VELOCITY ANALYSIS: PROBLEMS WITH SNELL WAVES

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Abstract

This paper discusses two problems associated with the use of Snell waves to estimate a velocity function from data: the first problem is related to diffractions in the t'-τ plane; the second to non-vertical Snell waves.

Introduction

In previous reports (see "Wave Equation Velocity Analysis," SEP-16, 181-204) the advantages of using Snell waves to estimate a velocity function from seismic data were mentioned. In particular it was shown that the Snell wave approach exploits the sensitivity of far offset data. Conventional velocity estimation is accurate for near offset traces only because of the paraxial approximation in the normal moveout (NMO) correction equation.

The original idea was to work in the offset-time h-t' plane. This plane is adequate for the velocity estimation part. However, if we could downward continue the data and focus the energy of both primary and multiple reflections, then our velocity estimation would not be contaminated by the multiples, which we would be able to mute out. An alternative was to perform the velocity estimation in the retarded time-depth t'-τ plane. Here we face two main difficulties, namely artificial diffracted energy and non-vertically incident waves. In particular, these problems arise when we downward continue the data using a velocity function which departs significantly from the true material velocities. In this paper we discuss some examples of problems associated with velocity estimation in the t'-τ plane.

Diffractions

We think the presence of diffracted energy when we downward continue data and display the t'-τ plane is worth consideration, since it is not familiar to most people and can be confused with energy from real events. The presence of diffracted energy is severe if the difference between the true velocity and the downward continuation velocity is appreciable—for example, a water solid bottom interface in the earth with water velocity migration. The diffracted energy, however, is likely to be due to cable truncations.

Figure 1 shows the synthetic data used in the following examples. The first frame is a synthetic common midpoint gather, and in the second gather a linear moveout (LMO) correction was applied.

Figure 2 shows an example of data in the t'-τ plane. In this example the data was downward continued in the frequency domain and wraparound was avoided by padding with zeros. This kind of display is suitable for an accurate velocity determination. From the aforementioned paper the relationship between the coordinates of the focuses in this plane and the velocities is

$$v_e^2 = \frac{1}{\frac{dt'}{d\tau}\left(\frac{1}{v_m^2} - p^2\right) + p^2} \qquad (1)$$

where p is the value of the ray parameter used for the LMO correction, t' is retarded time, τ is time-depth, v_m is the downward continuation velocity, and v_e is the desired velocity function.

In Figure 3 we muted the four near offset traces from the gather, and the data was not padded with zeros to emphasize both diffracted

