STEEPLY FAULTED REGIONS, ARTIFACTS, AND STABILITY IN THE 45-DEGREE ω-FINITE DIFFERENCE ALGORITHM

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Abstract

To test the hypothesis that possible artifacts ("reflections," coherent amplitude or phase lineups, etc.) were generated by steep interfaces or severe velocity contrasts in the 45-degree finite difference program, a series of test cases were migrated. Random number inputs with a severe (1:2) velocity contrast across a 90-degree (or near 90-degree) interface result in unstable migrations. No instability is detectable on random number migrations with dipping interfaces of 45 degrees or less (1:2 velocity contrast). The instability is greatly reduced by an ad hoc modification of the Crank-Nicolson equation that results in symmetric matrices. Artifacts are generated on real datasets when an interface dips more steeply than 45 degrees, even when the velocity contrast is small (6%).

Introduction

Reflection seismic data characterized by steep faults in the Wind River thrust region of Wyoming were migrated using our monochromatic 45-degree program (see Kjartansson, p. 1, this report). With these data, given the attributes of this computer program, it seemed appropriate to use migration velocity models which were discontinuous (in x) across known and hypothetical steep faults. The program has gradients only in z, not x and z. If vertical faults have even modest velocity contrasts across them, there is an implied reflection coefficient which will be large at grazing incidence (flat time horizons). The pitfall is that use of such a velocity model in a migration program can produce migrated data which confirms and accentuates the presumed fault(s).

Random number input

The first tests for artifacts used a random number dataset for input. Figure 1 is a section of random numbers with a Gaussian distribution (300 traces by 750 time points).

Migration instability is seen in Figure 2 (a random number migration with a centrally located vertical interface and a velocity contrast of 1:2). The removal in the frequency domain of evanescent energy (viscosity) is an optional parameter in the 45-degree algorithm; Figure 2 was generated with no viscosity. A low frequency, high dip instability creates a pattern of "hearts" on the data. A ⦚ "jagged" interface should be used to represent a lateral change in velocity that would otherwise be represented as a vertical interface.

Migration instability to a lesser degree is seen in Figure 3, a random number migration with an 87-degree dipping centrally-located interface, 1:2 velocity contrast, and viscosity. The "heart" pattern has become smeared; this is probably due to the introduction of asymmetry and viscosity.

