Chapter II
Derivation and Implementation of the Lateral Derivative Velocity Estimation Method

The lateral derivative velocity estimation method (LDM) can be developed using either a ray or a wave theory approach. The first section of this chapter uses the simpler former approach to develop an approximate traveltime equation for reflected energy in laterally varying media. From the traveltime equation it is possible to relate the estimated velocity to an event using conventional velocity estimation techniques with the correct velocity and the lateral derivatives of the correct velocity. The theory developed here is for interfaces which have little or no dip or for data which have been preprocessed by some migration operator (section 4.4) to reduce the dip effects.

The second section discusses the areas of applicability and the expected resolution of the lateral derivative method. The third section describes the implementation of the lateral derivative method. Because, in this method, the effect of the first derivative of the slowness is assumed negligible, the last section will examine the importance of the first derivative of velocity to see how large it must be before it is no longer negligible.

The basic drawback of the ray theory approach is that it is not easily generalized to handle more structurally complex areas where diffracted energy is a problem. Only by going to a wave theory approach can such problems be addressed. This is done in Chapter IV.

2.1. Traveltime equations for laterally varying media

Dix (1955) demonstrated that in a horizontally stratified medium the traveltimes to a given interface are very close to those of a medium in which the raypaths are straight and the velocity is replaced by the root-mean-square (RMS) velocity given by

$$v_{RMS}^2 = \frac{1}{t}\int_0^t v^2(t)\,dt \qquad (2.1)$$

Thus, for a common-midpoint gather, the traveltime to some interface is, to a good approximation, given by the hyperbolic moveout equation

$$t^2 = t_o^2 + \frac{f^2}{v_{RMS}^2} \qquad (2.2)$$

where f is the shot-receiver offset (separation), t_o is the zero-offset traveltime and v_{RMS} is the RMS velocity to the interface. For a stratified medium, the right-hand side of equation (2.2) can be shown to be the first two terms of a higher order expansion in f^2. However, for typical offsets and velocities encountered in seismic reflection work it is accurate to within a few percent (Taner and Koehler, 1969). Equation (2.2) also serves as the basis for nearly all standard velocity estimation procedures. If the traveltimes are known, then v_{RMS}^2 can be measured directly from the slope of the best fitting line in f^2,t^2-space (e.g. Dobrin, 1976). More conventionally, the velocity is estimated by measuring a coherency along hyperbolic trajectories in f,t-space. This method has an advantage over the f^2,t^2 method on low signal/noise data where the picking of traveltimes becomes a difficult task. Moreover, it has the attraction of requiring no human intervention. We will refer to these methods throughout this work as ***conventional*** velocity estimation.

The basic pitfall of conventional velocity estimation techniques is that they fail to acknowledge the fact that the velocity can change within a cable-length. That is, for a given midpoint they attribute the entire traveltime moveout to the velocity at the midpoint when the raypaths are actually sampling the velocity beneath all the midpoints encompassed by the cable-length. Therefore, it is intuitive that for a velocity estimation technique to work in a laterally varying medium it must couple the velocity estimate at a given midpoint with the surrounding midpoints.

The LDM provides this coupling by characterizing the slowness (1/velocity) in the vicinity of every midpoint as a truncated Taylor-series expansion in the midpoint direction. Consider the model shown in

Figure 2.1. Letting the velocity v be a function only of the midpoint direction y, i.e. assuming a ***vertically*** stratified medium, the traveltime to a given offset f can be expressed as

$$t = \int_{y_o-h}^{y_o+h} \frac{w(y)}{\sin \theta(y)}\, dy,$$

where θ is the angle the raypath makes with the vertical, h is the half-offset, and $w(y) = 1/v(y)$ is the slowness. For a first approximation, assume straight raypaths (or constant θ), which yields

$$t \simeq \frac{1}{\sin \theta} \int_{y_o-h}^{y_o+h} w(y)\, dy,$$

or

$$t \simeq \frac{(h^2+z^2)^{\frac{1}{2}}}{h} \int_{y_o-h}^{y_o+h} w(y)\, dy \qquad (2.3)$$

Expanding w(y) into a second order Taylor series in the midpoint direction gives

$$w(y) = w(y_o) + (y-y_o)\, w_y(y_o) + \frac{(y-y_o)^2}{2} w_{yy}(y_o),$$

where w_y is $\partial w/\partial y$, and performing the integration in equation (2.3) yields the traveltime equation

$$t \simeq 2\,(h^2 + z^2)^{\frac{1}{2}} \left(w + \frac{h^2}{6} w_{yy}\right). \qquad (2.4)$$

Thus, to a first approximation, the effect of the odd-order derivatives of the slowness on the traveltime is zero. Extending the Taylor-series approximation out to fourth order would yield a term of order h^4. However, there are also such terms due to vertical stratification which may be significant (e.g. May and Straley, 1979). Since these terms have been assumed to be negligible due to the vertical homogeneity of the model (Figure 2.1), it is unjustifiable to extend the Taylor series

model (Figure 2.1), it is unjustifiable to extend the Taylor series beyond second order unless these terms are included.

