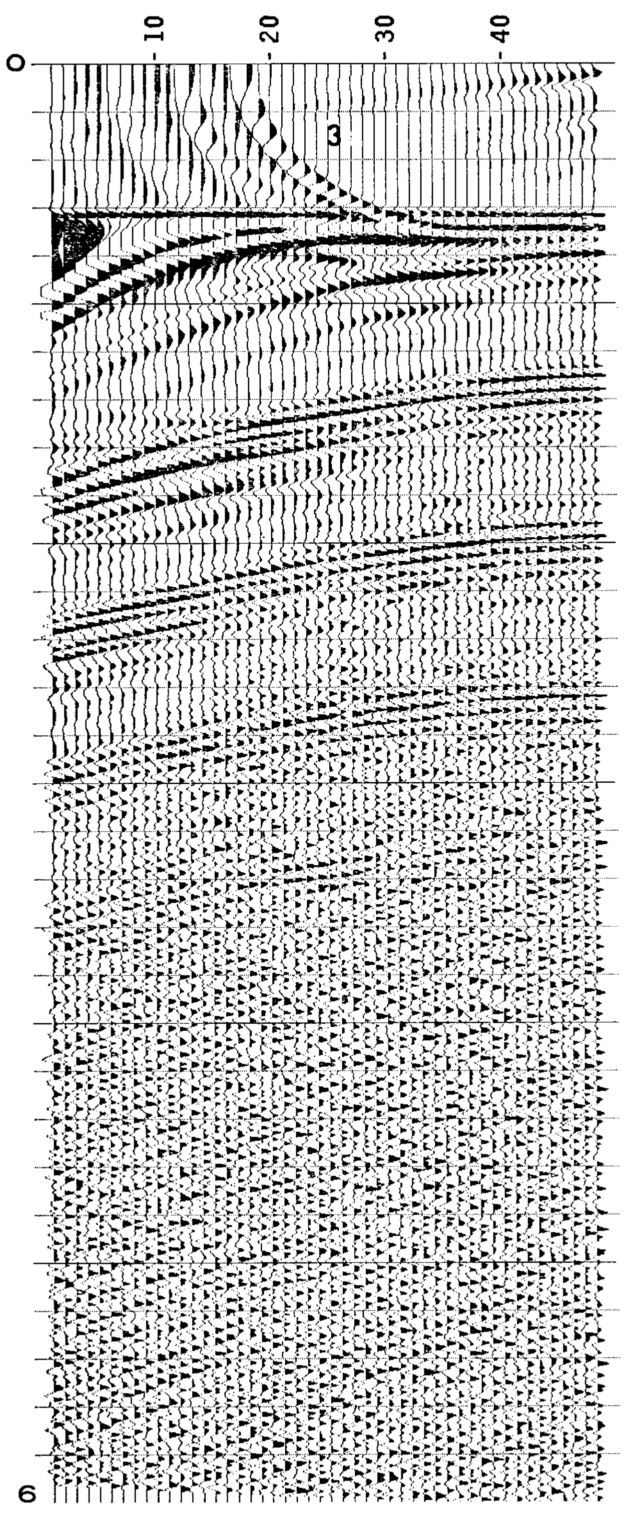
A few observations can be made on the two schemes described above for determining weights on a gather with, say, hyperbolic-moveout events. The first scheme attempts to increase the signal (f) to noise (g) ratio of the stack by assuming that the noise g is independent of f, and independent of itself from patch to patch. That is, there is a significant dip disparity in g between the patches which the data set is divided up into. The second scheme assumes there is a transformation that converts nonlinear moveout to linear moveout. Neither addresses the problem of truncation, so a second weight superomposed to take care of the boundary effects before stack is required. Most importantly, in both cases the weight is determined by the dip separation between events f and g: the greater the dip, the better the discrimination of f against g that is expected in the stack.

There is an important effect that is not being addressed in this paper, and that is the discretization of the data. As a result, events with high enough dips will be aliased, and the arguments for justifying choices of weights like (4) and (5) break down at a high enough dip of g. Intuitively one would want the weight applied to g to taper off to zero when it approaches the point of being aliased. For the stacking examples below, weights in high-dip portions of the gather will be selected to taper off in an ad-hoc manner. One factor that helps us here is that extrapolation and weighting is being performed on a gather moved out by the primary velocity function. Coherent events other than the primary have dips close to that of the primary; this fact reduces the chances of aliasing quite a bit.

Weight and Stack Examples

The field data used here as a test case has been used in previous SEP reports (Thorson, SEP-28 and Morley, SEP-29). Figure 3 is a typical common-shot gather from a survey recorded by GECO in the Barents Sea. Because of the shallowness of the subsurface dips here, the flat seafloor, and a strong desire to avoid CDP sorting, the common-shot gathers from here on are taken to be common-midpoint gathers. This causes the factor of two to disappear from the appropriate moveout equations. The gather of figure 3 is obviously dominated by water layer multiples, but if one sights sideways on the edge of the gather, faint primary events can be seen. The RMS velocity function used to correct figure 3 goes from 1480 m/sec to 2500 m/sec at 4 seconds (Thorson, SEP-28). An advantage to the Barents Sea data set is that the primary events are dipping in the subsurface while the seafloor is consistently flat. The success of a stack is easily visible -- that is, it is how well the dipping primaries stand out on the stacked section above the flat multiple energy.

