F-K MIGRATION FOR MULTI-OFFSET VERTICAL SEISMIC PROFILES

Robert W. Clayton and Mathew J. Yedlin

Abstract

The Born approximation is adapted to the geometry of vertical seismic profiles, to provide a frequency-wavenumber migration scheme for the reflected portion of the data. The method basically consists of three steps. First, a three-dimensional Fourier transform of the observed data is performed over time, shot location, and geophone location. Second, a two-dimensional stretch is applied to convert the data into a vertical-horizontal wavenumber domain. Finally, a two-dimensional inverse Fourier transform puts the migrated field back into the x-z domain. In the second step, one of the dimensions of the data is redundant, and hence, it is possible to stack along that direction.

Introduction

The basic geometry of a vertical seismic profile is shown in figure 1. The advantage of placing geophones down the hole, rather than keeping them on the surface as with a standard survey is that with the geophones closer to the horizons it is possible to get a more detailed stratigraphic picture. The up-hole times, for example, provide a direct estimate of the local velocities to tie seismic lines to well logs. The farther offsets of the source indicate the trend of the horizons away from the hole.

The data recorded in a vertical seismic profile consist of two parts: transmitted waves which proceed directly from the shot to the receiver, and reflected waves which bounce at least once off the geologic horizons before they are recorded. Inversion of the transmitted waves is basically a tomographic problem which we are not prepared to handle in this paper. The inversion of the reflected waves, on the other hand, is a migration problem, with a slight twist - the shot and geophone axes are orthogonal rather than parallel. We propose a migration scheme based on the Born approximation of the scattering equations (see "An Inversion Method For Acoustic Wave Fields", this report). One of the problems that occurs is that the geophone cannot distinguish between waves scattered from the right of the bore hole and waves scattered from the left. We will solve this problem by exploiting the redundancy in the data.

The Born Approximation

As can be seen from figure 1, we may consider the recorded data field Ψ as functions of the variables z_g, s, and t. [i.e. $\Psi(g=0,z_g,s,z_s=0,t)=\Psi(z_g,s,t)$]. What we now desire is a compact, concise method for extracting the position of our reflectors from the observed data field Ψ. Since the reflectors may be interpreted as scatterers, our objective can be reformulated in terms of scattering theory. These scatterers serve to perturb an outgoing, freely propagating wave field to generate a scattered wave field. Therefore, the observed data are a linear combination of an incident wave field (the direct wave) and a scattered wave field (the reflected wave). Mathematically, the above simple, physical idea may be stated in elegant form using the Born approximation expressed in Dirac notation. In the frequency domain

$$\langle g,z_g|G|s,z_s\rangle = \langle g,z_g|G_0|s,z_s\rangle$$

$$+ \int dx'\int dz'\langle g,z_g|G_0|x',z'\rangle\, \omega^2 V(x',z')\, \langle x',z'|G_0|s,z_s\rangle \qquad (1)$$

In this paper, we are assuming a local scattering potential of the form

$$V(x,z) = \frac{1}{v^2(x,z)} - \frac{1}{v_0^2}$$

where v_0 is a constant background velocity. This means we are effectively treating density as a constant, and consequently neglecting the angular dependence of the reflection coefficients.

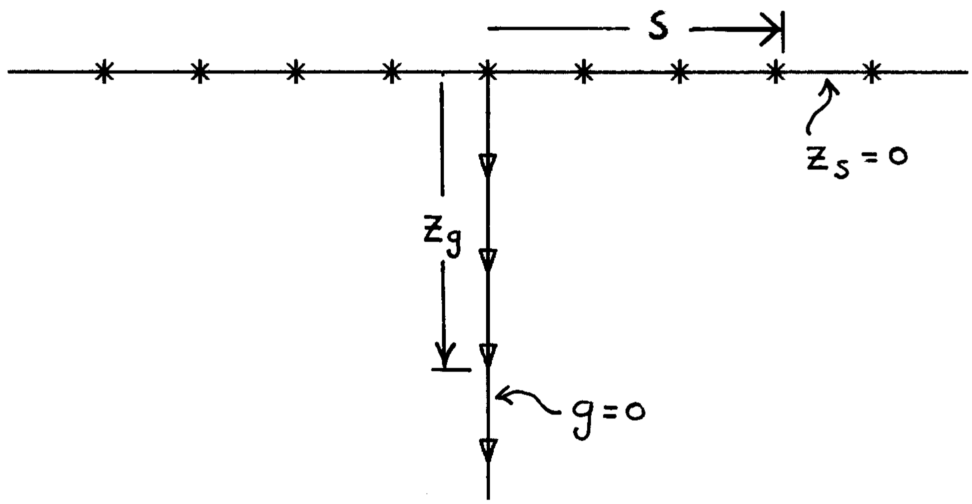

FIG. 1. Illustration of shooting and recording geometry in vertical seismic profiling