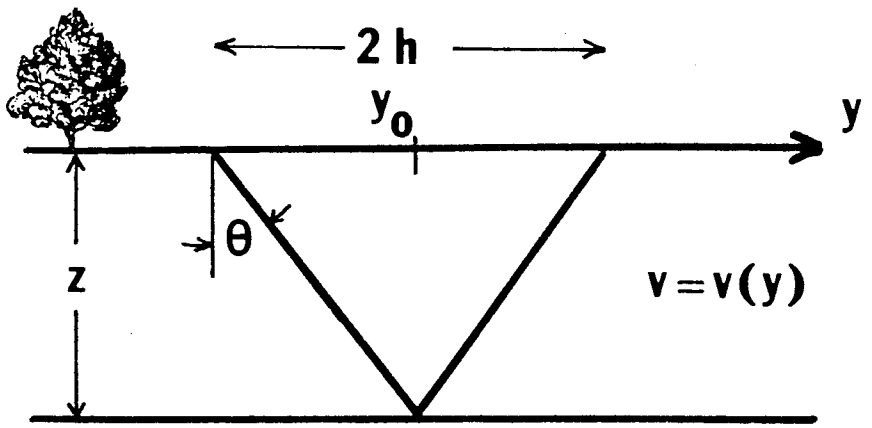


Figure 2.1. Model used for deriving traveltime equations in laterally varying media. The velocity v is assumed to vary only in the midpoint direction y.

Some insight can be gained in equation (2.4) by rewriting it as

$$t = \left[t_o^2 + (2hw)^2\right]^{\frac{1}{2}} \left(1 + \frac{h^2}{6}\frac{w_{yy}}{w}\right) \qquad (2.5)$$

and recognizing the first factor as the traveltime predicted by standard normal moveout. Using the approximation $(1 + x)^{\frac{1}{2}} \simeq 1 + x/2$, equation (2.5) can be written as

$$t = \left[(t_o^2 + (2hw)^2\right]^{\frac{1}{2}} \left[1 + \frac{h^2 w_{yy}}{3w}\right]^{\frac{1}{2}}$$

or

$$t^2 \simeq t_o^2 + 4h^2\left[w^2 + \frac{t_o^2 w_{yy}}{12w}\right] \qquad (2.6)$$

where the h^4 term has been neglected.

Comparing equations (2.2) and (2.6), it is easily seen that the relation between the conventional slowness estimate, w_c, [$1/v_{RMS}$ in equation (2.2)] to the true slowness in a laterally varying medium is given by

$$w_c^2 = w^2 + \frac{t_o^2 w_{yy}}{12w} . \qquad (2.7)$$

Equation (2.7) is the basic equation of the lateral derivative method. The second term on the right-hand side represents the first-order difference between the conventional slowness estimate and the actual slowness in a laterally varying medium. Thus, in regions where $w_{yy} < 0$, w_c will be less than the actual slowness w, and vice versa in regions where $w_{yy} > 0$. Note that the difference between the conventional and actual slowness depends on t_o^2, thus implying that the effect of the lateral velocity variation on the conventional velocity estimate goes as the square of the depth to the event. This fact is observed in the synthetic data in the following chapter.

To verify that the difference is of the correct sign consider the qualitative example shown in Figure 2.2. The upper frame (a) depicts a situation where there is a substantial second derivative of velocity at the midpoint y_o. The velocity is fastest at y_o and decreases on both sides of this midpoint. Consequently, the slowness w is at a minimum at y_o and increases away from y_o, as shown in Figure 2.2b. Thus, the second derivative of w is positive, which implies from equation (2.7) that the conventional slowness estimate will be greater than the actual slowness at y_o; in other words, the conventional velocity estimate will be less than the true velocity at y_o. Figure 2.2c shows that this is indeed the case. Sketched are the true arrival and the arrival for $v = v(y_o)$. Because the true event travels through slower media its arrival time is delayed relative to the hypothetical arrival for $v = v(y_o)$. Thus, the conventional velocity estimate will yield a slower velocity than the actual velocity at $v(y_o)$. Hence, the correction to the conventional velocity estimate due to w_{yy} agrees, at least

qualitatively, with what we expect from equation (2.7).

2.2. *Applicability and resolution of the lateral derivative method*

Having established a relation between the errors in conventional slowness estimates with the lateral changes in the true slowness, two questions of practical interest arise. These are: 1) to what range of earth models does equation (2.7) apply, and 2) what is the minimum lateral wavelength which can be resolved? Stated another way: if the velocity can be characterized as $v_o + v \sin(ky)$, 1) what is the minimum value of v below which the LDM does not improve upon conventional velocity estimation and the maximum value of v above which the method is invalid, and 2) what is the maximum value of k which can be resolved?

The lower limit of the magnitude of velocity variation needed before the LDM becomes useful is clearly related to the errors in picking the velocity estimates. That is, the amount of lateral velocity fluctuation must be greater than the errors in picking the velocity. Thus, it is important to know just how much lateral velocity variation is necessary before it affects the conventional velocity estimate beyond the noise level. To answer this question, first rewrite equation (2.5) as