The Barents Sea data will now be stacked with various weights. Our first weight is based on the criterion of equation (4): weighting is proportional to the dip of the multiple



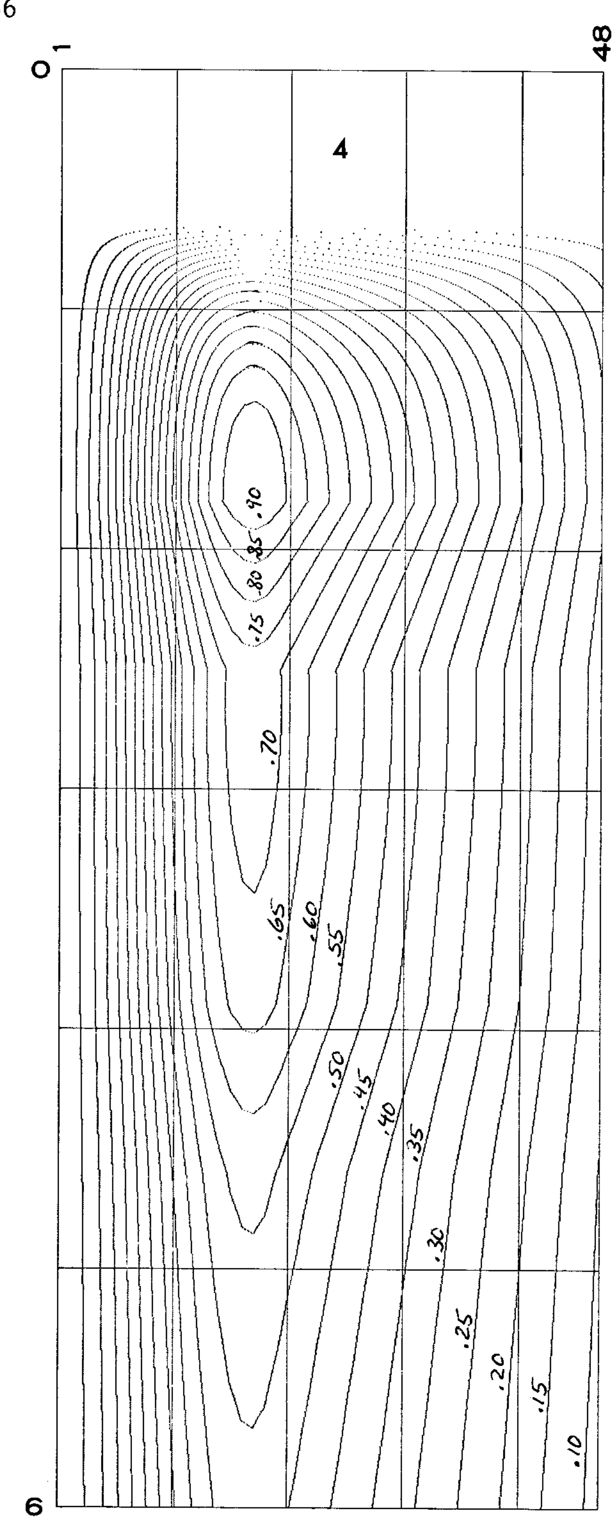


FIG. 3. GECO common-shot gather from the Barents Sea. 48 traces by 6 seconds, this gather is treated as if it were a sorted CDP gather (see text). Moveout correction to the primary velocity has been applied; the velocity function is given in (Thorson, SEP-28).

FIG. 4. Contour plot of stacking weight to apply to gathers like fig. 3. On the near offset side, weights for constant t are proportional to offset h or dip p of the multiple events. On the far offset side, a cosine taper 20 points wide has been applied. Contour interval 0.05.

event on the RMS-velocity-corrected gather of figure 3. Knowing the velocity for the multiple events, v_m = 1480 m/sec, and the RMS velocity function $v_p(t)$ for the primaries, the dip $p(h,t)$ of the multiples can be readily derived. On an uncorrected gather the relationship between the moveout curves of a multiple event and a primary event intersecting at the same point (h,t) is

$$t^2 = t_m{}^2 + \frac{4h^2}{v_m{}^2(t_m)} = t_p{}^2 + \frac{4h^2}{v_p{}^2(t_p)}$$

where t_m and t_p are zero-offset times of the multiple and primary, respectively. After the primary moveout transformation, $t \leftarrow t_p$, the moveout relation for the multiple becomes (v_m being constant):