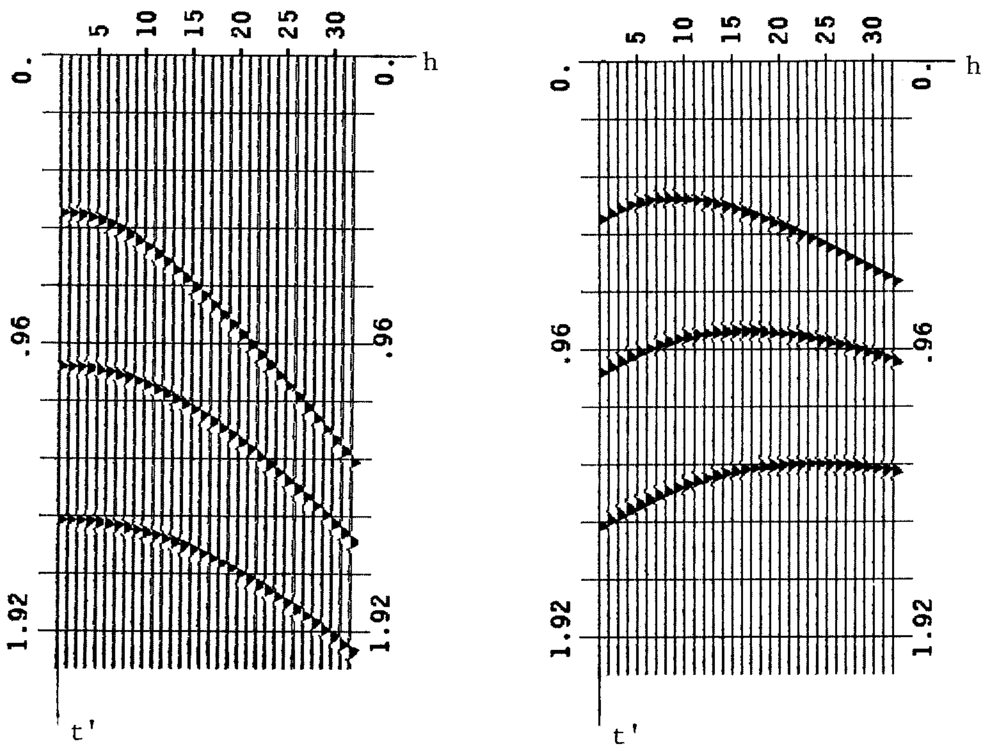


FIG. 1. Synthetic data gathers. (a) Common midpoint gather for a constant velocity earth model: 32 traces, offset = 100 m, sampling rate = 16 msec, velocity = 2500 m/sec. (b) Gather with an LMO correction applied for a value of p = 1/5000 sec/m.

energy and the effect of wraparound. The data was overclipped for the purpose of display. Note that the diffracted energy produces interference, creating spurious focuses easily confused with the true events. The presence of the artificial diffracted energy can be reduced partially by avoiding wraparound.

Slanted Snell Waves

The second problem associated with velocity estimation on the t'-τ arises when we no longer deal with vertically incident waves, but let the ray parameter p take any arbitrary value within the possible range allowed by the data. Figure 4 is the conventional downward continuation to different depths. When we use a value for p = 0, as the downward continuation proceeds, we are assured there will be a particular depth for

FIG. 2. This figure shows the downward continuation of the data of figure 1(a) for three different velocities. The data was zero-padded to avoid aliasing. The first frame (a) represents the data downward continued with a 15% lower velocity than the exact one; the second frame (b) had the right velocity; and the third frame (c) a 15% faster velocity. Note that we get better focusing in the second and third cases. We can observe some diffracted energy as steep slopes close to the focus.