$$t = t_H + t_{LV}, \tag{2.8}$$

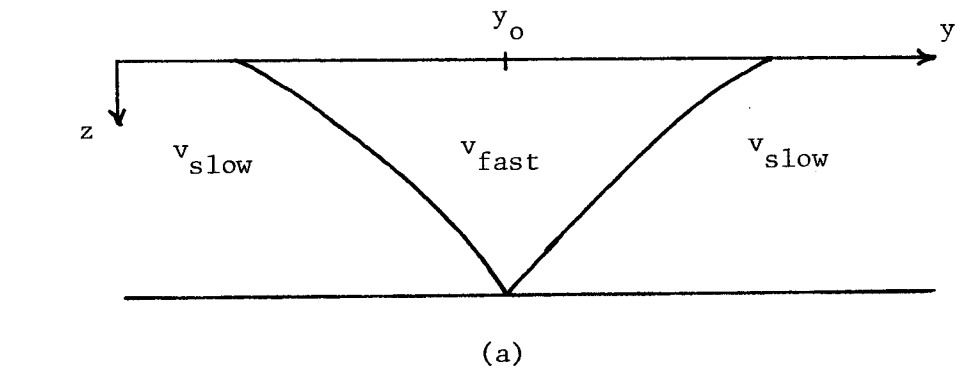
where

$$t_H = \left[t_o^2 + (2hw)^2\right]^{\frac{1}{2}}$$

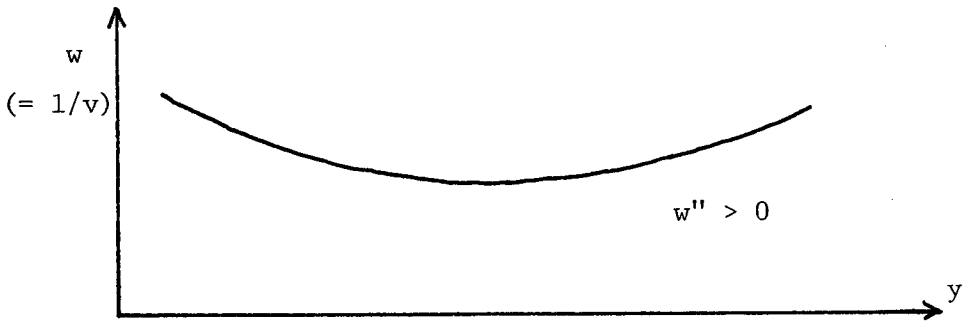
and

$$t_{LV} = \frac{h^2 w_{yy}}{6w} t_H . \tag{2.9}$$

t_H is the hyperbolic traveltime due to the velocity at a given midpoint and t_{LV} is the traveltime correction due to lateral velocity variations.



(a)



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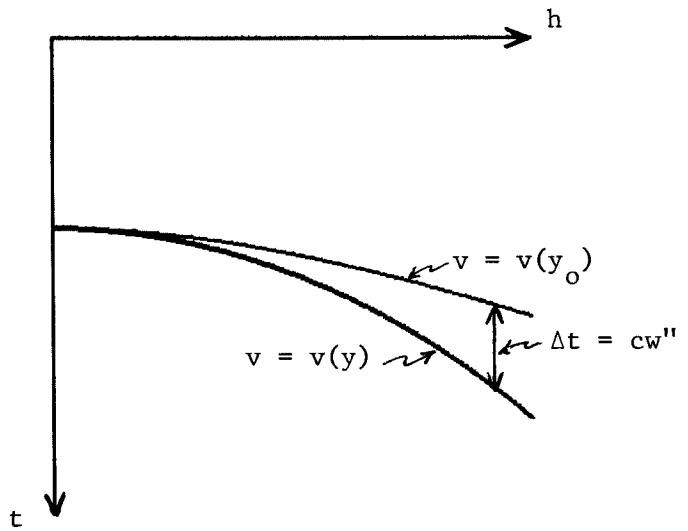

(c)

Figure 2.2. Qualitative verification of the second derivative traveltime correction in equation (2.5). a) An earth model where the velocity is faster beneath midpoint y_o than beneath the surrounding midpoints. Because of this w_{yy} is positive as shown in (b). c) The arrival times of a common-midpoint gather at y_o for the actual situation (bold line) and for the case where the media velocity is constant with the same velocity as that just below the midpoint. Since the media velocity is slower on either side of y_o the true traveltime is greater than the constant velocity traveltime and thus the required traveltime correction must be positive, which agrees with equation (2.5).

Letting the slowness at the midpoints $y \pm h$ be $w + \Delta w$, w_{yy} can be approximated as

$$w_{yy} \simeq \frac{2\Delta w}{h^2} \tag{2.10}$$

and equation (2.9) becomes

$$\frac{t_{LV}}{t_H} = \frac{\Delta w}{3w} . \tag{2.11}$$

To obtain an estimate for the value of Δw needed to produce an error in the conventional velocity estimate equal to the picking error, consider the following example. Let t_H = 2 sec. and assume that the velocity picks are on the order of ±200 ft/sec, which implies a Δt of ~±10 msec at a far offset of 8000 feet. Substituting t_{LV} = 0.010 and t_H = 2 into equation (2.11) yields $\Delta w/w$ = 0.015. For small values of Δw,

$$\frac{\Delta w}{w} \simeq - \frac{\Delta v}{v} \tag{2.12}$$

which implies that a lateral variation of only 1 to 2 percent can yield velocity errors on the order of the error in picking the stacking velocities. For example, for v = 8000 ft/sec, Δv is 120 ft/sec. Typically, velocity picks from coherency contour maps contain an error of at least 200 ft/sec, which is on the order of the minimum observable velocity variations across a cable-length. Thus, it is seen that the lower bound of lateral velocity variations before the LDM becomes useful is when the lateral fluctuations in velocity are greater than the error involved in picking the stacking velocities.

