Chapter 3
LATERAL VELOCITY VARIATION

Lateral variation in velocity is of great concern in areas with complex structural settings or rapid facies changes. With this motivation, we would like to extend the theory developed in Chapter 1 to incorporate lateral velocity variation. In particular, we will study the Sep(Y,H) operator and investigate terms that may be of significance.

3-1 The Double Square Root Equation

We will restrict ourselves to the case in which velocity does not vary within a cable length of shots and receivers. This will allow us to use (1-7) to extrapolate wavefields in media with lateral velocity variations. It will be convenient to express the associated vertical wavenumber k_z of (1-6) as

$$k_z = -\omega \left(\frac{1}{v^2} - \frac{k_x^2}{\omega^2} \right)^{1/2} \qquad (3\text{-}1)$$

Writing (3-1) once for receivers and once for shots, then summing, gives

$$k_z = -\omega \left[\left(\frac{1}{v(s)^2} - \frac{k_s^2}{\omega^2} \right)^{1/2} + \left(\frac{1}{v(g)^2} - \frac{k_g^2}{\omega^2} \right)^{1/2} \right] \qquad (3\text{-}2)$$

Using (1-28) we put (3-2) into midpoint-offset space

$$k_z = -\omega \left\{ \left[\frac{1}{v(s)^2} - \left(\frac{k_y - k_h}{2\omega} \right)^2 \right]^{1/2} + \left[\frac{1}{v(g)^2} - \left(\frac{k_y + k_h}{2\omega} \right)^2 \right]^{1/2} \right\} \quad (3\text{-}3)$$

For brevity, we will leave the variables s and g in the velocity function. For convenience, we will redefine the normalized wavenumbers

$$\begin{bmatrix} Y \\ H \end{bmatrix} = \frac{1}{2\omega} \begin{bmatrix} k_y \\ k_h \end{bmatrix} \quad (3\text{-}4a,b)$$

We will make the following further definitions:

$$M(s) = \frac{1}{v(s)^2} \quad (3\text{-}5a)$$

and

$$M(g) = \frac{1}{v(g)^2} \quad (3\text{-}5b)$$

where M is the square of the slowness function.

Upon substitution of (3-4) and (3-5), (3-3) takes the form

$$k_z = -\,\omega \left\{ \left[M(s) - (Y - H)^2 \right]^{1/2} + \left[M(g) - (Y + H)^2 \right] \right\} \quad (3\text{-}6)$$

Omitting the scaling frequency ω , we define the double square root operator in laterally varying media as

$$DSR(Y,H) = \left[M(s) - (Y - H)^2 \right]^{1/2} + \left[M(g) - (Y + H)^2 \right]^{1/2} \quad (3\text{-}7)$$

3-2 The Separable Approximation

We defined the improved conventional processing by the NewSep(Y,H) operator. Let us derive this operator using the newly defined DSR(Y,H) given by (3-7). We will make use of the shortcut derivation (2-20):

$$NewSep(Y,H;H_0=\hat{H},Y_0=0) = DSR(Y,\hat{H}) + DSR(0,H) - DSR(0,\hat{H})$$

Making all the relevant substitutions

$$NewSep = \left[M(s) - (Y - \hat{H})^2 \right]^{1/2} + \left[M(g) - (Y + \hat{H})^2 \right]^{1/2} \quad (3\text{-}8)$$

$$+ \left(M(s) - H^2 \right)^{1/2} + \left(M(g) - H^2 \right)^{1/2}$$

$$- \left(M(s) - \hat{H}^2 \right)^{1/2} - \left(M(g) - \hat{H}^2 \right)^{1/2}$$

The third and fourth square roots are NMO-like terms similar to (1-35) ; i.e. the stacking operator. Abandoning them we will be left with the retarded migration part:

$$Mig = \left[M(s) - (Y - \hat{H})^2 \right]^{1/2} + \left[M(g) - (Y + \hat{H})^2 \right]^{1/2} \qquad (3\text{-}9)$$

$$- \left(M(s) - \hat{H}^2 \right)^{1/2} - \left(M(g) - \hat{H}^2 \right)^{1/2}$$