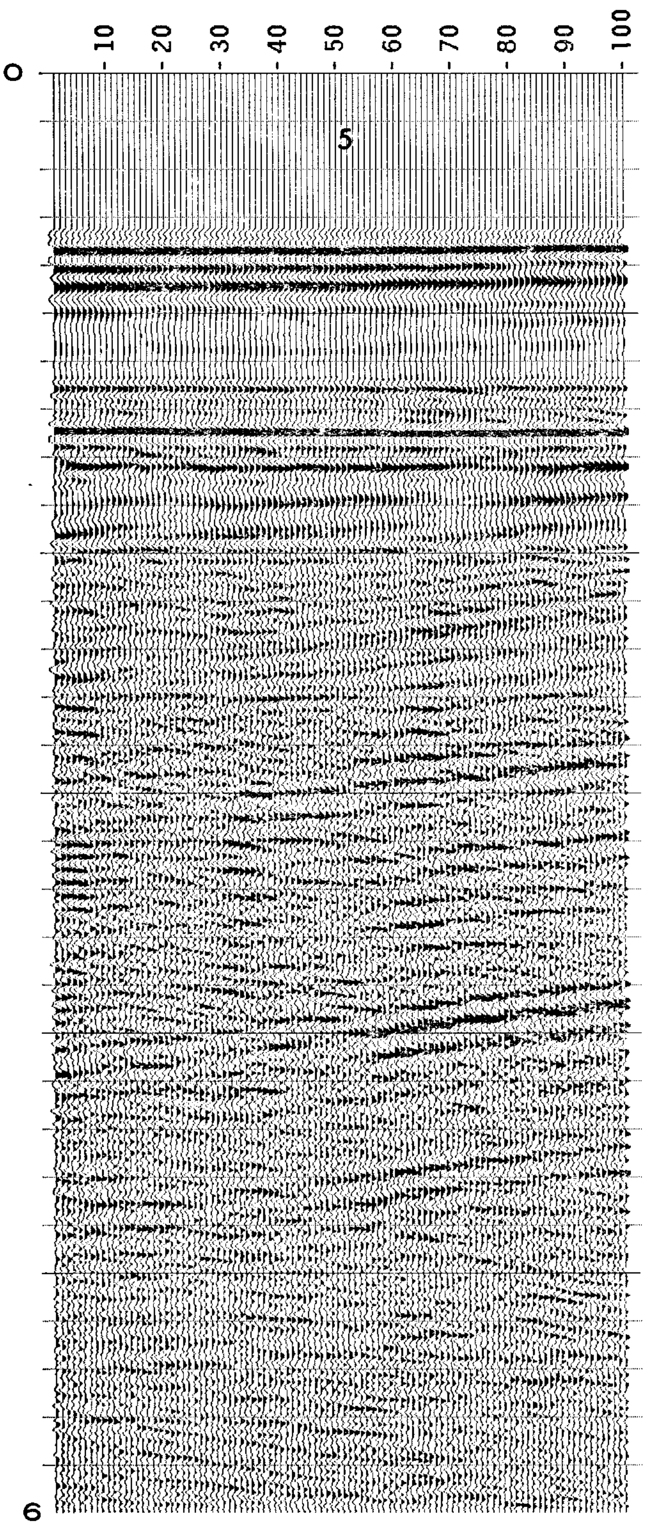
$$t^2 = t_m{}^2 + 4h^2\left[\frac{1}{v_m{}^2} - \frac{1}{v_p{}^2(t)}\right]$$

If the relative derivative v'_p/v_p with respect to t is assumed to be small, then

$$p = \frac{dt}{dh} \approx 4\frac{h}{t}\left[\frac{1}{v_m{}^2} - \frac{1}{v_p{}^2(t)}\right] \tag{6}$$

Dip is proportional to h for fixed t, and our first selection for a weighting function, W proportional to p, is identical to the weighting implied by the Jacobian of the x^2-t^2 transformation discussed in the last section. Equation (6) will now be used as a weight for stacking, with one modification. An additional taper at the far offsets must be superimposed to take care of the truncation at the far-offset edge of the data, and for that matter any aliasing error, though the gather seems not to have much of a spatial aliasing problem. The combined weight function $W(t,h)$ is contoured in figure 4, and a stack of 100 gathers using this weight is shown in figure 5. For comparison, a uniformly-weighted stack of the same data is in (Thorson, SEP-28, p. 314). It is evident that a large contribution from the multiple is present on the stack at 1.5 seconds. This is from the peakiness of the weighting function (fig. 4), whose maximum is approximately at trace 15. Either the h-proportional weighting is too drastic or the ramping at far offsets is, but both combine to give a resultant weighting that seems to favor a small part of the gather.

Utilizing the weight of equation (4), a weight proportional to p^2, doesn't help the problem. A modification of (4) can generate more reasonable weights in the following way. The argument that led to relation (4) assumed that the multiple g contributed uncorrelated noise to the stack with a variance proportional to $1/p$. Let us assume that an additional component of uncorrelated noise is present (more like real noise), whose variance is constant over the gather and is equal to $1/p_0$, where p_0 is some constant reference dip. Instead of