An upper limit for the magnitude of lateral velocity variations is obtained from equations (2.5) and (2.6). Equation (2.5) will be a valid approximation to the traveltime so long as

$$\left| \frac{h^2 w_{yy}}{6w} \right| \ll 1. \tag{2.13}$$

Using equation (2.10) to write w_{yy} in terms of the slowness variation across the cable-length and setting the maximum value of the left-hand side of equation (2.13) equal to 0.1, we obtain

$$\frac{\Delta w}{w} = \frac{\Delta v}{v} = 0.3 \qquad (2.14)$$

This represents about one order of magnitude in velocity variation above the minimum velocity variation which is observable in the traveltimes. However, this relation is subject to the validity of equation (2.6) which depends on

$$\frac{t_o^2 w_{yy}}{12w} \ll w^2$$

or

$$\frac{\Delta w}{w} \ll \frac{6h^2}{v^2 t_o^2} = 1.5 \frac{h^2}{z^2} . \qquad (2.15)$$

Setting the maximum allowable ratio of $\Delta w/w = 0.3$ [from equation (2.14)] implies that the LDM begins to break down at depths greater than one cable-length or roughly 10,000 feet. Thus, the usable range of the LDM is for velocity variations which satisfy

$$0.015 < \frac{\Delta v}{v} < 0.3 \qquad (2.16)$$

and $z < \sim 10{,}000$ feet.

The final question to consider is, what is the minimum resolvable lateral wavelength using the LDM? Writing w(y) in terms of its Fourier components, $W(k_y)$, equation (2.3) becomes

$$t = \frac{(h^2 + z^2)^{\frac{1}{2}}}{h} \int_{-h}^{h} d\sigma \int_{-\infty}^{\infty} W(k_y)\, e^{ik_y(y_o+\sigma)}\, dk_y ,$$

where $\sigma = y - y_o$. Performing the integration over σ yields

$$t = (h^2 + z^2)^{\frac{1}{2}} \int_{-\infty}^{\infty} \frac{\sin(k_y h)}{k_y h} W(k_y)\, e^{ik_y y_o}\, dk_y . \qquad (2.17)$$

Now expanding $\sin(k_y h)$ as a series out to third order gives

$$t = (h^2 + z^2)^{\frac{1}{2}} \int_{-\infty}^{\infty} [1 - (k_y h)^2/6] \, W(k_y) \, e^{ik_y y_o} \, dk_y \, . \qquad (2.18)$$

Inverse transforming equation (2.18) leads to equation (2.4) and thus implies that the second-order Taylor-series approximation is valid so long as

$$1 - \frac{(k_y h)^2}{6} \simeq \frac{\sin(k_y h)}{k_y h} \, . \qquad (2.19)$$

This will be true so long as $k_y h < \pi/2$, or $\lambda_y < 4h = 2f$. Thus, the minimum obtainable lateral resolution is on the order of two cable-lengths.

2.3. Implementation of the LDM

The lateral derivative velocity estimation method, based on equation (2.7), can be implemented in many ways. Because this equation is nonlinear it is necessary to either linearize it or to solve it in an iterative manner. In this section the linearization approach will be considered. We will linearize equation (2.7) in two ways. The first is convenient for examining the stability and some of the other characteristics of the LDM. The second is more accurate and is the one which will actually be implemented.

In order to examine the stability of the inversion process, let

$$w(y) = \bar{w} + \tilde{w}(y), \qquad (2.20)$$

where $\bar{w}$ is constant in midpoint and $\tilde{w}(y) << \bar{w}$. A good value for $\bar{w}$ is the average of the conventional slownesses over midpoint. Substituting equation (2.20) into (2.7) gives

$$w_c^2 = \bar{w}^2 + 2\bar{w}\tilde{w} + \frac{t_o^2}{12\bar{w}} \tilde{w}_{yy}$$

or

$$\frac{w_c^2 - \bar{w}^2}{2w} = \tilde{w} + \frac{t_o^2}{24\bar{w}^2}\tilde{w}_{yy} , \qquad (2.21)$$

where squared terms in $\tilde{w}$ have been ignored. Denoting the midpoint index as j and finite differencing equation (2.21) leads to the tridiagonal system of equations

$$f_j = c_j\tilde{w}_{j-1} + (1 - 2c_j)\tilde{w}_j + c_j\tilde{w}_{j+1}, \qquad j=1,2,\ldots,n \qquad (2.22)$$

where

$$c_j = \frac{t_o^2(j)}{24\bar{w}^2\Delta y^2} , \qquad f_j = \frac{w_c^2(j) - \bar{w}^2}{2\bar{w}} ,$$

and n is the number of midpoints. Because c_j is always positive, the matrix inversion will be generally unstable. This is easily seen by examining the eigenvalues of the matrix (with zero-slope boundary conditions) for the constant coefficient case ($c_j = c$), which are

$$\lambda_j = 1 - 4c \cos^2\left(\frac{j\pi}{2n}\right), \qquad j=1,2,\ldots,n . \qquad (2.23)$$

Hence, if c is not less than 1/4, the possibility of zero or very small eigenvalues arises; consequently, the inversion of the system of equations for $\tilde{w}$ will be unstable. Unfortunately, in practice c will always be greater than 1/4 and so something must be done to stabilize equation (2.22).