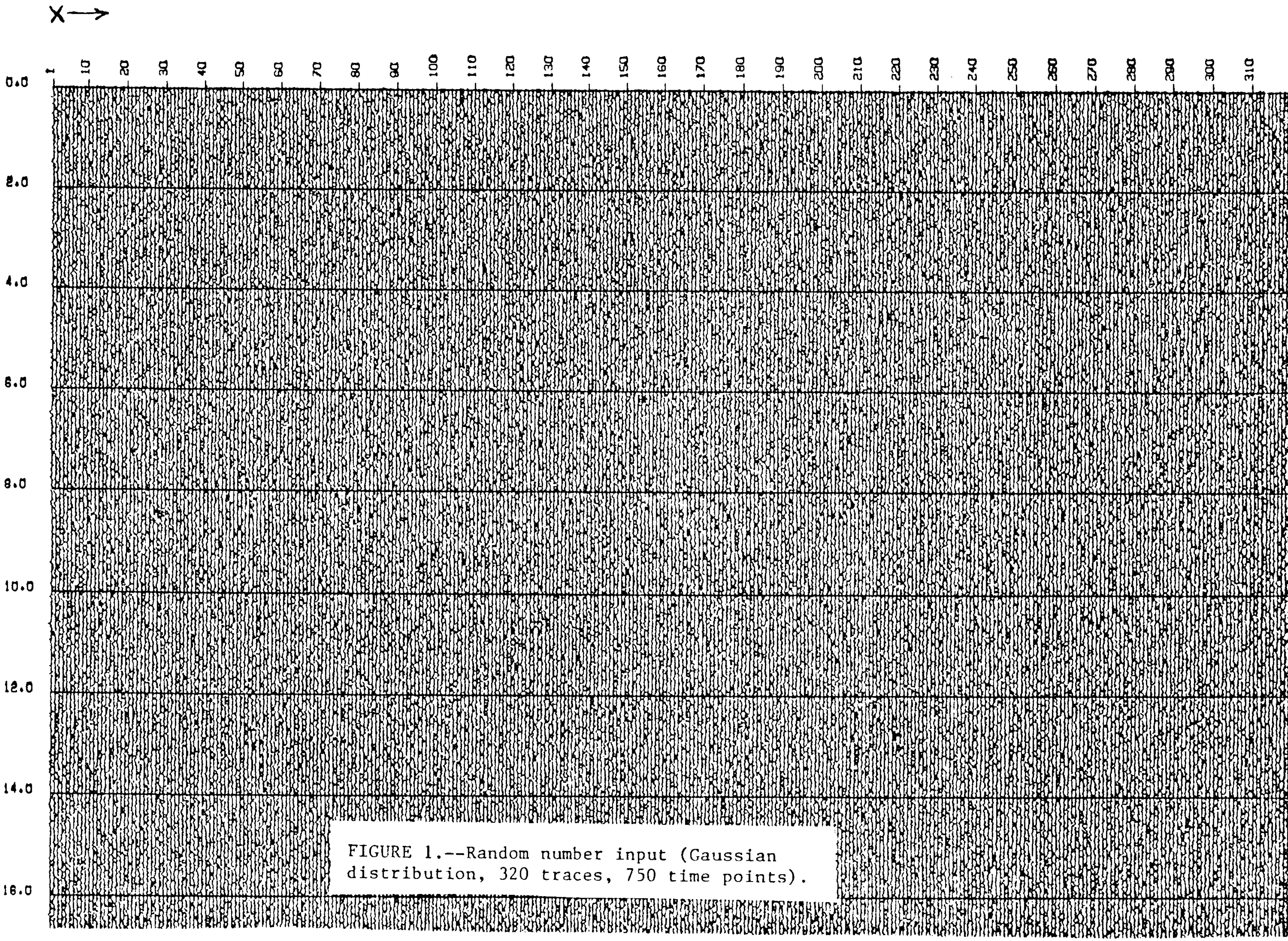

FIGURE 1.--Random number input (Gaussian distribution, 320 traces, 750 time points).

FIGURE 2.- Unstable migration of random numbers: no viscosity, velocity contrast 1:2, centrally located 90-degree interface.