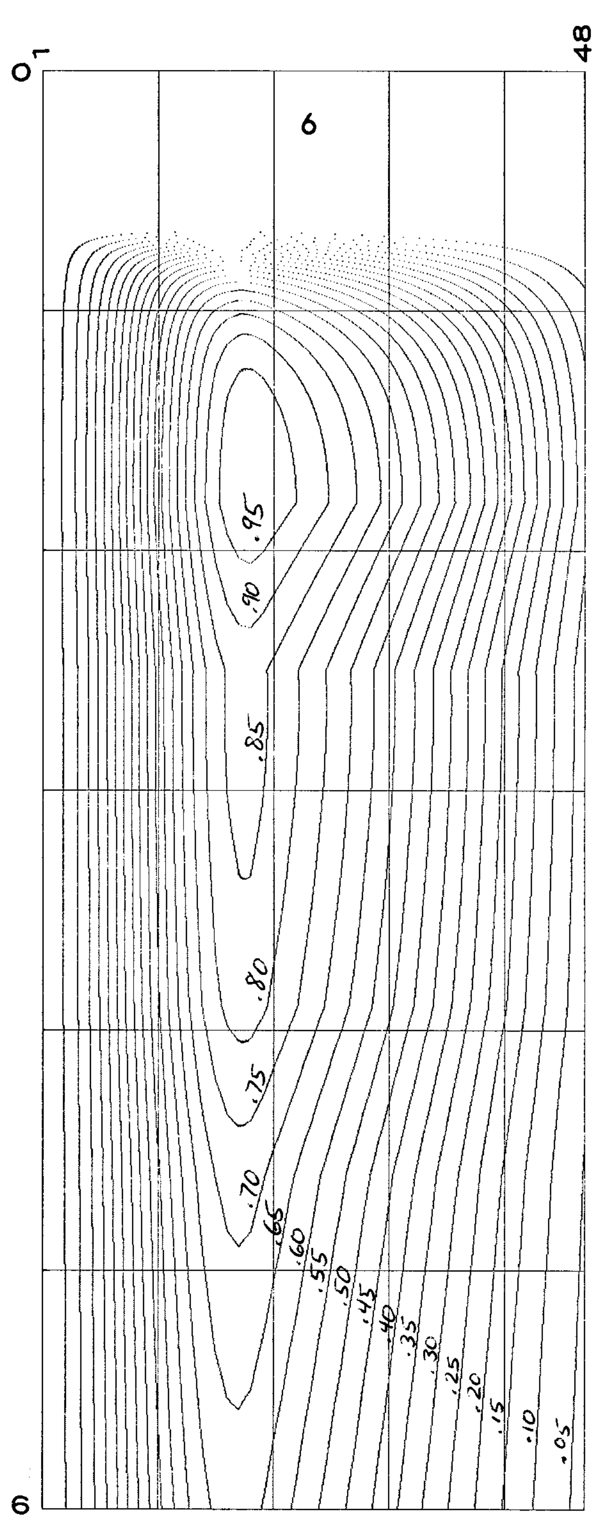
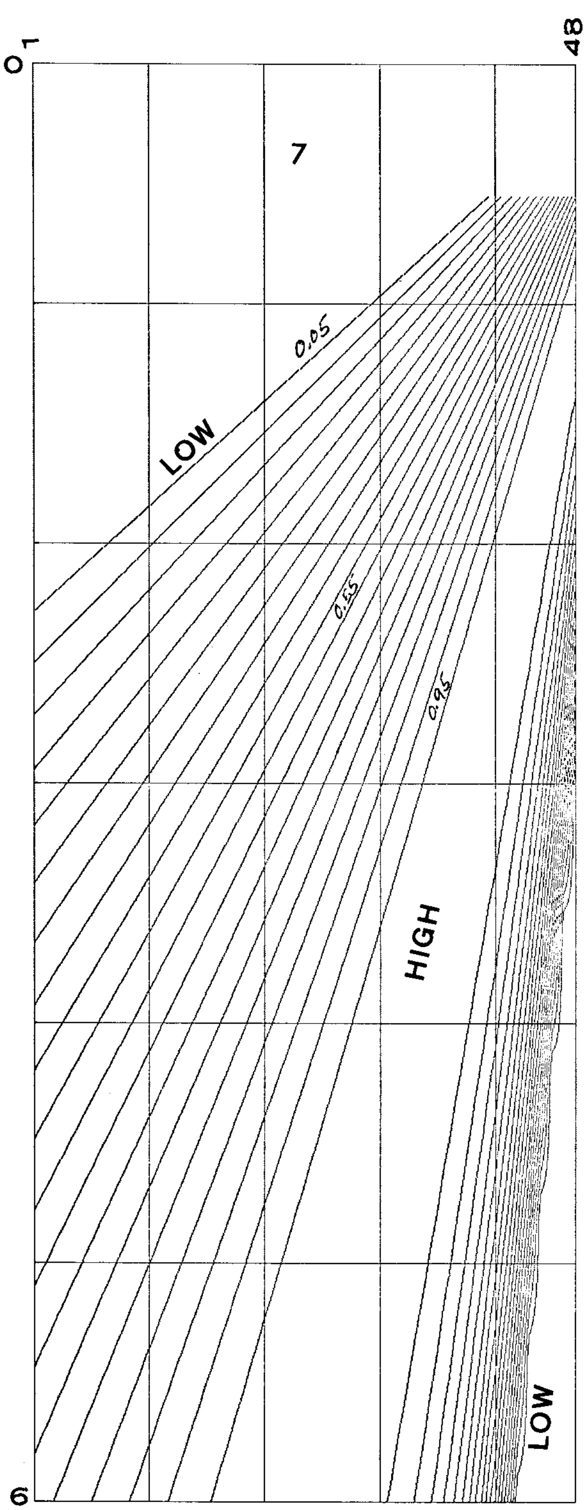


FIG. 5. Stack of 100 gathers using weight of figure 4. Primaries dip to the left while water multiples are flat. The strongest portion of the first multiple comes in a few hundred milliseconds below its zero offset arrival time.

FIG. 6. Contour plot of weight derived from equation 7. Dips p range from 0 to 15 msec/trace; "noise" dip p_o is set to 7 msec/trace. Contour interval 0.05.