The first term in (1) represents the direct wave which has propagated from a shot with coordinates (s,z_s) to a receiver at the location (g,z_g). The quantity G_0 is known as the Green's function of the outgoing freely propagating wave field. It is the impulse response of the medium which has properties that do not vary as a function of any of the coordinates. The second term on the right-hand side of (1) is an approximation to the scattered wave field. The first bracketed term, $\langle x',z'|G_0|s,z_s\rangle$ propagates the wave field from the source to the scatterer at x',z', which has a local strength V(x', z') This scattered wave is then propagated from the scatterer to the geophone at (g,z_g). An integral over all scatterers located at x',z' determines all scattered waves arriving at the receiver. The combination of the two wave fields yields an approximate impulse response, G, for the medium in which the scatterers are imbedded. That is, we've successfully constructed a more complicated impulse response G from G_0, which we already know.

The use of the Born approximation will impose several limitations on the migration method. First, we assume a constant background velocity. Second, we have not included the effect of a free surface, and consequently multiples are not properly treated by the algorithm. Third, since we are using a scattering method, we cannot incorporate the direct wave in the inversion.

Scattering in the Frequency-Wavenumber Domain

With (1) as our paradigm, it will be more convenient to do subsequent calculations in the frequency-wavenumber domain. Let us define our recorded data to include only scattered waves. That is

$$\Psi(z_g,s,\omega) = \langle g,z_g|G-G_0|s,z_s\rangle \tag{2}$$

By equation (1), we may relate Ψ to the local scattering potential

$$\Psi(z_g,s,\omega) = \int dx'\int dz'\langle g,z_g|G_0|x',z'\rangle\ \omega^2 V(x',z')\ \langle x',z'|G_0|s,z_s\rangle \tag{3}$$

Now we do a double Fourier transform over variables z_g and s in (3) to obtain

$$\Psi(l_g,k_s,\omega) = \int dx'\int dz'\langle g,l_g|G_0|x',z'\rangle\ \omega^2 V(x',z')\ \langle x',z'|G_0|k_s,z_s\rangle \tag{4}$$

where k_s, l_g, and ω are the wavenumbers dual to s, z_g, and t. For the geometry of the vertical seismic profile g=0 and z_s=0, so that

$$\Psi(l_g,k_s,\omega) = \int dx'\int dz'\langle 0,l_g|G_0|x',z'\rangle\ \omega^2 V(x',z')\ \langle x',z'|G_0|k_s,0\rangle \tag{5}$$

To proceed, it is necessary to obtain expressions for the Green's operators. This has been done for the right-hand Green's operator (cf. "An Inversion Method For Acoustic Wave Fields", Appendix A, this report) and the result is

$$\langle x',z'|G_0|k_s,z_s\rangle = \frac{i}{(2\pi)^{\frac{1}{2}}}\ \frac{e^{-ik_s x'-i\nu_s|z'-z_s|}}{-2\nu_s} \tag{6}$$

where

$$\nu_s = \left(\frac{\omega^2}{v^2} - k_s^2\right)^{1/2}$$

By symmetry between x and z, we can also find the left-hand Green's operator

$$\langle g, l_g | G_o | x', z' \rangle = \frac{i}{(2\pi)^{\frac{1}{2}}} \frac{e^{-i\eta_g |x'-g| + il_g z'}}{-2\eta_g} \tag{7}$$

where

$$\eta_g = \left(\frac{\omega^2}{v^2} - l_g^2\right)^{1/2}$$

Substitution of (6) and (7) into (5) with g=0 and z_s=0 results in

$$\Psi(l_g, k_s, \omega) = -\frac{1}{2\pi}\int dx' \int dz' \frac{e^{il_g z' - i\eta_g |x'|}}{-2\eta_g} \omega^2 V(x'z') \frac{e^{-ik_s x' - i\nu_s |z'|}}{-2\nu_s} \tag{8}$$

Since there are no scatterers for z' < 0, the modulus sign may be removed. Such is not the case for the modulus of x', since scatterers may be present for x' > 0 and x' < 0. We can remove the modulus sign, however, by writing V(x',z') as V(x',z') H(x') + V(x',z') H(-x'). Then, the right-hand side of equation (8) becomes the sum of two integrals, or

$$\Psi(lg, k_s, \omega) = \frac{1}{2\pi}\int dx' \int dz' e^{-i(k_s x' + \eta_g x')} e^{i(l_g z' - \nu_s z')} \omega^2 \frac{V(x', z')}{-4\nu_s \eta_g} H(x')$$

$$+ \frac{1}{2\pi}\int dx' \int dz' e^{-i(k_s x' - \eta_g x')} e^{i(l_g z' - \nu_s z')} \omega^2 \frac{V(x', z')}{-4\nu_s \eta_g} H(-x') \tag{9}$$