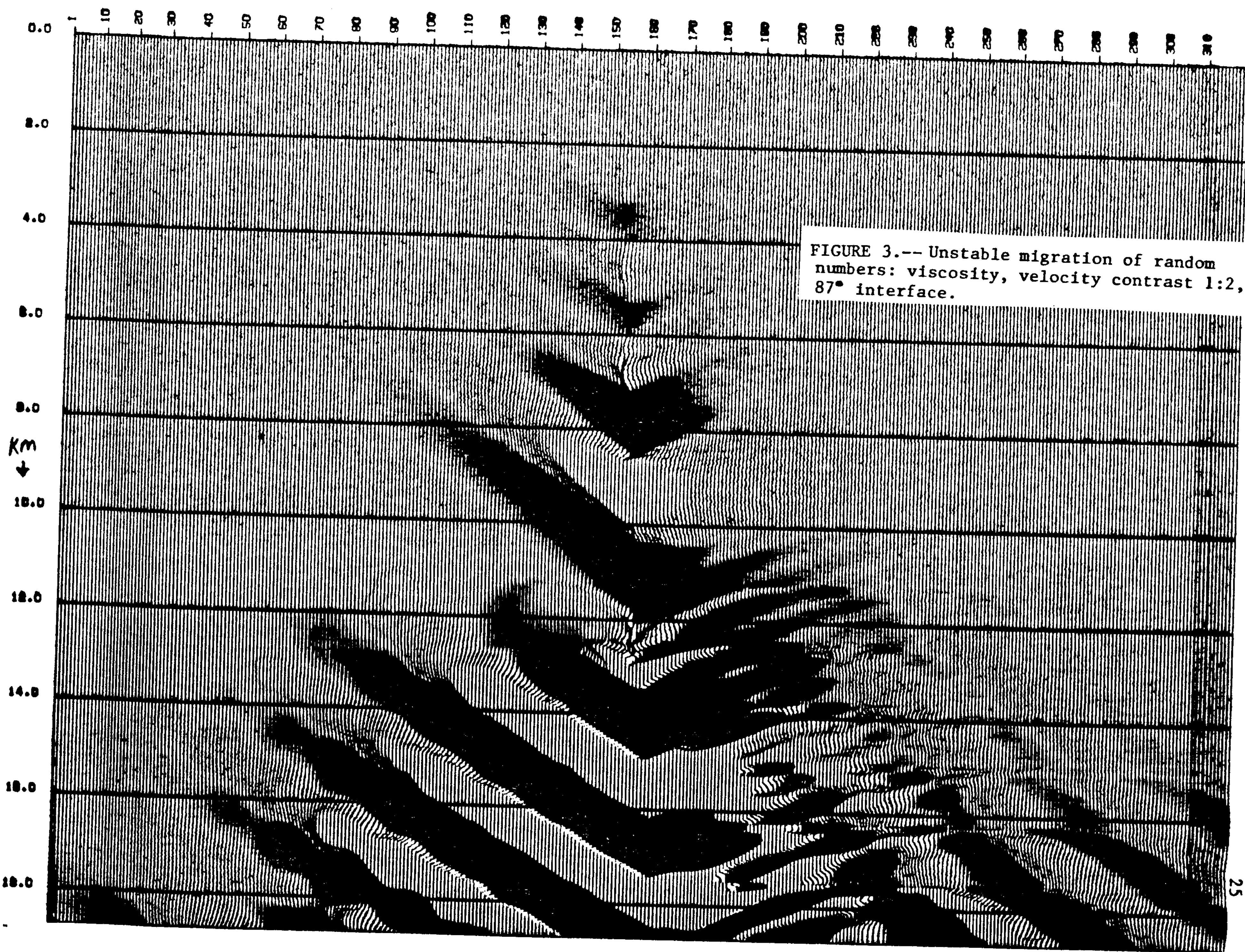

FIGURE 3.-- Unstable migration of random numbers: viscosity, velocity contrast 1:2, 87° interface.