One possible way of avoiding the inherent instability is to alter equation (2.22) so that the eigenvalues never go to zero. A means of doing this readily suggests itself by considering the Fourier transform of equation (2.21),

$$F(k_y) = (1 - sk_y^2)\,\tilde{W}(k_y), \qquad (2.24)$$

where

$$s = \frac{t_o^2}{24\bar{w}^2} = \frac{\bar{z}^2}{24}$$

is assumed to be constant, k_y is the midpoint wavenumber, and $\bar{z} = t_o/\bar{w}$. Two plots of the second derivative transfer function between F and W for different values of t_o are shown as the light curves in Figure 2.3. To keep the inversion process unconditionally stable it is necessary to keep this transfer function from becoming zero. To accomplish this consider making an adjustment to equation (2.24) of the form

$$F(k_y) = (1 - sk_y^2 + \alpha k_y^4)\, \tilde{W}(k_y), \qquad (2.25)$$

where $\alpha > 0$. The presence of the k_y^4 term is to alter the transfer function to what is shown as the dark curves in Figure 2.3 (labeled 4th derivative TF).

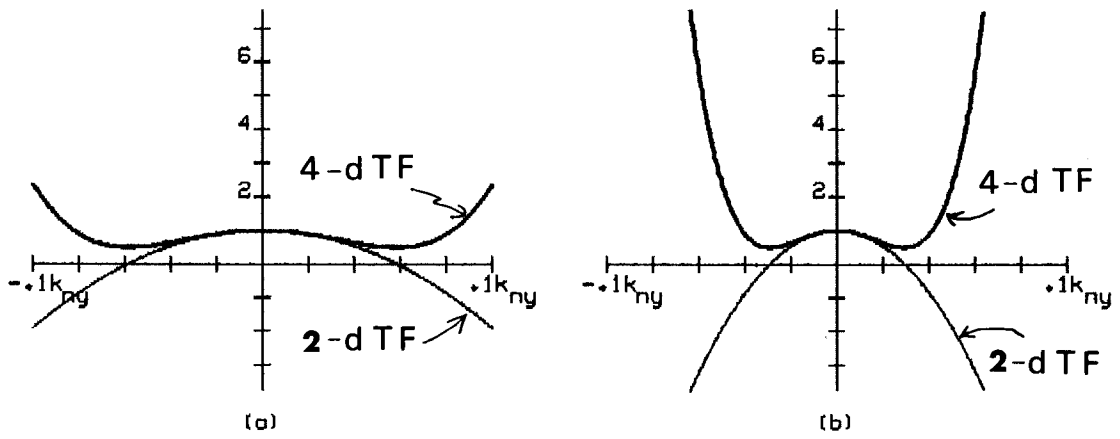

Figure 2.3. Lateral derivative transfer functions for the second-derivative method [light curve; equation (2.24)] and the fourth-derivative method [dark curve; equation (2.25)]. a) $t_o = 1.0$, $\bar{v} = 1/\bar{w} = 8000$ ft/sec, and $\epsilon = 0.5$. b) $t_o = 2.0$, $\bar{v} = 1/\bar{w} = 8000$ ft/sec, and $\epsilon = 0.5$. The limits of the k_y-axis are from $-.1k_{ny}$ to $.1k_{ny}$ where $\Delta y = 300$ feet. A cable-length of 10000 ft would correspond to $.06k_{ny}$ and the resolution limit of two cable-lengths would be $.030k_{ny}$. Note that for large values of t_o (z) the fourth-derivative correction begins to impinge upon the resolution limit.

The constant α depends on the value of the minimum of the transfer function. Denoting this minimum as ϵ, where $0 < \epsilon < 1$, then α is easily determined to be

$$\alpha = \frac{s^2}{4(1-\epsilon)} \tag{2.26}$$

The choice of ϵ is somewhat subjective and is discussed in the following chapter. Inverse transforming equation (2.25) gives the modified equation

$$f(y) = \tilde{w} - s\,\tilde{w}_{yy} + \alpha\,\tilde{w}_{yyyy} \tag{2.27}$$

where s is given in equation (2.24) and α in equation (2.26).

From Figure 2.3 it is seen that the effect of adding on a fourth-derivative term is to attenuate the high wavenumbers. Moreover, the attenuation affects lower wavenumbers for larger values of z ($= t_o/2\bar{w}$). It is easily shown that the fourth-derivative term does not affect the resolution limit of two cable-lengths as long as $z < \sim 1$ to 2 cable-lengths. This is at the upper limit for the applicability of the LDM so no harm is done by adding on the fourth-derivative term.

A final matter to discuss before the alternative linearization of equation (2.7) is the impulse response of equation (2.27). In the frequency domain, the amplitude spectrum of the impulse response, $\tilde{W}_{ir}$, is given by

$$\tilde{W}_{ir}(k_y) = \frac{1}{1 - sk_y^2 + \alpha k_y^4} \tag{2.28}$$

Plots of the frequency and space-domain impulse responses are shown in Figure 2.4. The space-domain impulse response is very similar to a sinc function which comes from the rectangle-function appearance of the amplitude spectrum. The wavelength of the impulse response depends upon the depth to the reflector and increases as it increases.