In (9), we recognize each of the double integrals as Fourier transforms of the one-sided potentials, V(x',z')H(x') and V(x',z')H(-x'). Hence,

$$\Psi(l_g,k_s,\omega) = \frac{-\omega^2}{4\nu_s\eta_g}\left[V(-k_s-\eta_g,l_g-\nu_s) * \tilde{H}(-k_s-\eta_g)\delta(l_g-\nu_s)\right.$$

$$\left. + V(-k_s+\eta_g,l_g-\nu_s) * \tilde{H}(k_s-\eta_g)\delta(l_g-\nu_s)\right]$$

$$\equiv \frac{-\omega^2}{4\nu_s\eta_g}\left[J_1 + J_2\right] \qquad (10)$$

with $\tilde{H}$, the Fourier transform of the Heaviside function. From equation (10), it is apparent that the observed wave field depends on the sum of two one-sided scattering potentials (J_1 and J_2), each evaluated along a different shell. To invert this equation, we first make the identification $k_x = -k_s-\eta_g$ and $k_z = l_g-\nu_s$, and implement this change of variables by the stretches

$$l_g = l_g(k_x,k_z,k_s) = \frac{k_z^2-k_x^2-2k_xk_s}{2k_z} \qquad (11)$$

$$\omega = \omega(k_x,k_z,k_s) = v\left[\left(k_x+k_s\right)^2 + l_g^2\right]^{\frac{1}{2}} \qquad (12)$$

In this stretch, k_s is redundant, and is treated as a parameter. An alternate way of implementing the coordinate transformation is to stretch k_s and ω, and treat l_g as a parameter. With the substitutions outlined above, we multiply both sides of equation (10) by $-4\nu_s\eta_g/\omega^2$, and inverse transform over k_x and k_z.

$$\frac{1}{2\pi}\int dk_x\int dk_z\frac{-4\nu_s\eta_g}{\omega^2}\, e^{-ik_xx'-ik_zz'}\,\Psi(l_g,k_s,\omega) = J_1(x',z',k_s) + J_2(x',z',k_s) \qquad (13)$$

The inverse transform of J_1 is simply

$$J_1 = V(x',z')\, H(x')$$

The inverse transform of J_2 is given by the expression

$$J_2(x',z',k_s) = \frac{1}{2\pi}\int dk_x \int dk_z e^{-ik_x x' - ik_z z'} V(-2k_s - k_x, k_z)$$

$$\frac{1}{2\pi}\int dk_x \int dk_z e^{-ik_x x' - ik_z z'} \delta(k_z)\tilde{H}(2k_s + k_x)$$

To evaluate this expression, we first do the integral over k_z, and use the shift theorem to obtain

$$J_2(x',z',k_s) = e^{4ik_s x'} V(-x',z')H(x') \tag{14}$$

We have evaluated each of the terms on the right side of (16) (J_1 and J_2), and can now combine the results to get the final answer

$$I_1 \equiv \frac{1}{2\pi}\int dk_x \int dk_z \frac{-4\nu_s \eta_g}{\omega^2} e^{-ik_x x' - ik_z z'} \Psi(l_g, k_s, \omega) =$$

$$V(x',z')H(x') + e^{4ik_s x'} V(-x',z')H(x') \tag{15}$$

In equation (10), we chose to invert using the shell defined by $k_x = -k_s - \eta_g$ and $k_z = l_g - \nu_s$. We could also have chosen the shell $k_x = -k_s + \eta_g$ and $k_z = l_g - \nu_s$. The result in this case is

$$I_2 \equiv \frac{1}{2\pi}\int dk_x \int dk_z \frac{-4\nu_s \eta_g}{\omega^2} e^{-ik_x x' - ik_z z'} \Psi(l_g, k_s, \omega) =$$

$$V(x'.z')H(-x') + e^{4ik_s x'} V(-x',z')H(-x') \tag{16}$$

Either of equations (15) or (16) will provide the final answer to the inversion problem. In equation (15) for example, both the potential and the Heaviside function are real, and therefore, by identifying real and imaginary parts, we can solve for the left and right halves of the scattering potential

$$V(x',z')H(-x') = \frac{\mathrm{Im}\ I_1}{\sin 4k_s x'} \tag{17}$$

and

$$V(x',z')H(x') = \text{Re } I_1 - V(x',z')H(-x') \quad \cos 4k_s x' \tag{18}$$

We note immediately that no solution exists for k_s=0. This means that the geophone cannot distinguish scattered waves arriving laterally from the right or left, when the incident plane waves are leaving the shot vertically.

