Chapter IV

REFLECTIONS DUE TO CONTRAST IN Q

Reflection and transmission coefficients are derived for anelastic materials by matching displacements and tractions across the interfaces just as in the elastic case. The stress at any point in a linear material may be found by convolving the strain with a modulus filter; the requirements of causality and physical realizability are satisfied when the integral of the modulus is an impedance function [Claerbout, 1976]. Specializing to monochromatic plane waves at normal incidence, with an interface at $z = 0$, we have

$$\sigma(\omega) \quad = \quad m(\omega) * \epsilon(\omega) \quad = \quad -\rho c^2 \frac{\partial U(\omega,z)}{\partial z} \tag{4.1}$$

where σ is stress, ϵ is strain, U is displacement, m is the modulus filter, ρ is density, and c is a velocity-like quantity, defined by

$$c^2(\omega) \quad = \quad \frac{m(\omega)}{\rho} \tag{4.2}$$

Equation (4.1), when combined with the equilibrium equation, leads to a wave equation, which has the same form as the usual wave equation, except c enters as a filter in the time domain or as a frequency-dependent complex function in the frequency domain. Plane-wave solutions to the wave equation may be written as the incident, reflected, and transmitted wave displacements:

$$U_i \quad = \quad \exp\left[i\omega\left(t - \frac{z}{c_1}\right)\right] \tag{4.3a}$$

$$U_r \quad = \quad R \exp\left[i\omega\left(t + \frac{z}{c_1}\right)\right] \tag{4.3b}$$

$$U_t \quad = \quad T \exp\left[i\omega\left(t - \frac{z}{c_2}\right)\right] \tag{4.3c}$$

At the interface, $z=0$, continuity of the displacements implies that

$$U_i + U_r = U_t \tag{4.4}$$

or

$$T = 1 + R \tag{4.5}$$

Substituting equation (4.1) into (4.2), and imposing continuity on the stresses, we get

$$\rho_1 c_1 - R\rho_1 c_1 = T\rho_2 c_2 \tag{4.6}$$

This combined with (4.5) gives

$$R = \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2} \tag{4.7}$$

The form of c depends on the particular material. The response of most rocks is well approximated by the constant Q formulation [Kjartansson, 1979], where c has the form

$$c = c_0 \left(\frac{i\omega}{\omega_0}\right)^{\gamma} \tag{4.8}$$

where ω_0 is an arbitrary reference frequency and γ is related to Q by

$$\frac{1}{Q} = \tan(\pi\gamma) \tag{4.9}$$

Substitution of (4.8) into (4.7) gives

$$R = \frac{\dfrac{\rho_1 c_{01}}{\rho_2 c_{02}}\left(\dfrac{i\omega}{\omega_0}\right)^{\gamma_1-\gamma_2} - 1}{\dfrac{\rho_1 c_{01}}{\rho_2 c_{02}}\left(\dfrac{i\omega}{\omega_0}\right)^{\gamma_1-\gamma_2} + 1} \tag{4.10}$$

This shows that when the Q for both media are the same, the reflection coefficient is real and independent of frequency. The power series expansion for the natural logarithm, given by

$$\frac{1}{2}\ln x = \frac{1-x}{1+x} + \frac{1}{3}\left(\frac{1-x}{1+x}\right)^3 + \frac{1}{5}\left(\frac{1-x}{1+x}\right)^5 + \ldots\ldots \tag{4.11}$$

may be used to rewrite (4.10):

$$\frac{1}{2}\ln\left(\frac{\rho_1 c_{01}}{\rho 2 c_{02}}\right) + \frac{1}{2}(\gamma_1-\gamma_2)\ln\left(\frac{i\omega}{\omega_0}\right) = R + \frac{1}{3}R^3 + \frac{1}{5}R^5 + \ldots\ldots \tag{4.12}$$

When R is small we can neglect third and higher powers of R. Then equation (4.12) reduces to

$$R = \frac{\rho_1 c_{01}-\rho_2 c_{02}}{\rho_1 c_{01}+\rho_2 c_{02}} + \frac{1}{2}(\gamma_1-\gamma_2)\ln\left(\frac{\omega}{\omega_0}\right) + i\frac{\pi}{4}(\gamma_1-\gamma_2)\mathrm{sgn}(\omega) \tag{4.13}$$

Thus the reflection may be treated as a sum of two contributions: a real frequency-independent part and a frequency-dependent part that depends on the Q contrast and is similar to a Hilbert transform of the incident wave, except that it is one-sided (causal) in the time domain.

Discussion

McDonal et al. [1958] measured attenuation in water-saturated shale *in situ*. They observed Q values of about 30 for P-waves and 10 for S-waves. The laboratory results of Winkler and Nur [1979] indicate that Q may be an order of magnitude more sensitive than velocity to changes in conditions such as saturation or pore and confining pressures, and that P-wave attenuation in partially saturated rocks may be much greater than in fully saturated or dry rocks. This raises the possibility that a substantial portion of the reflections observed in some areas are caused by changes in Q rather than elastic impedance.