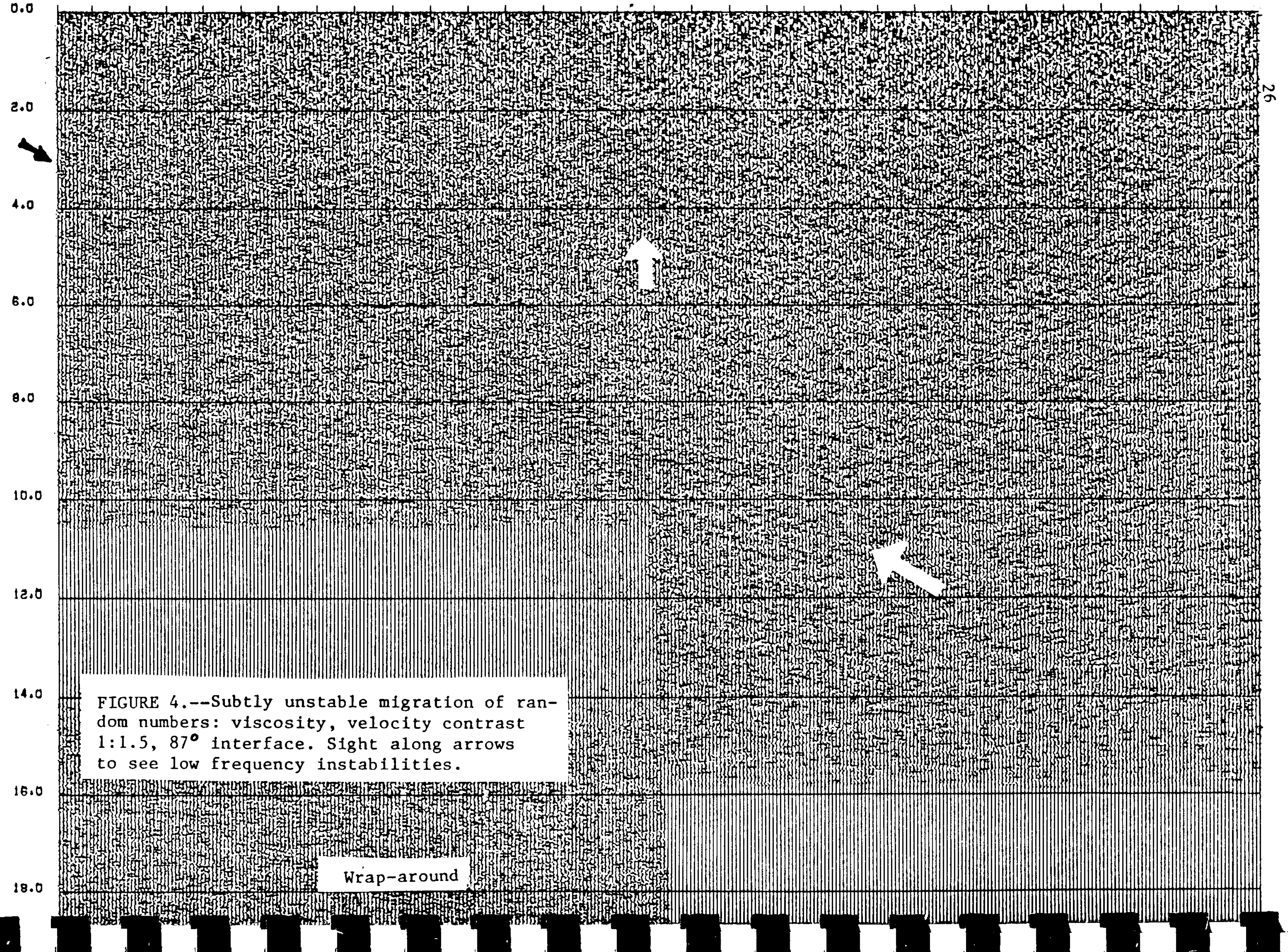

FIGURE 4.--Subtly unstable migration of random numbers: viscosity, velocity contrast 1:1.5, 87° interface. Sight along arrows to see low frequency instabilities.

Apparent stability was achieved by lowering the velocity contrast to 1:1.5 (Figure 4). (Viscosity and an 87^{o} dipping fault plane were used, as in Figure 3.) This result seems to be stable; however, there are two areas wherein a small amount of low frequency instability appears to be sneaking in. One (linear) area lies between the two dipping arrows; the other area lies above the vertical arrow. The two blank rectangles are normal and expected: both represent the end of the dataset. The block of "data" in the lowest lefthand corner is due to wraparound from the top of the time section. (This 45-degree program is periodic in time.)

Absolute stability resulted with lowering the dip of the interface to 45 degrees. Figure 5, a random number migration, used a 45-degree dipping interface (starting at trace 62), viscosity, and a velocity contrast of 1:2. This represents the extreme model for the lateral velocity variation dataset. (The Wind River thrust's dip is believed to be between 30 and 35 degrees; the velocity contrast across it is less than 1:2). There is stability and no apparent artifacts in this *random number* migration. There are lower frequencies in the faster velocity medium, and higher frequencies in the slower medium, as theory predicts. Since this plot is in z (km) not t (seconds), this result is expected because

frequency * wavelength = velocity

As velocity increases, wavelength also increases for a given frequency content.

The velocity *model* used in the real data migrations (Bloxsom and Ottolini, this report) produced no artifacts and was stable in a random number migration. The velocity model for Figure 6 used the lower (linear) dipping interface of Figure 11 from Bloxsom and Ottolini, this report (see Figures 8 and 11.) The velocity contrast was 1:2, with the higher velocity medium above, as usual. No viscosity was used. There are no lineups of phase or amplitude along the discontinuity; we also observe the same frequency content characteristic seen in Figure 5.