In order to obtain good results in the deconvolution it is

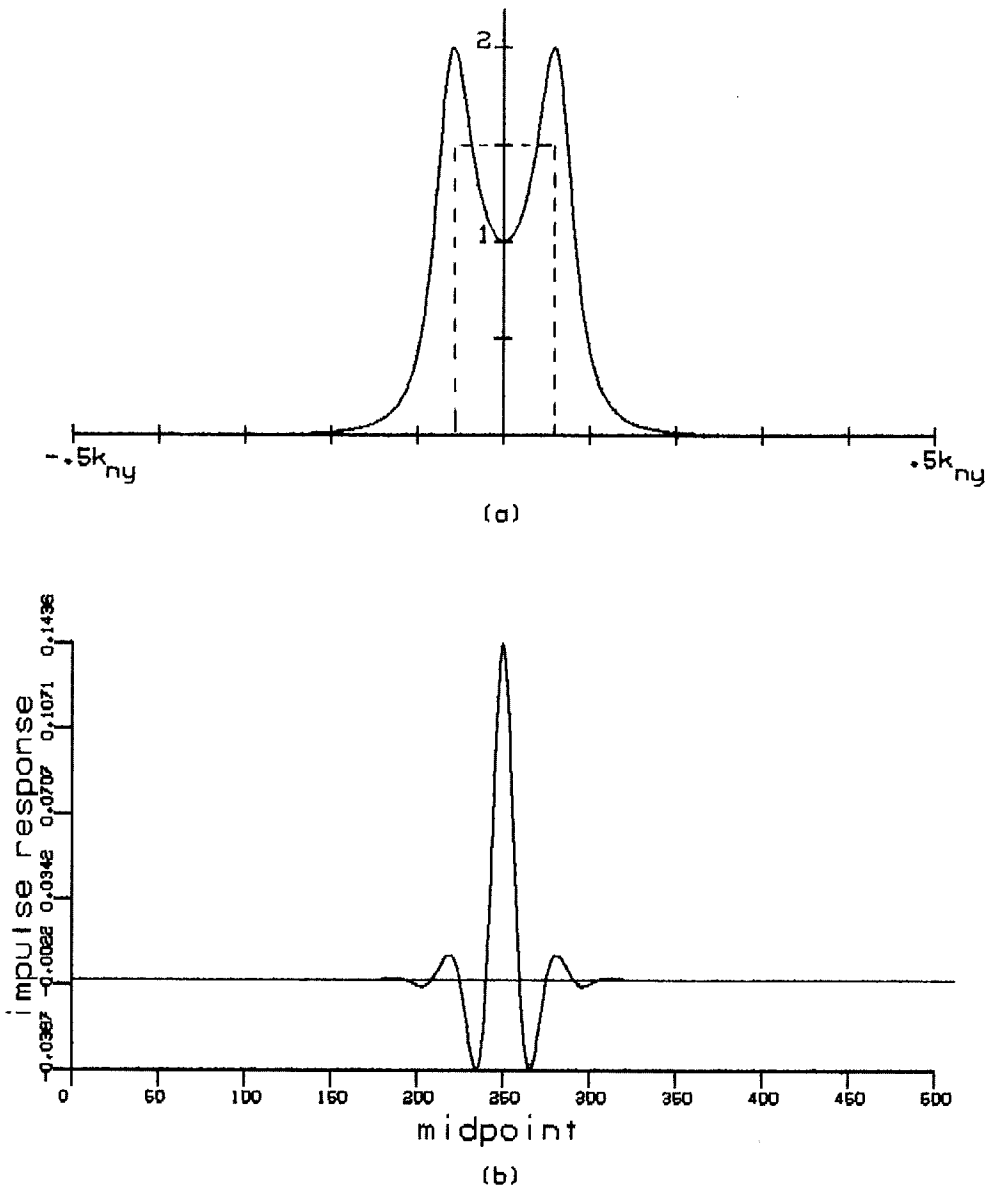

Figure 2.4. a) Amplitude spectrum of the lateral derivative operator [equation (2.28)]. The dashed box represents a crude approximation to the spectrum to determine the dominant wavelength of the impulse response in b. b) Impulse response of the lateral derivative equation (2.28) for an impulse at y = 250. The parameters used are the same as in Figure 2.3a; i.e. t_o = 1.0 sec, $\bar{w}$= 1/8000. sec/ft, and α = 0.5.

necessary to have the length of the dataset be several times as wide as the characteristic wavelength of the impulse response. For a rough estimate we can approximate the amplitude spectrum with the dashed rectangle function as shown in Figure 2.4a. The rectangle function is non-zero between the k_y's corresponding to the maximum lobes of the amplitude spectrum. This yields wavelengths in the sinc-like function on the order of one to two cable-lengths. Thus, to avoid any problems with boundary conditions it is necessary that the section be several cable-lengths long. Moreover, this implies that the lateral velocity results will be invalid near the boundaries of the model unless the model satisfies the boundary conditions. The boundary conditions are arbitrary and for the results presented in the following chapter they are zero first and zero second derivatives.