Referring to the second order square root expansion provided in Appendix A (Equation A-3), we have

$$\left[M(s) - (Y - \hat{H})^2 \right]^{1/2} = \left(M(s) - \hat{H}^2 \right)^{1/2} \qquad (3\text{-}10a)$$

$$\cdot \left[1 + \frac{\hat{H}\ Y}{M(s) - \hat{H}^2} - \frac{M(s)\ Y^2}{2 \left(M(s) - \hat{H}^2 \right)^2} \right]$$

and

$$\left[M(g) - (Y + \hat{H})^2 \right]^{1/2} = \left(M(g) - \hat{H}^2 \right)^{1/2} \qquad (3\text{-}10b)$$

$$\cdot \left[1 - \frac{\hat{H}\ Y}{M(g) - \hat{H}^2} - \frac{M(g)\ Y^2}{2 \left(M(g) - \hat{H}^2 \right)^2} \right]$$

For simplicity, define

$$C(s) = \left(M(s) - \hat{H}^2 \right)^{1/2} \qquad (3\text{-}11a)$$

$$C(g) = \left(M(g) - \hat{H}^2 \right)^{1/2} \qquad (3\text{-}11b)$$

Substituting (3-10) into (3-9) together with the definitions (3-11) we have

$$Mig \approx C(s) \left[1 + \frac{\hat{H}Y}{C(s)^2} - \frac{M(s)\ Y^2}{2\ C(s)^4} \right]$$

$$+ C(g) \left[1 - \frac{\hat{H}\ Y}{C(g)^2} - \frac{M(g)\ Y^2}{2\ C(g)^4} \right]$$

$$- C(s) - C(g)$$

Simplifying, we get the final expression

$$\text{Mig} \approx \hat{H}\, Y \left[\frac{1}{C(s)} - \frac{1}{C(g)} \right] - \frac{Y^2}{2} \left[\frac{M(s)}{C(s)^3} + \frac{M(g)}{C(g)^3} \right] \qquad (3\text{-}12)$$

The second term on the right is a typical 15-degree-like migration equation which is adjusted for offset via (3-11). But the first term is entirely a stranger to us:

$$\text{NewMig} = \left[\frac{1}{C(s)} - \frac{1}{C(g)} \right] \hat{H}\, Y \qquad (3\text{-}13)$$

Substituting (3-4a) and (3-11) we have

$$k_z = - \omega \; \text{NewMig}$$

$$k_z = - \omega \left\{ \frac{1}{\left(M(s) - \hat{H}^2 \right)^{1/2}} - \frac{1}{\left(M(g) - \hat{H}^2 \right)^{1/2}} \right\} \hat{H} \frac{k_y}{2\,\omega} \qquad (3\text{-}14)$$

Notice that the new definitions of Y and H (3-4) are the old Y and H (1-30) divided by v. We may also define the new $\hat{H}$ as the old $\hat{H}$ (2-13) divided by v; thus

$$\hat{H} = \frac{2\,h}{v^2\, t}$$

Using the new definition for $\hat{H}$ and assuming it is small

$$\left[M(s) - \hat{H}^2 \right]^{1/2} \approx \left[M(s) \right]^{1/2} = \frac{1}{v(s)}$$

$$\left[M(g) - \hat{H}^2 \right]^{1/2} \approx \left[M(g) \right]^{1/2} = \frac{1}{v(g)}$$