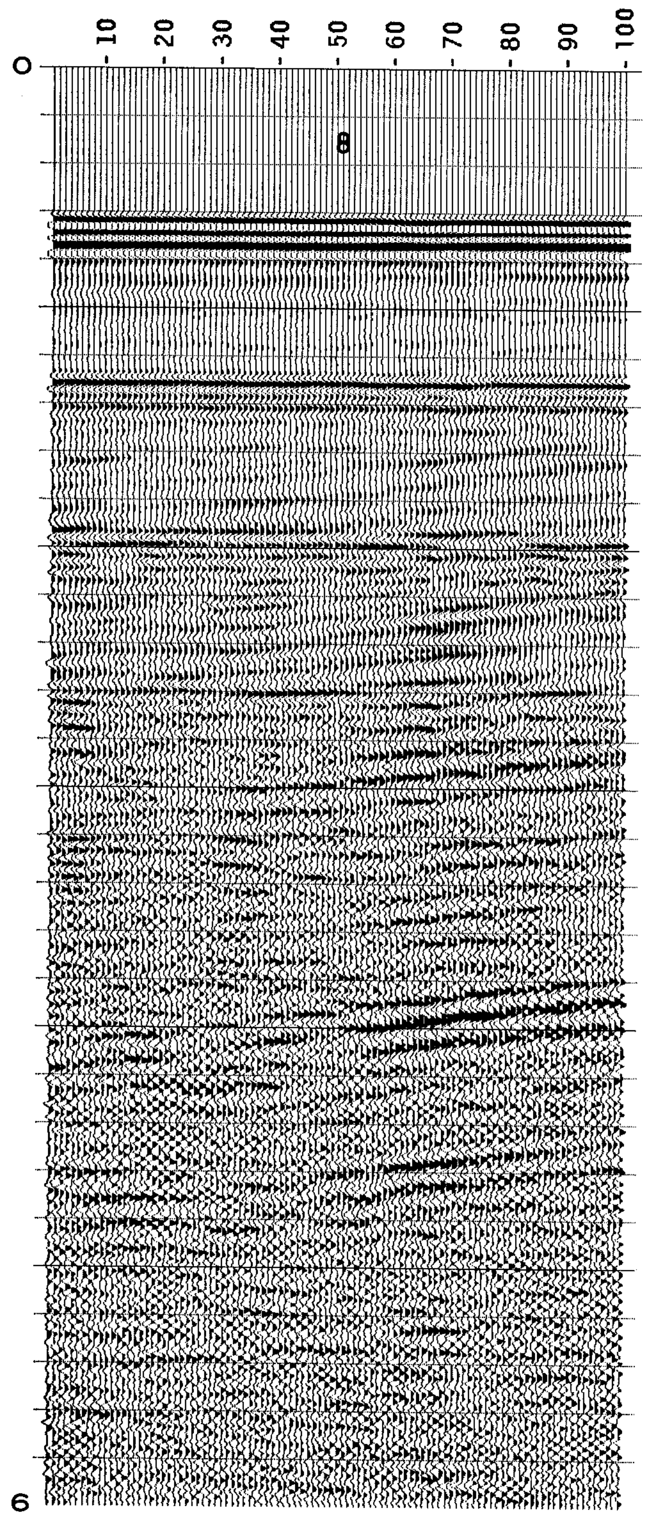


FIG. 7. Contour of weight that is constant along radial lines from $h=0$, $t=0$. The far offsets have a very wide taper applied to them while the near offset weight drops off sharply.

FIG. 8. Stack with the weight of figure 7. In contrast to figure 5, the multiples on the stack are restricted to their zero-offset times, where they make a strong contribution to the stack.

(4), one can then derive as a weight

$$W = \frac{c}{1/p^2_o + 1/p^2} \quad (7)$$

This weight is contoured in figure 6. For the parameters chosen it is identical in appearance to figure 4, and its stack has the same appearance as the stack of figure 5. The far-offset taper as it is used here is inadequate to remove the contribution of the multiple from the stack.

A last example confirms this observation. Figure 7 is the contour of a weighting function that obviously tapers off smoothly the far-offset traces, at the expense of truncating the near-offset events sharply. The corresponding stack (figure 8) shows multiples from about offset trace 15 to be subdued, while the multiples from the near offsets are strong on the stack. An argument for extrapolating on the far traces now presents itself: to allow the broadening of the far-offset taper. Even though this extrapolation is performed on supercritical reflections, extending the far offsets and subsequently weighting them cannot but help the stack in these cases.

It is interesting to note that weighting functions with a similar appearance are estimated by Western Geophysical's "Optimum Weight CDP Stacking" process. That is, weighting increases roughly as the square of the offset, then tapers off at the far offset edge of the data. In this process (Western, 1978), uncorrelated noise and correlated multiples are taken into account in a data-dependent determination of the weights. Western's process does not specifically rely on a hyperbolic moveout function for multiples as this paper does.

Extrapolation, Weighting Schemes

Two final stacking experiments will now be described. The first involves extrapolating traces on both edges of each gather, and then stacking with a p-dependent weight. The algorithm used for extrapolating new traces is the same as that described in (Thorson, SEP-28). Briefly, the object is to extrapolate the dominant multiple events, and since the extrapolation takes place in the frequency domain it is necessary to move out the gather to flatten the multiples. The traces then satisfy a little closer the stationarity requirement for the process. Traces are transformed to the ω domain, and for each frequency a short prediction error filter is calculated on data in the x direction, using in this case only the 12 traces nearest each edge of the gather. The prediction error filter (or its opposite, the prediction filter) can be used to extrapolate 12 steps out. Finally, inverse Fourier transform and moveout-correct the gather to the primary velocity function and it is ready to stack. Figure