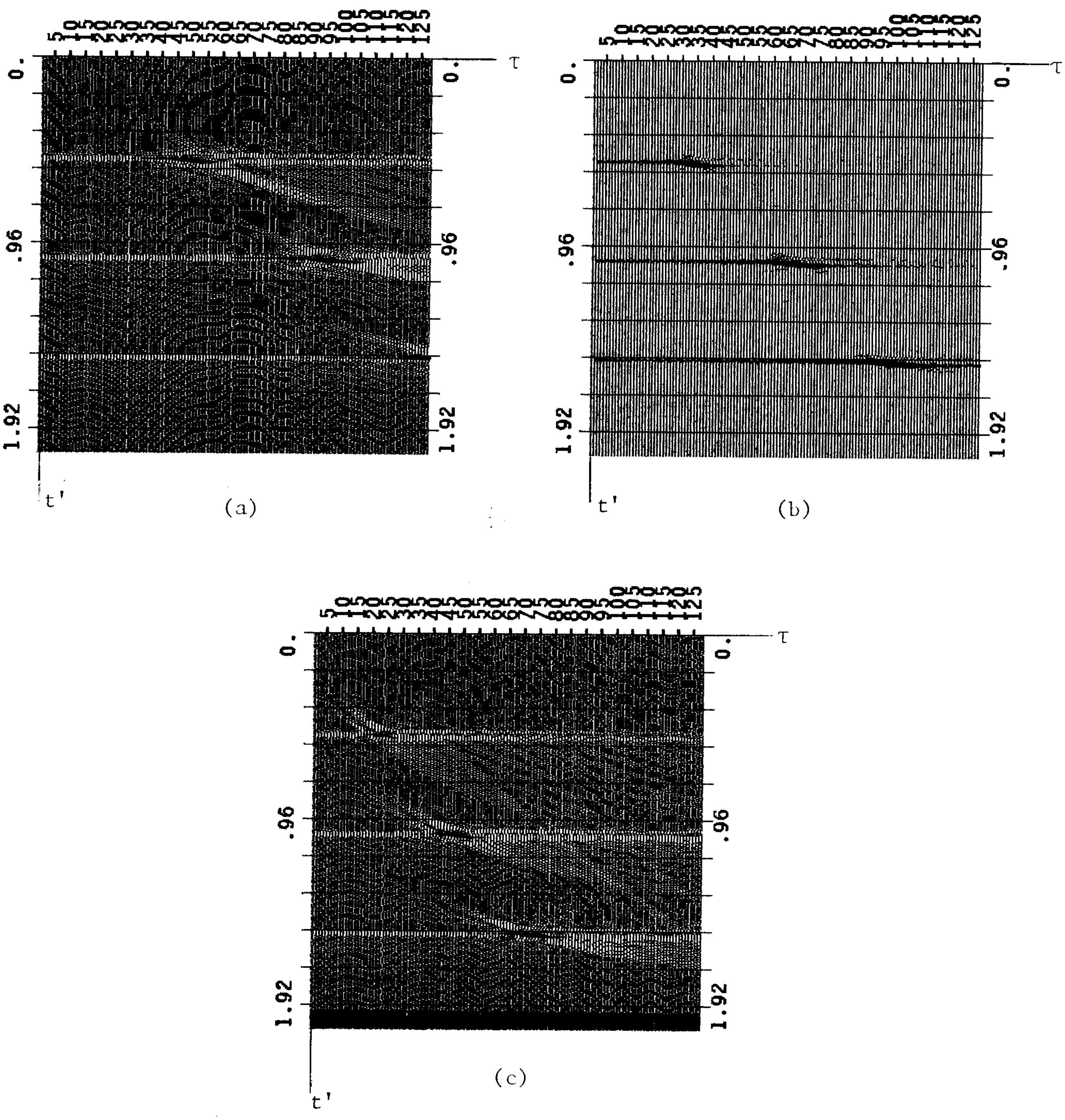
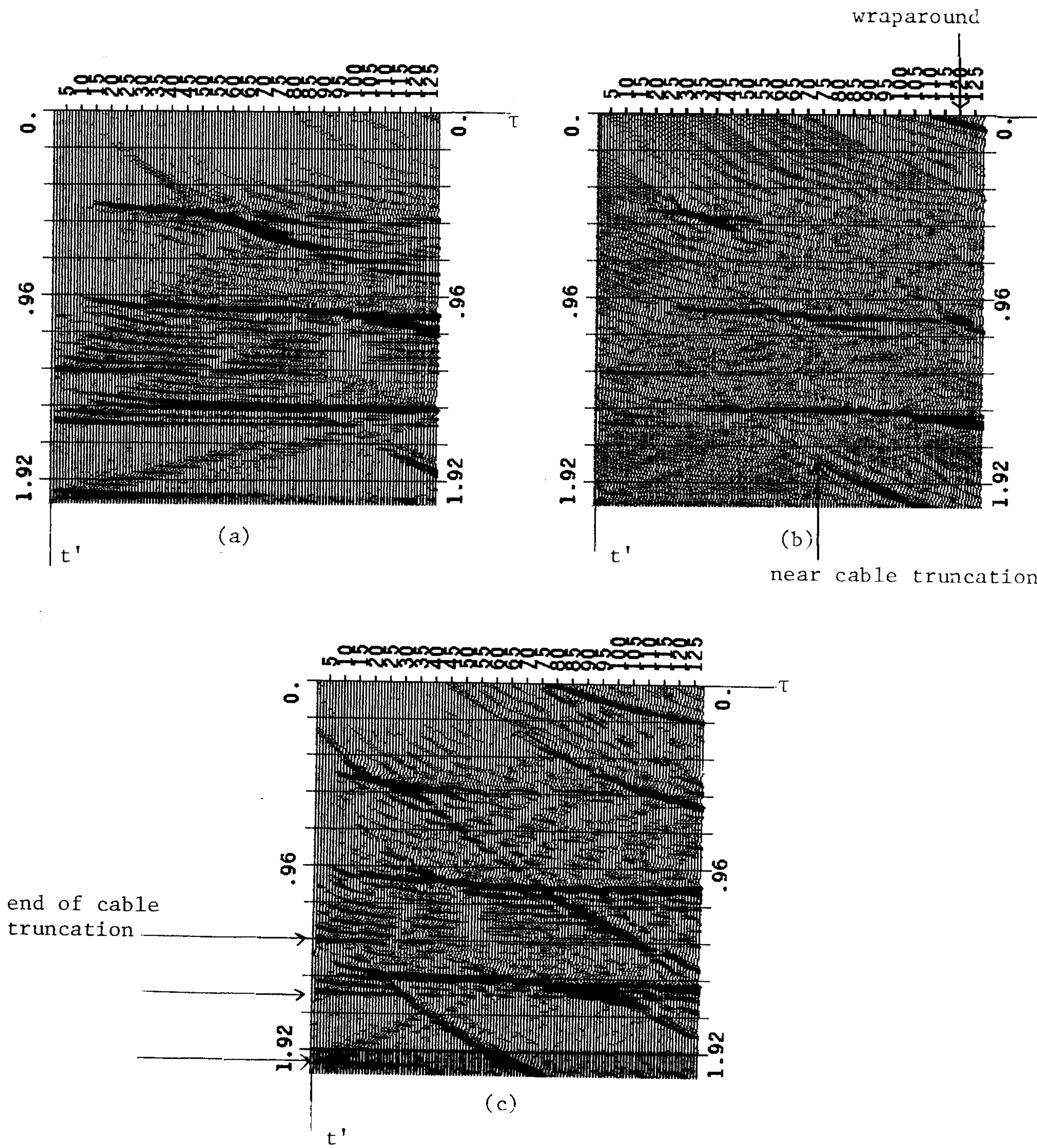