(3-14) becomes

$$k_z = - \omega \left[v(s) - v(g) \right] \left(\frac{2\ h}{v^2\ t}\right) \left(\frac{k_y}{2\ \omega}\right)$$

$$k_z = - \frac{h}{t} \frac{v(s) - v(g)}{v^2} k_y$$

which transforms to

$$P_z = - \frac{h}{t} \frac{v(s) - v(g)}{v^2} P_y \qquad (3\text{-}15)$$

where v is considered to be the average of v(s) and v(g). This equation implies a pure lateral shift. Notice the coefficient is strongly dependent upon the offset value and the lateral velocity gradient. For zero offset this equation means no operation. However, in a laterally varying medium, the thin lens term of the migration process still needs to be applied even to zero offset. If we imagine a point scatterer in a medium with lateral velocity , Equation (3-14) implies the following: If the top of the diffraction hyperbola on a common offset section (h_1) is located under midpoint (y_1) , then on a common offset section (h_2) the top will be located under a different midpoint (h_2) .The lateral shift ($y_2 - y_1$) will depend upon how strong the lateral velocity gradient is and how far apart the two offsets are ($h_2 - h_1$) .

Let us investigate the significance of (3-15). During the processing of the field dataset described in Section 2-4, it was observed that some of the dipping events showed evidence that some lateral phase shift from one common offset to another was still present after moveout correction and partial migration. An example is shown in Figure 3-1. One possible explanation is that there may be lateral variation in velocity due to overpressurized shale (Claerbout, personal comm.) or prominent structural complexity.

First let us transform (3-15) to NMO coordinates. The transformation defined by (2-27) will simply change the coefficient of (3-15):

$$P_z = -\frac{h}{t'}\left[1+\left(\frac{2h}{v\,t'}\right)^2\right]^{-1/2}\frac{v(s)-v(g)}{v^2}\,P_y \qquad (3\text{-}16)$$

We let

$$C = \frac{h}{t'\left[1+\left(\frac{2h}{v\,t'}\right)^2\right]^{1/2}}\,\frac{v(s)-v(g)}{v^2} \qquad (3\text{-}17)$$

Earlier in the derivation of (3-15) we made a small-offset approximation. Similarly, (3-17) becomes

$$C = \frac{h}{t'}\left[1 - \frac{1}{2}\left(\frac{2\ h}{v\ t'}\right)^2\right]\frac{v(s) - v(g)}{v^2} \tag{3-18}$$

For the sake of a crude estimation, let us consider a constant lateral velocity gradient. This will allow us to write an analytical solution to (3-16). Using (3-17) we have

$$P_z = -\ C\ P_y \tag{3-19}$$

Fourier transforming in t and y , $P = P(k_y,z,\omega)$, we obtain

$$P_z = -i\ C\ k_y\ P \tag{3-20}$$

The solution to (3-20) is

$$P(k_y,z,\omega) = P(k_y,0,\omega)\ e^{-i\ C\ k_y\ z} \tag{3-21}$$

Given the phase shift $\Phi = C\ k_y\ z$, we can estimate the lateral velocity gradient. Solving for v(s)-v(g) in (3-18) and substituting $k_y = (\frac{\omega}{v})\ \sin(\theta)$ and $z = vt'/2$, we obtain

$$v(s) - v(g) = \Delta v = \frac{2\ \Phi\ v^2}{h\left(1 - \frac{2\ h^2}{v^2\ t'^2}\right)\sin(\theta)\ \omega} \tag{3-22}$$

where θ can be regarded as the average dip over the distance s-g.

Let us plug some numbers into the expression (3-22). Referring to Figure 3-1, between midpoints 100-150, we have

$$\Phi = \pi \text{ radians}$$

$$h = 5200 \text{ ft}$$

$$t' = 2.5 \text{ sec}$$

$$v(t') = 7500 \text{ ft/sec}$$

$$\theta \simeq 15^{0}$$

$$\omega \simeq 2\pi \times (25 \text{ Hz})$$

The resulting value for Δv is approximately 500 ft/sec per 1000 ft of lateral distance. Even this crude estimate gives an idea as to the significance of (3-15). A closer study of this term may lead to a procedure by which one can accurately estimate lateral variation in velocity. In particular, the statics problem can now be reconsidered to take offset dependency into account.