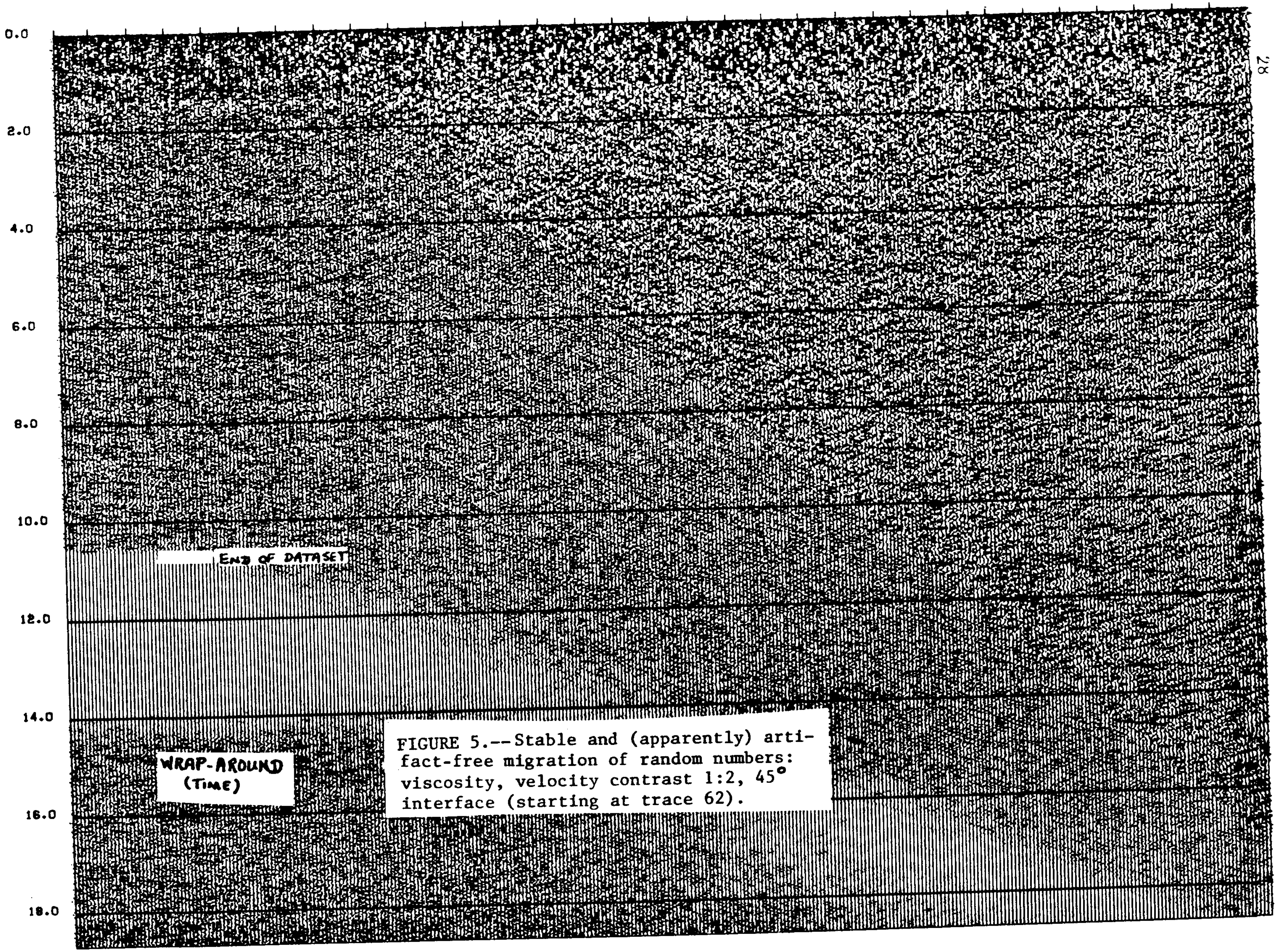

FIGURE 5.--Stable and (apparently) artifact-free migration of random numbers: viscosity, velocity contrast 1:2, 45° interface (starting at trace 62).