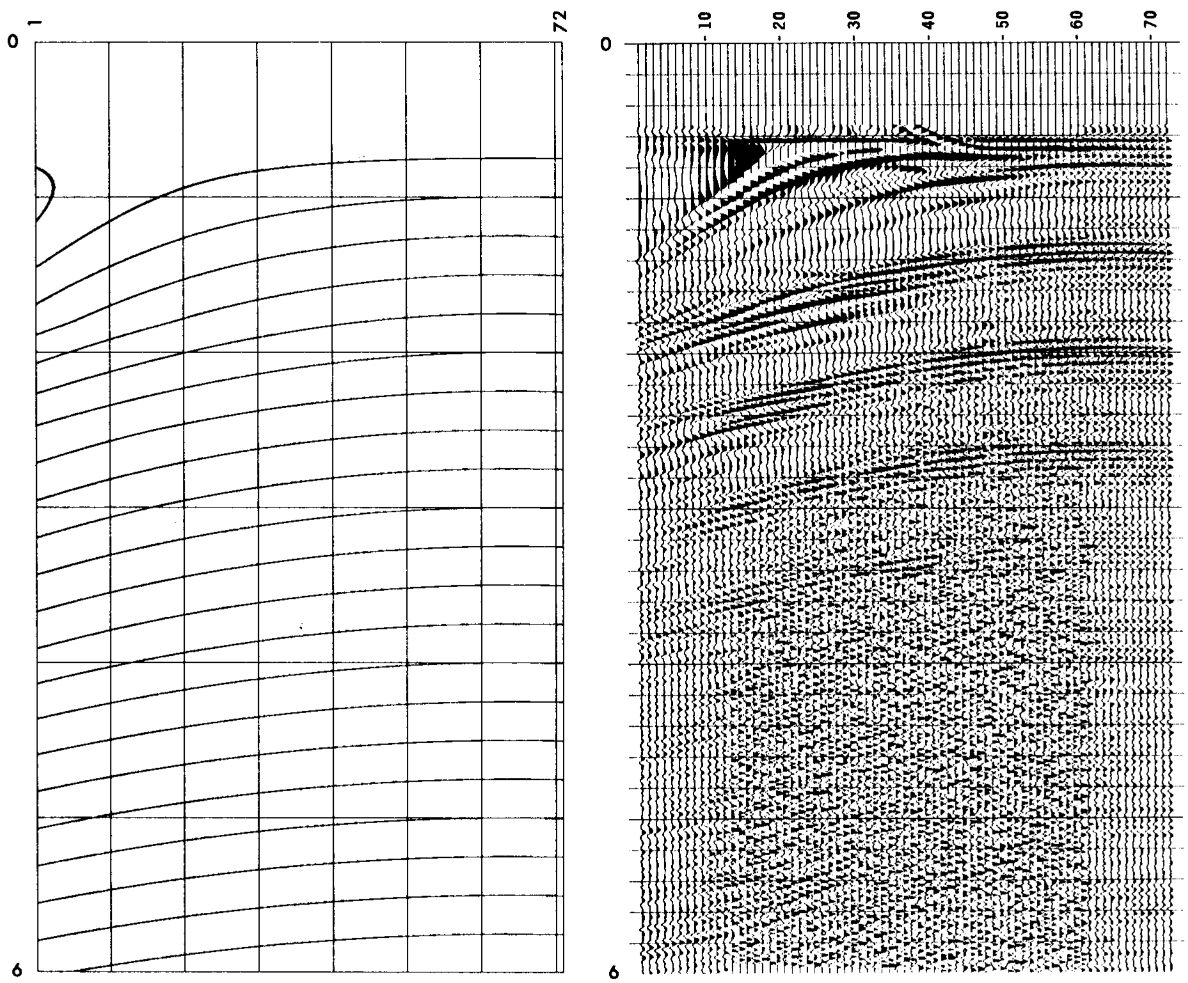


FIG. 9. An extrapolated gather at primary velocity moveout. The original gather of figure 3 was padded on the left and right side with 12 extrapolated traces. The extrapolation was performed by a prediction error filter estimation in the frequency domain (Thorson, SEP-28). The contour plot on the right displays lines of constant multiple-moveout time. The traces were extrapolated while the gather was moved out with respect to multiple velocity.

9 is an example of such an extrapolated gather. The stationarity assumption is a strong constraint on the extrapolation: even though the character of the real traces changes from shallow to deep, the extrapolated traces have a uniform appearance over the time axis.

Figure 11 is a stack of gathers prepared like the one shown in figure 9. The p-dependent weight of equation 7 was used in the stack, with a wider taper now applied to the far-offset traces. The weight still peaks at traces in the center of the gather, resulting