The validity of the linearization depends upon $\tilde{w} << \bar{w}$. In areas where there is a regional gradient in the actual slowness, keeping $\bar{w}$ constant may invalidate the strong inequality. Therefore, for the actual implementation of equation (2.7), we will use the following linearization

$$w(y) = \bar{w}(y) + \tilde{w}(y), \tag{2.29}$$

where $\bar{w}$ is the conventional slowness estimate. Substituting equation (2.29) into (2.7) yields

$$-\frac{t_o^2}{24\bar{w}^2}\bar{w}_{yy} = \tilde{w} + \frac{t_o^2}{24\bar{w}^2}\tilde{w}_{yy}. \tag{2.30}$$

Using what we have learned from the previous linearization, the implementation of equation (2.30) is straightforward. First, a conventional velocity analysis is done across the section to give $\bar{w}(y) = w_c(y)$. Before computing the second derivative of the conventional slowness estimates for the left-hand side of equation (2.30), it is necessary to first smooth the estimates over at least a cable-length to avoid problems with noise. Moreover, it is best to estimate $\bar{w}_{yy}$ by using a finite-differencing spacing of at least one fourth a cable-length. Next, equation (2.30) is stabilized by adding a fourth-derivative term

as in equation (2.27) to give

$$-\frac{t_o^2}{24\bar{w}^2}\bar{w}_{yy} = \tilde{w} + \frac{t_o^2}{24\bar{w}^2}\tilde{w}_{yy} + \alpha(y)\frac{\partial^4\tilde{w}}{\partial y^4}, \qquad (2.31)$$

where

$$\alpha(y) = \frac{1}{4(1-\epsilon)}\left[\frac{t_o^2(y)}{24\bar{w}^2(y)}\right]^2 .$$

Finite differencing the $\tilde{w}$-derivatives and denoting the midpoint index with the subscript j leads to the following pentadiagonal system of equations

$$-s_j\frac{\partial^2\bar{w}_j}{\partial y^2} = d_j(\tilde{w}_{j-2} + \tilde{w}_{j+2}) + (c_j-4d_j)(\tilde{w}_{j-1} + \tilde{w}_{j+1}) + (1-2c_j+6d_j)\tilde{w}_j, \qquad (2.32)$$

where

$$s_j = \frac{t_o^2(j)}{24\bar{w}^2(j)}, \quad c_j = \frac{s_j}{\Delta y^2}, \quad d_j = \frac{s_j^2}{4(1-\epsilon)\Delta y^4}$$

and $(1,-4,6,-4,1)/\Delta y^4$ has been used as the fourth-derivative operator. The derivative of $\bar{w}$ has been written out explicitly to avoid confusion with the midpoint subscript j. Finally, the system of equations is solved by using a pentadiagonal matrix solver like the one given in the appendix. The following table summarizes the entire algorithm.

Outline of the LDM algorithm

1. Estimate w_c to a given event with a conventional hyperbolic velocity estimation procedure.
2. Smooth w_c over a cable-length.

3. Compute the second derivative of w_c using a finite-differencing spacing of 1/4 of a cable-length.
4. Compute $\alpha(y)$ from $t_0(y)$ and $w_c(y)$ and add on a stabilizing fourth-derivative term [see equation (2.31)].
5. Solve the pentadiagonal system of equations to get $\tilde{w}(y)$.
6. Obtain the total slowness by adding $w_c(y)$ to $\tilde{w}(y)$.

In the next chapter, this algorithm will be applied to three different synthetic models of varying complexity and one field dataset. Before doing this, however, we will first take a brief look at the validity of neglecting the first derivative of the velocity.

2.4. Importance of the lateral velocity gradient

The straight-ray approximation in the derivation of the traveltime equation (2.4) implies that the effect of the first derivative (in the midpoint direction) of the slowness on the traveltime is zero. In this section we will consider the effect of the first derivative of the velocity, v_y, on the traveltime. Although a constant v_y does not imply a constant w_y, it is more convenient to work with the former since an analytic expression can be written for the traveltime (see Dix, 1952, p. 146).

To get an estimate of just how large a gradient is tolerable, consider a model like that shown in Figure 2.5. The velocity v is assumed to be linearly increasing in the +y direction with a gradient of β ft/sec/ft and constant in depth. The straight-ray approximation implies that the estimated velocity at the midpoint y_0 is close to $v(y_0)$ and approximates the true raypaths shown in Figure 2.5a with those in 2.5b.

As an example, consider $v(y_0)$ = 8000 ft/sec and with a constant velocity gradient of β. Figure 2.6 shows a plot of the estimated velocity versus β using a conventional f^2,t^2 method. For β less than .2, the estimated velocity is within 1% of the correct velocity of 8000 ft/sec. The error increases to 5% at β = .5, 10% at β= .8, and 20% at β = 1.1. Thus, for velocity gradients less than, say, .5 ft/sec/ft, we