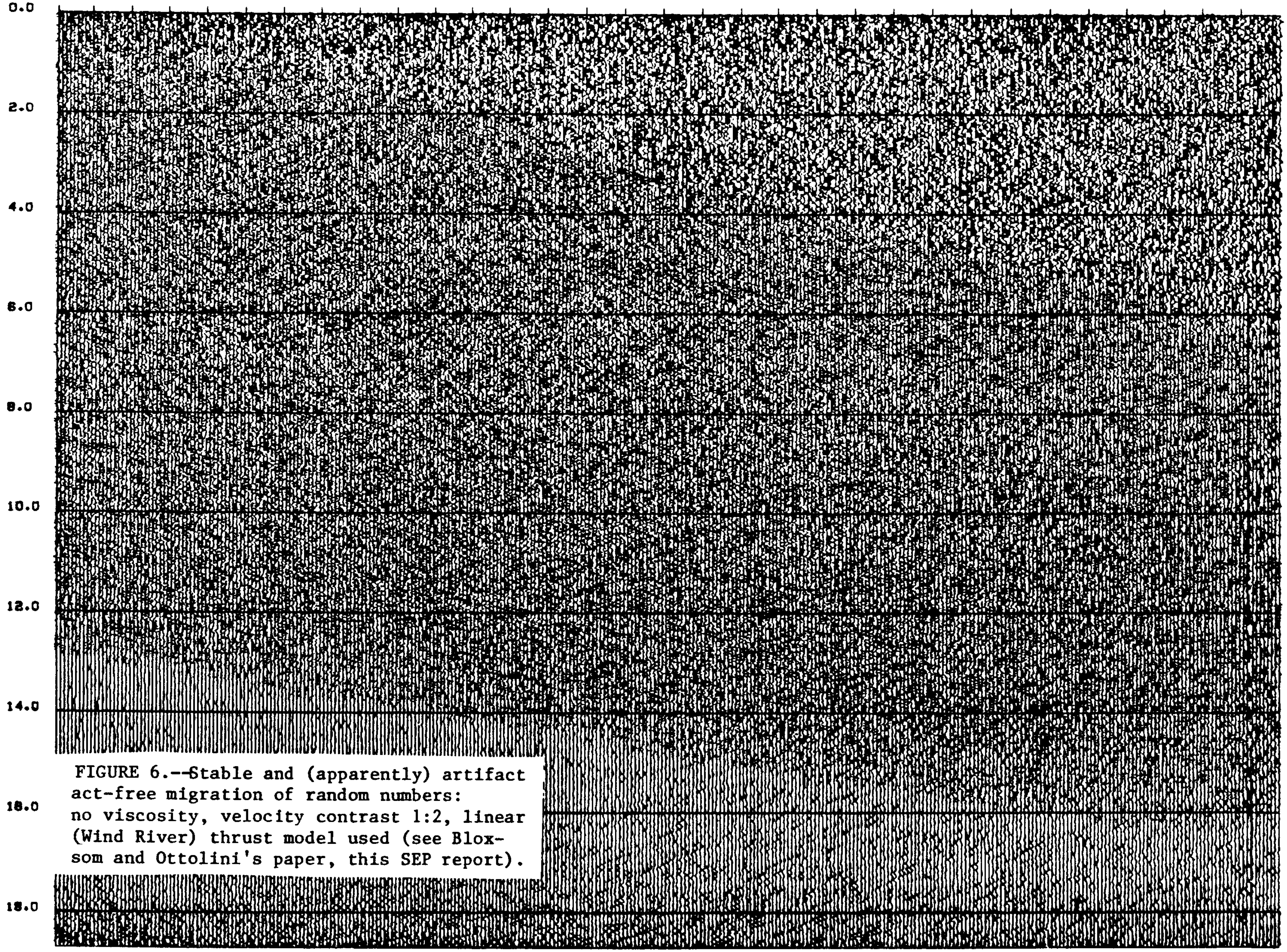

FIGURE 6.--Stable and (apparently) artifact-free migration of random numbers: no viscosity, velocity contrast 1:2, linear (Wind River) thrust model used (see Bloxsom and Ottolini's paper, this SEP report).

Our 45-degree algorithm uses the Crank-Nicolson scheme discussed by Claerbout (1976, pp. 184-189). [See Equation (10), Kjartansson, this report, p. .] The matrices in the matrix equation generated by the Crank-Nicolson scheme become asymmetric with lateral variation in velocity. An ad hoc modification made the matrices symmetric. The results are Figures 7, 8, and 12. In Figure 7, we see that the instability is reduced many orders of magnitude. Figure 7 was generated with a symmetric matrix, viscosity, a 1:2 velocity contrast, and an 87-degree dipping interface. The only difference between this figure and Figure 3 is the symmetry (and non-symmetry) of the matrix. In Figure 7, between 8 and 10 km depth in the center of the plot, there is only a faint darkening, suggestive of the "heart" pattern seen earlier. This subtle instability is potentially worse than the black or white stability cases seen earlier (wherein stability or instability is *instantly* obvious). Interpreters would not want to suggest a geologic reason for any of these dark streaks on the migrated data.

The effect that viscosity has on stability is shown in Figure 8. Instability has returned, though of much less magnitude than Figure 4. Figure 8 was generated with a symmetric matrix, no viscosity, a velocity contrast of 2:1, and an 87-degree dipping interface. This result shows that symmetry in the matrix helps but is not sufficient (without viscosity) for stability, using the current algorithm. (With viscosity, the result is Figure 7.)

This problem is being currently investigated and we expect to find a modification of the finite difference algorithm that will be absolutely stable for all velocity models (whether they are physically sensible or not).