FIG. 3. This figure shows the downward continuation of the data of figure 2(a) for three different velocities. The data was not zero-padded in this case and the four near traces were muted. The velocities used for the downward continuation are the same as those in figure 2 respectively. Annotated in the figures is the diffracted energy. In particular, observe the effects of wraparound and cable truncations.



each event where most of the energy will be collapsed to the zero offset trace. If we look at the t-τ plane we expect, therefore, to always distinguish the place where this focusing is optimum. The quality of focus will depend principally on errors in the downward continuation velocity function employed and the bandwidth of the data.

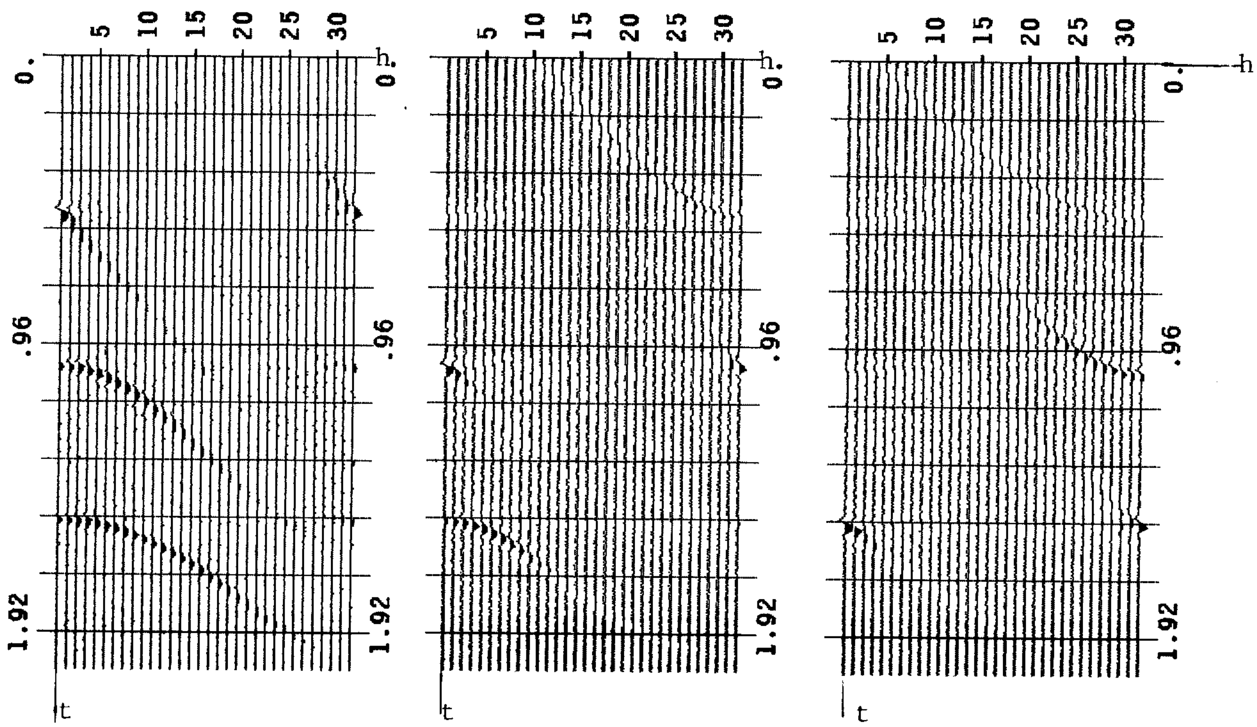


FIG. 4. This figure shows the downward continuation of the data of figure 2(a) using the correct velocity for the first (a) second (b) and third (c) event depths. Since we are working with a value of p = 0 we expect all of the energy for an event to collapse to the zero offset for a given depth. When we display the zero-offset traces for different depths we expect to be able to identify precisely where the perfect focus occurred.

