Continuation of Moveout Corrected Radial Traces

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An attractive equation for downward continuing field profiles is found by simultaneous use of

Radial traces
Moveout correction
Muir expansion

Ray methods of downward continuation have always offered the advantage that spatial aliasing is defined with respect to moveout corrected data. This new approach seems to bring the same advantage to wave equation methods. Likewise, the effect of cable truncation should be reduced.

The Coordinate System of Moveout Corrected Radial Traces

The radial coordinate r is defined at the earth surface by

$$r \quad = \quad \frac{2h}{v\,t_{surface}} \quad = \quad \sin\vartheta \tag{1}$$

For brevity of exposition we will henceforth set $v = 1$. Presumably, the theory presented here can be rederived for any stratified velocity. We will first define the interpretation coordinates precisely, then re-express the scalar wave equation in those coordinates. Refer to figure 1.

We start with the wave equation in Cartesian variables, namely (g,t,z). The variables of data interpretation are (r,d,z'). The variable z' is the same as z; it is introduced solely to reduce confusion during partial differentiation. Elementary geometry combining equation (1) with figure 1 gives:

$$g(r,d,z') \quad = \quad (2d-z)\tan\vartheta \quad = \quad (2d-z)\frac{r}{\sqrt{1-r^2}} \tag{2a}$$

$$t(r,d,z') \quad = \quad (2d-z)\frac{1}{\cos\vartheta} \quad = \quad (2d-z)\frac{1}{\sqrt{1-r^2}} \tag{2b}$$

$$z(r,d,z') \quad = \quad z' \tag{2c}$$

Back solve

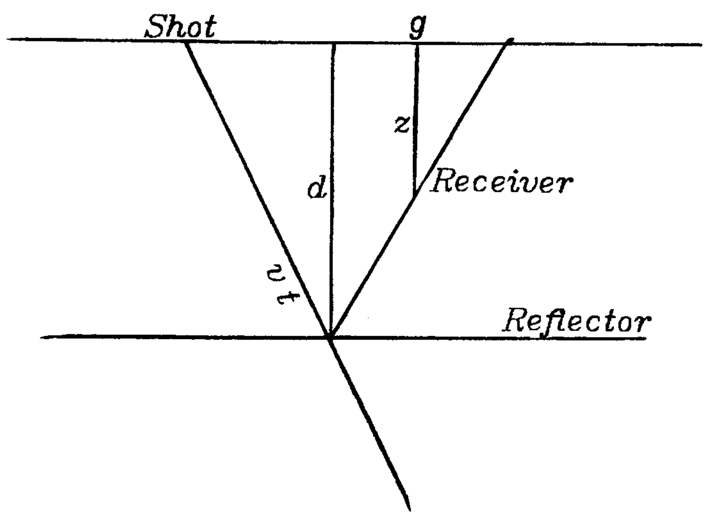

FIG 1. Geometry of downward continued receiver.

$$r(g,t,z) \quad = \quad \sin\vartheta \quad = \quad \frac{g}{t} \tag{3a}$$

$$d(g,t,z) \quad = \quad \frac{1}{2}(z + t\,\cos\vartheta) \quad = \quad \frac{1}{2}(z + \sqrt{t^2 - g^2}\,) \tag{3b}$$

$$z'(g,t,z) \quad = \quad z \tag{3c}$$

Chain Rule

Next we define a mathematical function Q used to represent the wavefield in the radial trace coordinates.

$$P(g,t,z) \quad = \quad Q(r,d,z') \tag{4}$$

The chain rule for partial differentiation gives

$$\frac{\partial P}{\partial g} = \frac{\partial Q}{\partial r}\frac{\partial r}{\partial g} + \frac{\partial Q}{\partial d}\frac{\partial d}{\partial g} + \frac{\partial Q}{\partial z'}\frac{\partial z'}{\partial g} \tag{5a}$$

$$\frac{\partial P}{\partial t} = \frac{\partial Q}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial Q}{\partial d}\frac{\partial d}{\partial t} + \frac{\partial Q}{\partial z'}\frac{\partial z'}{\partial t} \tag{5b}$$

$$\frac{\partial P}{\partial z} = \frac{\partial Q}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial Q}{\partial d}\frac{\partial d}{\partial z} + \frac{\partial Q}{\partial z'}\frac{\partial z'}{\partial z} \tag{5c}$$