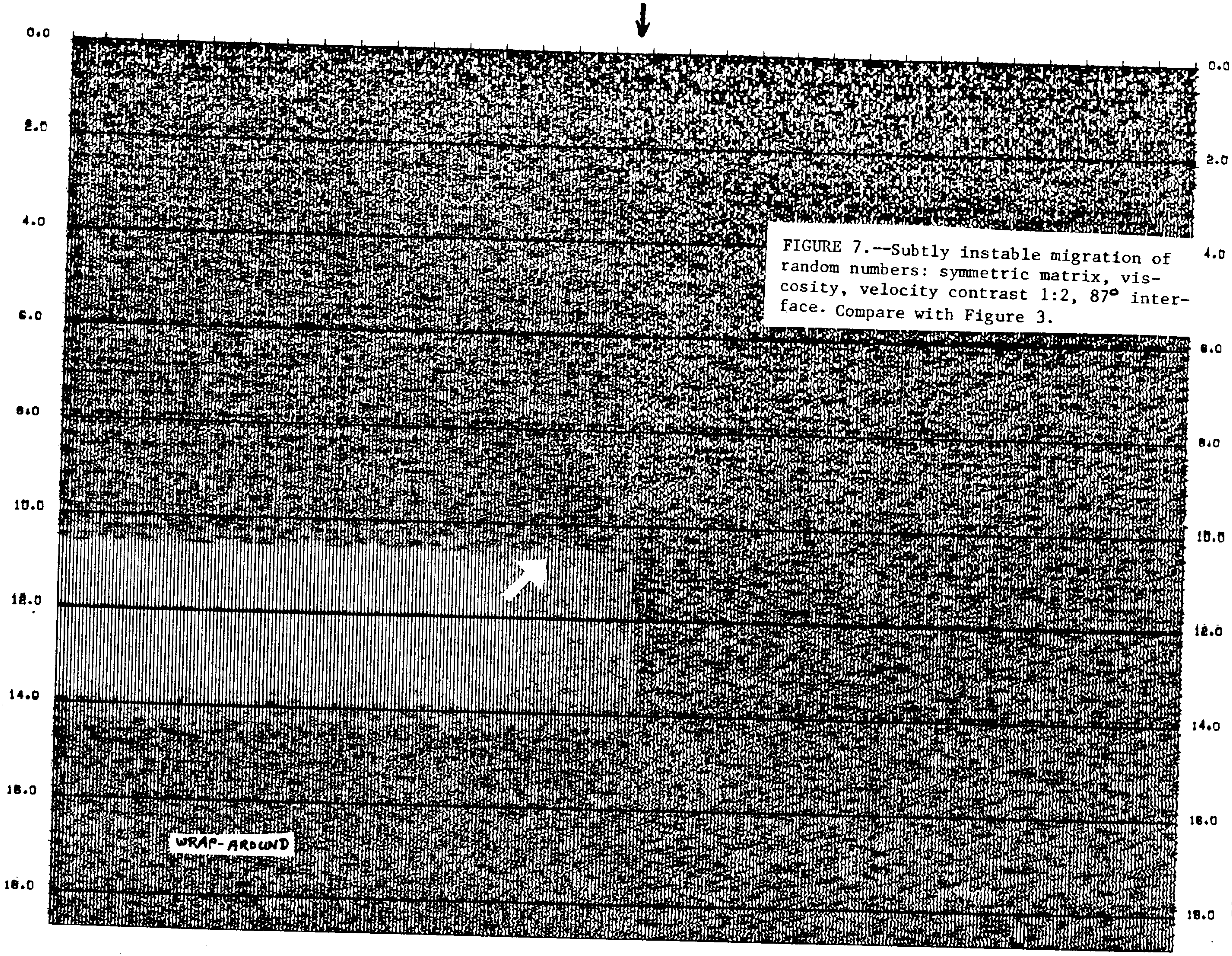

FIGURE 7.--Subtly instable migration of random numbers: symmetric matrix, viscosity, velocity contrast 1:2, 87° interface. Compare with Figure 3.