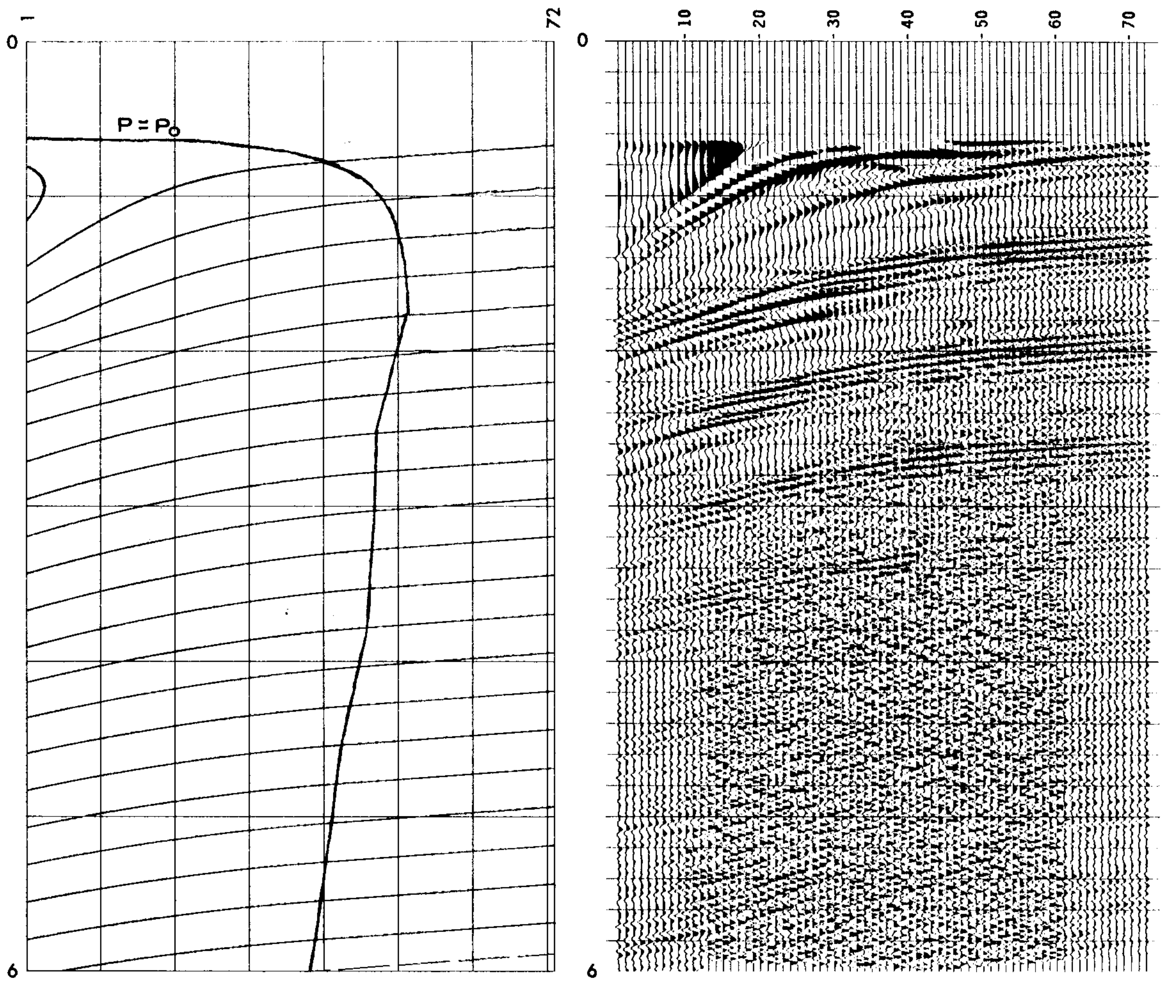
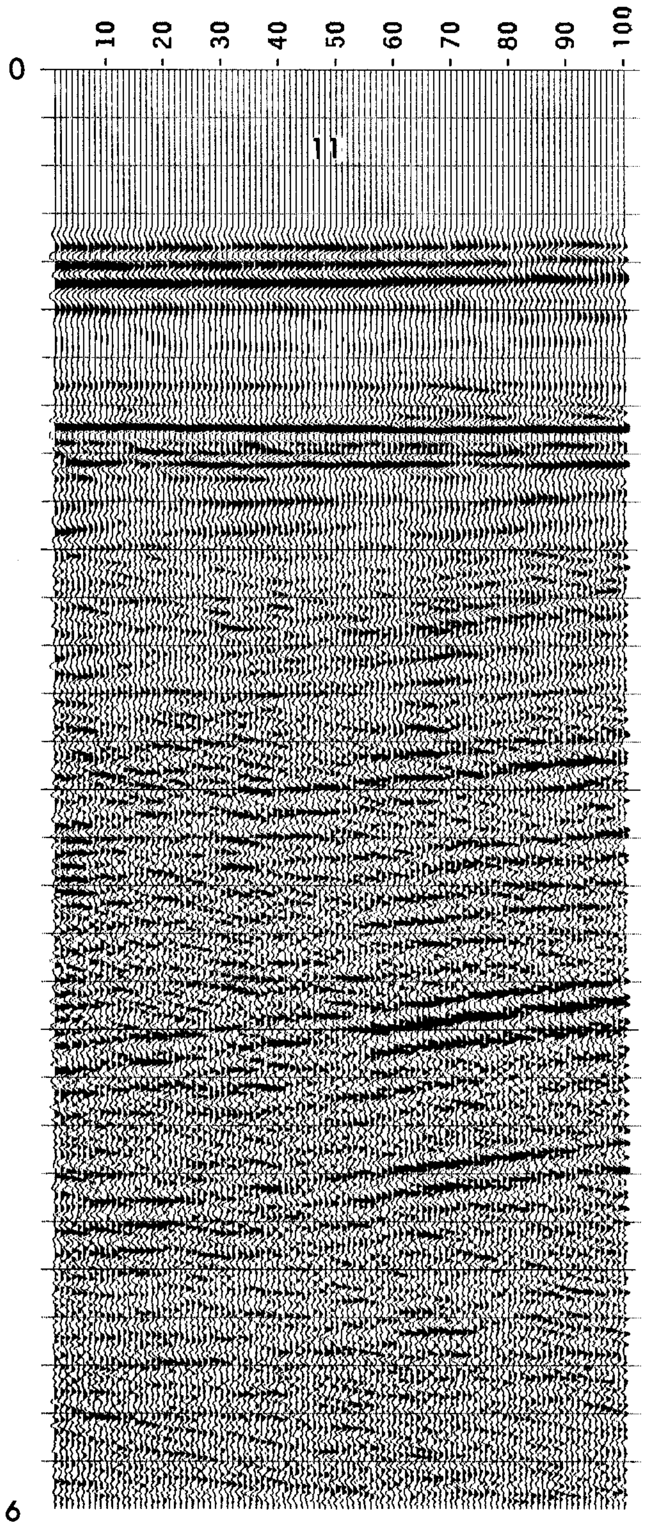


FIG. 10. The same gather of figure 9, but moveout-corrected with a combination of linear moveout and multiple-to-primary moveout. The contour plot on the right represents lines of constant multiple time on the gather (the contour plot overlays the gather). The solid vertical curve through the contours represents the line of constant dip p_o on the multiple events. Here, p_o is 0.0007 sec/m. The area on the left side of the curve has the normal multiple-to-primary moveout of figure 9, while the area on the right side has a constant linear moveout of p_o.

in a strong contribution from the multiples there. Of course the extrapolation on the near offset traces contributes virtually nothing to the stack since the p-proportional weights are so small on the near offsets. The stack is very much like that of figure 5.