Geometrical Interpretation of Theory

As a check on our theory, we can see if the previous transformations, equations (11) and (12), agree with simple geometrical considerations. Consider a particular vertical seismic profile experiment for a given shot-receiver configuration, illustrated in figure 2.

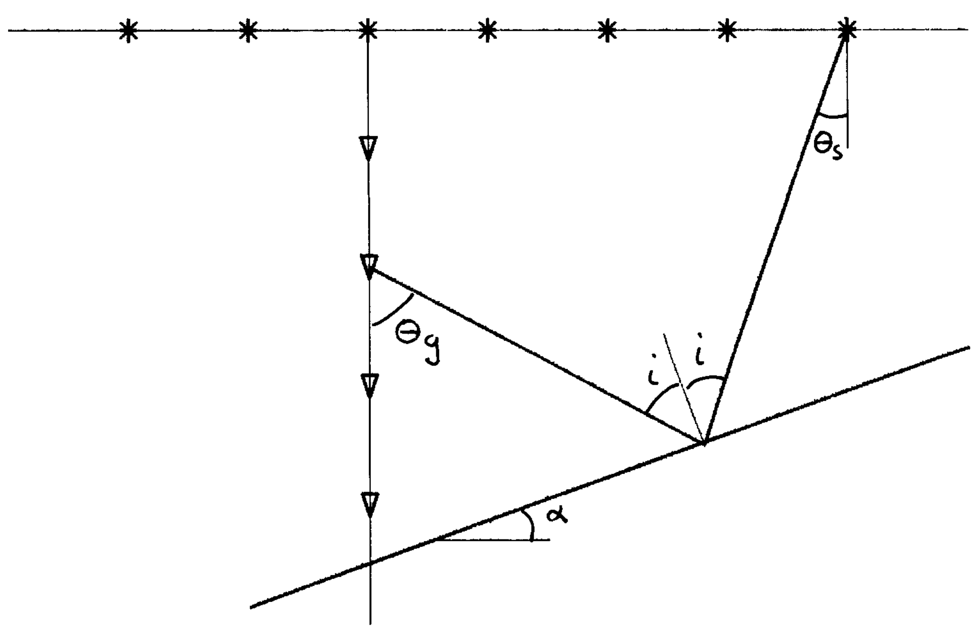

FIG. 2. Illustration of angular relationships of shot and subsurface geophone in the vicinity of a dipping reflector.

Following a development analagous to that taken by Clayton in SEP-20, we find that, by the law of reflection

$$\theta_g - \theta_s = 2\alpha \tag{19}$$

Similarly, if we denote the incident angle by i, then

$$i = \frac{\theta_g + \theta_s}{2} \tag{20}$$

Solving for θ_g and θ_s in terms of i and α, we obtain

$$\theta_g = i + \alpha \quad \text{and} \quad \theta_s = i - \alpha \tag{21}$$

From the coordinate transformations used in the migration, we had defined

$$k_x = -\eta_g - k_s \quad \text{and} \quad k_z = l_g - \nu_s \tag{22}$$

If those transformations are correct, the ratio of k_x to k_z must be the tangent of α, as shown in figure 2. To demonstrate that this is indeed the case, we set

$$k_s = -\frac{\omega}{v}\sin\theta_s \quad \text{and} \quad l_g = -\frac{\omega}{v}\cos\theta_g$$

In the above equations, the negative sign is associated with k_s due to the clockwise sense of θ_s. The same sign is associated with l_g because we are considering waves travelling upwards from the reflector to the geophone. Then, the relations (22) become

$$k_x = \frac{\omega}{v}\sin\theta_s - \frac{\omega}{v}\sin\theta_g \tag{23}$$

$$k_z = -\frac{\omega}{v}\cos\theta_g - \frac{\omega}{v}\cos\theta_s \tag{24}$$

The ratio of (23) to (24) is

$$\frac{k_x}{k_z} = \frac{\sin\theta_g - \sin\theta_s}{\cos\theta_g + \cos\theta_s} \tag{25}$$

Substitution of the relations (21) into equation (25) results in

$$\frac{k_x}{k_z} = \frac{\sin(i+\alpha) - \sin(i-\alpha)}{\cos(i+\alpha) + \cos(i-\alpha)} = \tan\alpha \tag{26}$$

Thus, we know that the transformation defined in (22) will properly image dipping reflectors. The above geometrical argument gives identical results if the defining equations for the other shell are used, with the disposition of shots and receivers interchanged.