VELOCITY ESTIMATION IN LATERALLY VARYING MEDIA

A DISSERTATION

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ABSTRACT

In areas of large lateral variations in velocity, conventional velocity estimation techniques fail and often result in absurd interval velocities because they assume a stratified earth model. The errors in conventional velocity estimation due to lateral velocity variations are shown to be related to the second lateral derivative of the RMS slowness ($1/v_{RMS}$). This relation is obtained by assuming straight raypaths and approximating the vertical RMS slowness to a given interface as a second order Taylor series expansion in the midpoint direction. Under these approximations, the effect of the first lateral derivative of the slowness on the traveltime is negligible. Knowing the stacking velocity to a given event as a function of midpoint, the lateral derivative method (LDM) can correct for the velocity estimation errors due to lateral velocity variations down to wavelengths on the order of two cable-lengths. Letting Δv be the velocity variation over one cable-length, the LDM is applicable when $\Delta v/v > \sim 2\%$. The assumptions of the method break down, however, when $\Delta v/v > \sim 30\%$.

Linearizing the equation relating the conventional velocity estimates and the true velocity results in a set of equations whose inversion is unstable. However, stability is easily achieved by adding a non-physical fourth derivative term which affects only the higher spatial wavenumbers that are beyond the lateral resolution of the LDM. The velocity estimation procedure, then, is to solve a pentadiagonal system of equations in which the input data are the conventional velocity estimates and the zero-offset traveltimes to a given event.

Synthetic models designed to test the assumptions of the LDM show that none of the assumptions leads to spurious results. Even in the worst possible cases, the LDM results show an improvement over conventional results. A test of the lateral derivative method on field data where there is a lateral velocity variation due to seafloor topography gives a result which is substantiated by a depth migration.

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