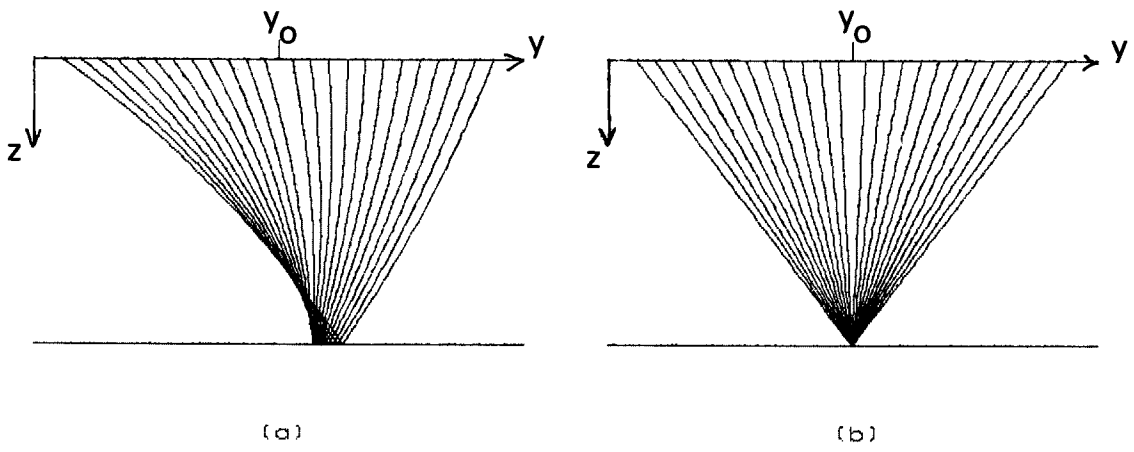

Figure 2.5. Medium with a horizontal velocity gradient. a) True raypaths. b) Assumed straight raypaths by the lateral derivative method.

can expect the lateral derivative effect to be negligible on the traveltime. For example, with a cable-length of 8000 feet, the velocity may differ linearly by 4000 ft/sec at opposite ends of the spread and still a conventional velocity procedure will yield a good approximation to the value at the midpoint.

It is interesting to note that the velocity estimates in Figure 2.6 are always lower than the correct midpoint velocity. This is true for all reflector depths and midpoint velocities. This presents an interesting paradox. If the velocity increases linearly in the midpoint direction above a horizontal interface then the zero-offset (stacked) section will show a dipping event with less traveltime on the faster end of the line. One might interpret the section as a dipping horizon and consequently correct the estimated velocities by multiplying by cos(dip) (see Levin, 1971). However, because of the lateral gradient all the velocity estimates are too low to begin with, as indicated in Figure 2.6. Thus, multiplying them by cos(dip) will further increase the error. The paradox is easily resolved if the velocity analysis is done often enough in the midpoint dimension.

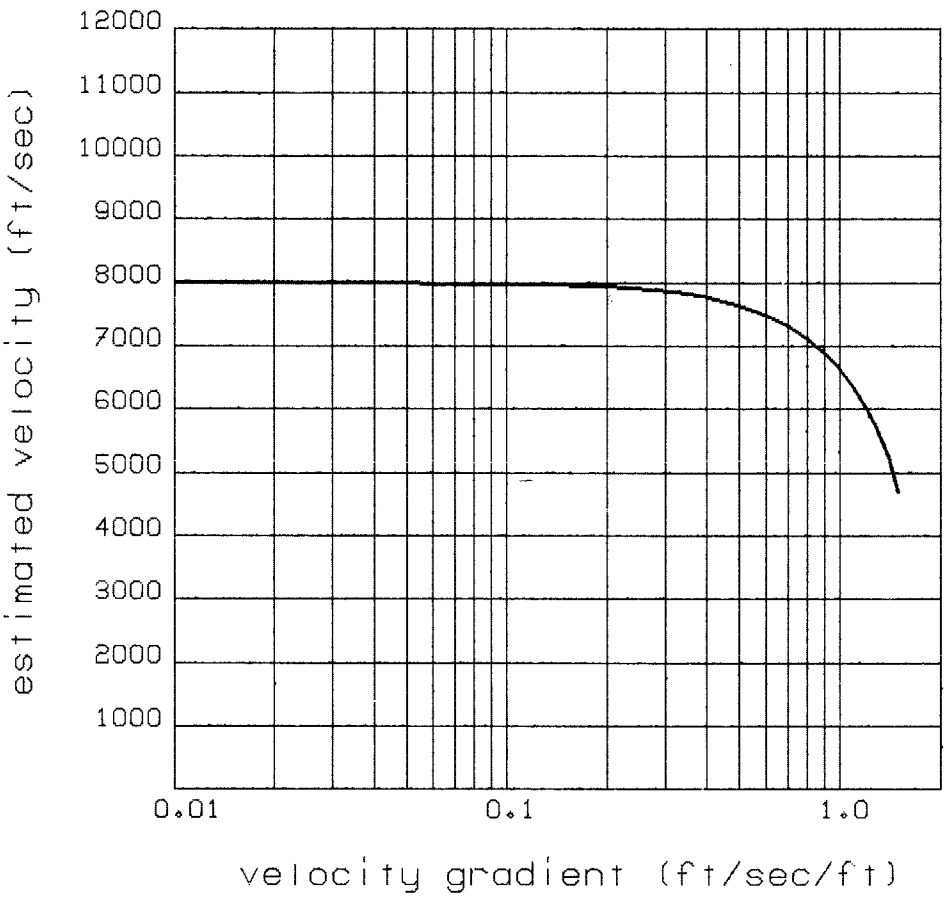

Figure 2.6. Estimated velocity using conventional velocity analysis for the model shown in Figure 2.5 at midpoint y_0 for different lateral gradients β. The correct media velocity at this midpoint is 8000 ft/sec. For this example the true and estimated velocities begin to substantially diverge at β = 0.5 ft/sec/ft. Note that the estimated velocity is always less than the true velocity.