Substitute (3) into (5)

$$\frac{\partial}{\partial g} = \frac{1}{t}\partial_r + \frac{-g}{2\sqrt{t^2-g^2}}\partial_d = \frac{1}{t}\partial_r - \frac{\tan\vartheta}{2}\partial_d \tag{6a}$$

$$\frac{\partial}{\partial t} = -\frac{g}{t^2}\partial_r + \frac{t}{2\sqrt{t^2+g^2}}\partial_d = \frac{-\sin\vartheta}{t}\partial_r + \frac{1}{2\cos\vartheta}\partial_d \tag{6b}$$

$$\frac{\partial}{\partial z} = \frac{1}{2}\frac{\partial}{\partial d} + \frac{\partial}{\partial z'} \tag{6c}$$

Wave Equation

The scalar wave equation is

$$\partial_{zz} + \partial_{gg} = \partial_{tt} \tag{7}$$

Substitution from (6) gives

$$\left[\frac{1}{2}\partial_d + \partial_{z'}\right]^2 + \left[\frac{1}{t}\partial_r - \frac{\tan\vartheta}{2}\partial_d\right]^2 = \left[\frac{-\sin\vartheta}{t}\partial_r + \frac{1}{2\cos\vartheta}\partial_d\right]^2 \tag{8}$$

Neglect the gradients of the coordinate frame itself. They just cause slow amplitude variations which get ignored at the end anyway. From here on we can forget about the distinction between z and z'. Cancelation gives

$$\partial_{zz} + \partial_{dz} = -\frac{\cos^2\vartheta}{t^2}\partial_{rr} \tag{9}$$

Re-expressing the coefficient in terms of the independent variables.

$$\partial_{zz} + \partial_{dz} = -\left[\frac{1-r^2}{2d-z}\right]^2 \partial_{rr} \tag{10}$$

Extrapolation Equations

In the early papers, it would be argued that ∂_{zz} was small. If neglected (10) would be a 15 continuation equation. Or the term could be estimated to give a 45 equation. Now of course we use a Muir continued fraction square root expansion. That is, notice that the following recurrence, if it converges, it converges to a form like (9).

$$Z_{(n+1)} = \frac{-\left[\dfrac{1-r^2}{2d-z}\right]^2 \partial_{rr}}{D + Z_{(n)}} \tag{11}$$

The starting value $Z_{(0)} = \partial_z = 0$ is a so-called 5 equation. With this equation, downward continuation requires that nothing be done. But note equation (2a,b) which says that when transforming back from the radial trace coordinates to the physical coordinates, there is a time shift on the d-axis by an amount z.

In the early papers it was apparently not envisioned that the data would be transformed to radial traces. Now we can recognize this as an important simplifying convenience. This is also the first time that we see *both* moveout correction and the Muir expansion.

Since the data is moveout corrected, the r derivatives should be small. The accuracy of the downward continuation is not troubled by having a large product of offset angle times geophone spacing, as would be the straightforward application of the Muir expansion to field profiles.

REFERENCE

Claerbout, J.F. and S.M. Doherty, *Downward Continuation of Moveout Corrected Seismograms*, Geophysics, Vol 37, No 5, October 1972

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1. You are on the island of Princes and Pawns. Princes always tell the truth, and pawns always tell lies. You cannot tell them apart from appearance. You come upon three natives sitting by the sea. You ask the first native, "Are you a prince or a pawn?" He answers, but you could not hear his answer. You turn to the second native, who says, "The first guy said he was a prince, and so am I." The third native speaks up and says, "Don't believe him. Those two are pawns and I am a prince." Which of the natives are princes?

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2. What is a ten digit number that contains all ten digits and is prime?

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3. Is it possible to cut a cube into 27 smaller cubes with less than six straight cuts?

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4. A man was looking at a portrait. Someone asked him, "Whose picture are you looking at?" He replied: "Brothers and sisters have I none, but this man's father is my father's son." Whose picture was the man looking at?

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5. A man was looking at a portrait. Someone asked him, "Whose picture are you looking at?" He replied: "Brothers and sisters have I none, but this man's son is my father's son." Whose picture was the man looking at?

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6. Twenty-four red socks and twenty-four blue socks are lying in a drawer in a dark room. What is the minimum number of socks I must take out which will guarantee that I have at least two socks of the same color?

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7. There are a certain number of red socks and an equal number of blue socks in a drawer in a dark room. It turns out that the minimum number of socks I must pick in order to be sure of getting at least one pair of the same color is the same as the minimum number I must pick in order to be sure of getting at least two socks of different colors. How many socks are in the drawer?

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8. A man owned no watch, but he had an accurate clock at home which he sometimes forgot to wind. Once when this happened he went to the house of a friend, passed the evening with him, went back home, and set his clock. How could he do this without knowing beforehand the length of the trip?