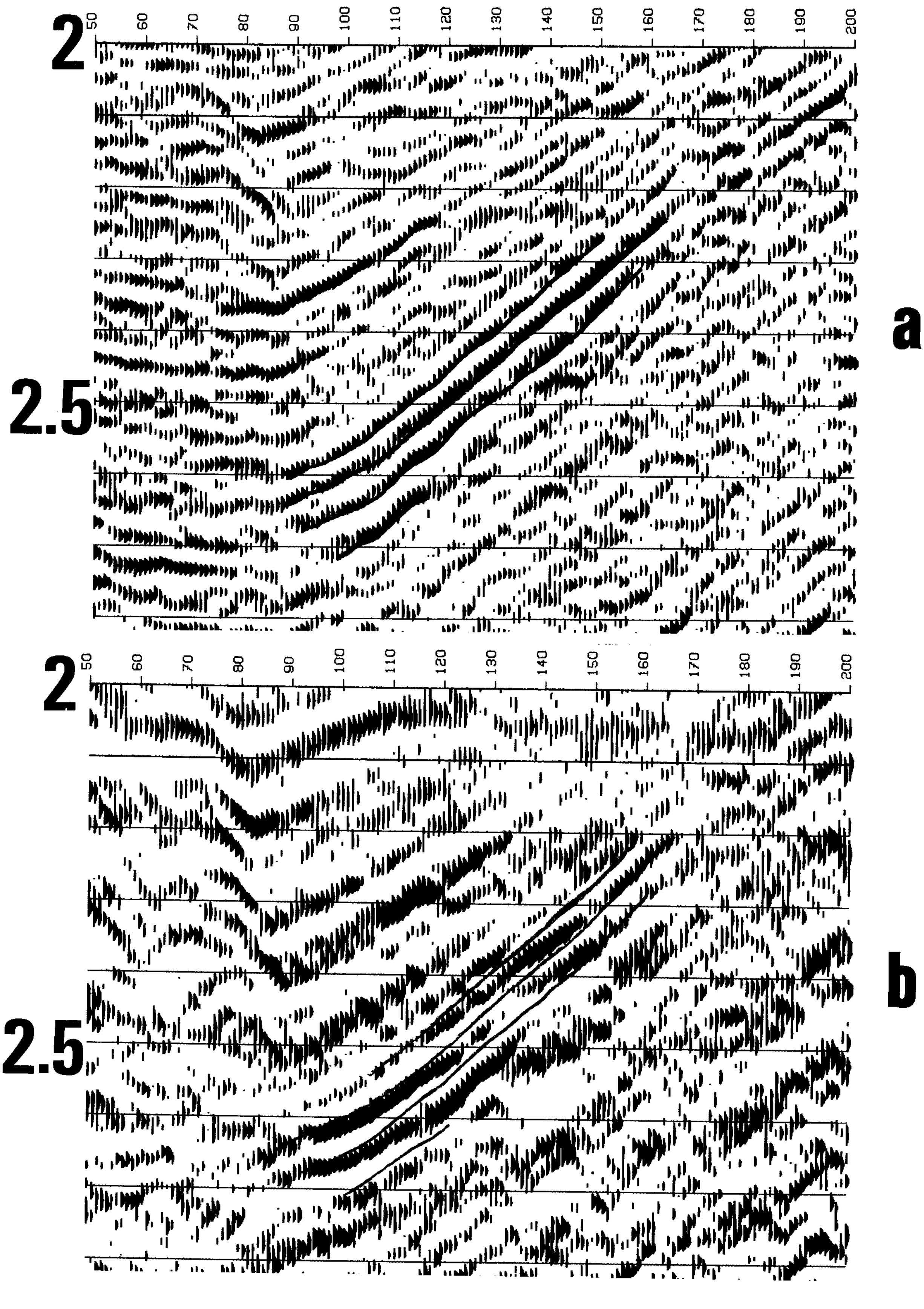

FIG. 3-1. Portions of the two moveout-corrected and partially migrated common offset sections from the Gulf data. The event marked is located differently in the two sections, possibly due to lateral variation in velocity.