When we allow p to be nonzero, we need to change the condition which defines the plane we want to display from the data in the (h,t',τ) volume (here h stands for half-offset). At first it seems that the condition:

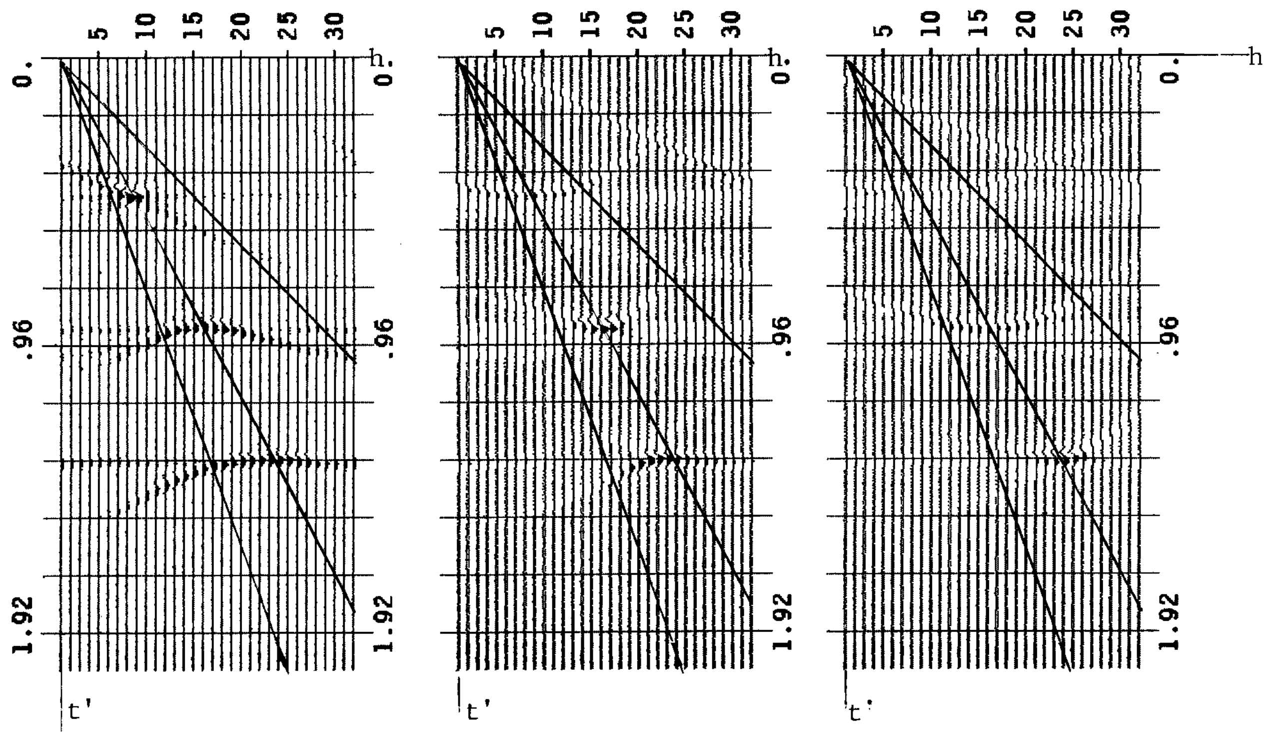


FIG. 5. This figure shows the case of nonzero p. The data was LMO-corrected using $p = 1/5000$ sec/m. As in the previous figure the correct velocity was used for the downward continuation. However, we need to define a function of offset and depth which will allow us to find the right plane to display as the downward continuation proceeds. The figures show three lines corresponding to three different constant velocities differing from the true velocity by an error of 15%. The effect we will get is clear if we choose a wrong offset function when we display our data on the t'-τ plane.

$$\bar{h} = \frac{1}{2}\int_0^{t'} \frac{pv^2}{1 - p^2v^2}\, dt' \qquad (2)$$

was satisfactory. However, as can be concluded from Figure 4, the offset function defined by this condition turns out to be particularly sensitive to velocity. This is especially critical if we are to work in the t'-τ plane. In other words, the offset function does not guarantee that we are going to get most of the energy focused at the particular offset that we display at a given depth. Since any error in velocity will imply the energy will focus somewhere else, the quality of focus and hence the velocity estimation will be diminished. The problem is that finding the right plane to display apparently involves not one but two conditions.

Conclusion

The problem of velocity estimation using the wave equation deserves more consideration. We have analyzed two approaches to the problem of estimating the velocity function in two different planes of the data volume. The original idea of doing the whole analysis in the offset-time domain remains the best. However, the wave equation offers the potential of merging processes which are conventionally done separately, and in this respect we still need to find solutions.