Our last example is an attempt to allow the primary events on the near offset traces to contribute to the stack, while at the same time reducing the contribution from the multiples



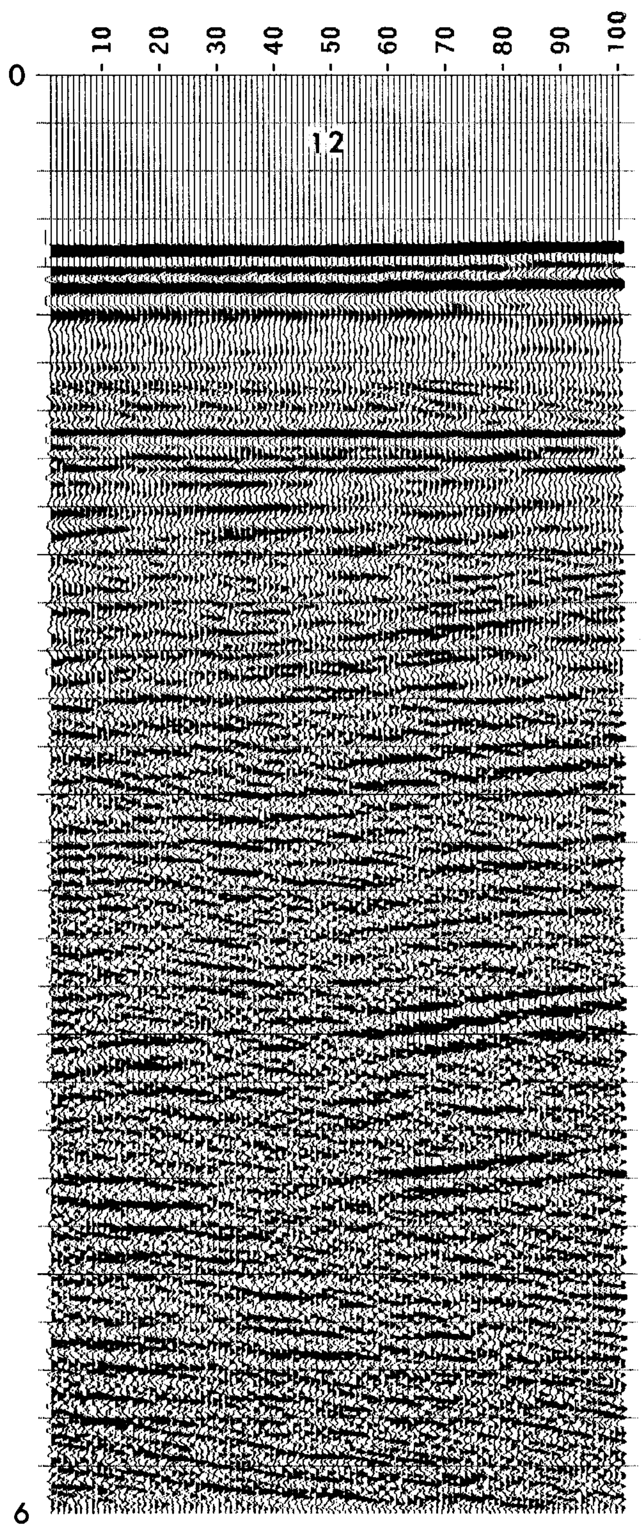


FIG. 11. Stack of 100 extrapolated gathers moved out with primary velocity. See figure 9 for a plot of gather #1. The weight used in stacking is that shown in figure 6, appropriately expanded to 72 traces from 48 traces wide.

FIG. 12. Stack of 100 gathers, extrapolated and moved out with the function of figure 10: linear moveout on the near offsets, and primary velocity moveout on the far offsets. A uniform weight was used, with small tapers on the left and right side of the gather.

at near offset. We shall start with the extrapolated gathers described in the previous example. There, the multiple events can be seen to flatten out completely and actually "turn over" on the negative offsets (figure 9). Rejecting multiples with this kind of behavior requires a stacking weight of zero. This is a drastic weight to apply to primaries as well as multiples. Alternatively, let us try to leave a linear moveout on the near offset traces in order to allow the multiples to destructively interfere with each other upon stacking. We are relying on the fact that on the original data, the zero offset trace is not recorded and there is a small amount of differential moveout between primary and multiple on the near-offset side of the gather. An example of such a hybrid moveout function is shown in figure 10. The dark line on the contour plot represents a curve of constant multiple dip p_0 on a gather corrected for primary velocity. The dip p_0 is a minimum slope allowed the multiple events: to the left of the curve the normal primary moveout function is used, while on the right linear moveout is applied at the constant slope p_0. The plot of the gather in figure 10 should be compared to the same gather in figure 9, on which a normal moveout was applied.

Figure 12 is a uniformly-weighted stack of Barents Sea gathers moved out with the function of figure 10. A uniform weight (with small tapers at each edge) can now be used because the near-offset multiples cancel due to their linear moveout. The stack looks similar to that of figure 11, but with slightly more resolution on a primary at 1.6 seconds. The strong multiples at 1.5 seconds are again from events in the middle of the gather, so at least this method can attenuate the contribution of near-offset multiple events to the stack without having to apply a p-dependent weight.

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