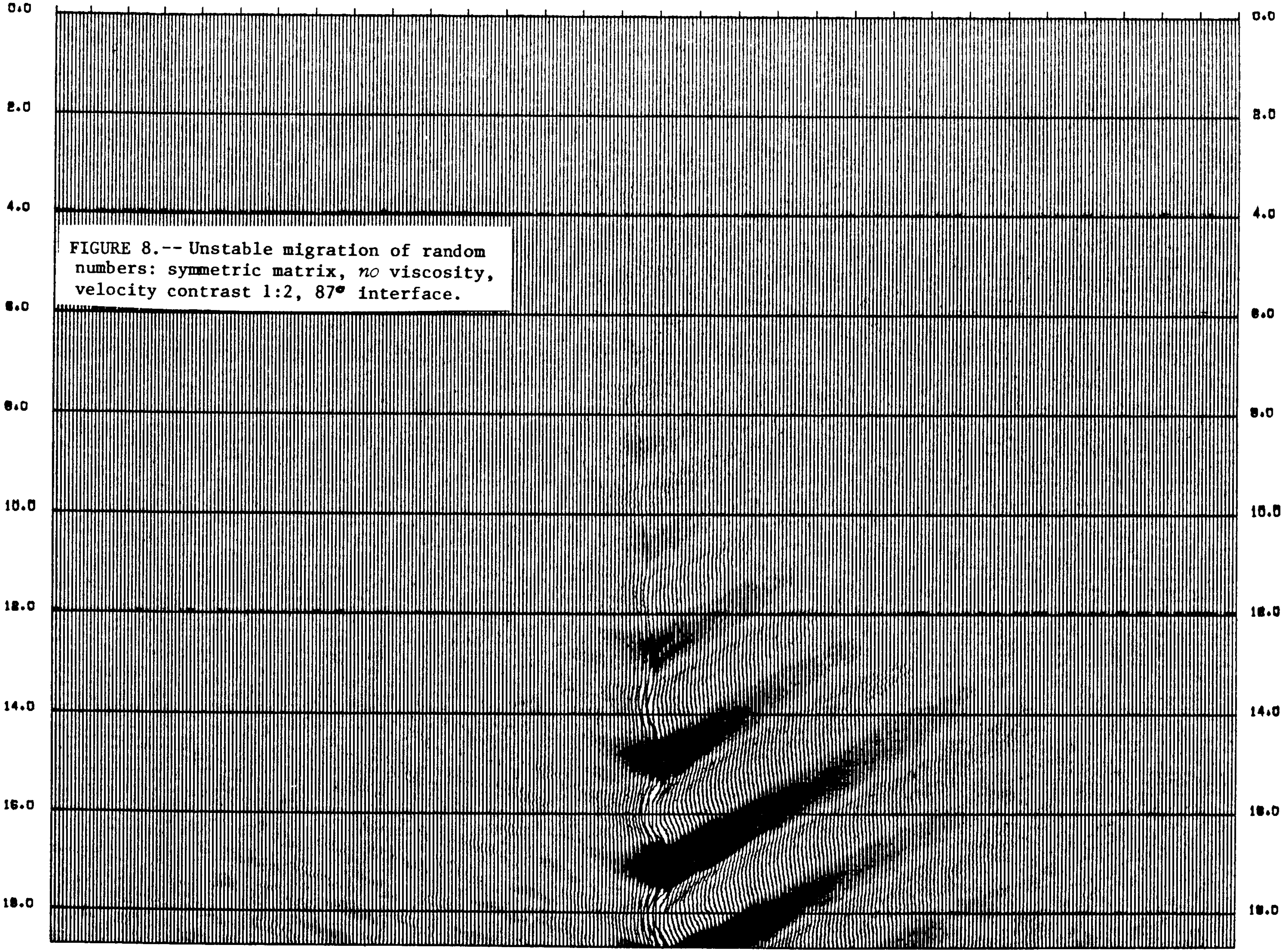

FIGURE 8.-- Unstable migration of random numbers: symmetric matrix, *no* viscosity, velocity contrast 1:2, 87° interface.

Real data

Real seismic data are now examined for artifacts and stability in regions of steeply dipping faults. When real data is migrated with velocity interfaces dipping more than 45 degrees, even though the velocity contrasts are small (6%), spurious reflections from the interface are generated. Figure 9 is the velocity model used: the interval velocities came directly from the stacking velocities (not geologic concepts of earth velocity, as did the models in Bloxsom and Ottolini's paper). The interface on the far left dips more steeply than 45 degrees from 3-19 km. Figure 10 is the migrated section, with artifacts. The artifacts generated by the far left interface are clearly seen. This situation imposes a greater practical limitation for the range of models that should be used.

Figures 11 and 12 show that when the dips of the interfaces become 45° at *most*, there are no apparent artifacts. Figure 11 is the model used: the heavy black lines show the interfaces modified to dip only 45 degrees. Compare Figure 10 and 12 to see that the artifacts on Figure 10 at the left are *absent* on Figure 12. With more array processor program memory, we will be able to handle *lateral* as well as vertical *gradients* in velocity, which will solve both of the problems.

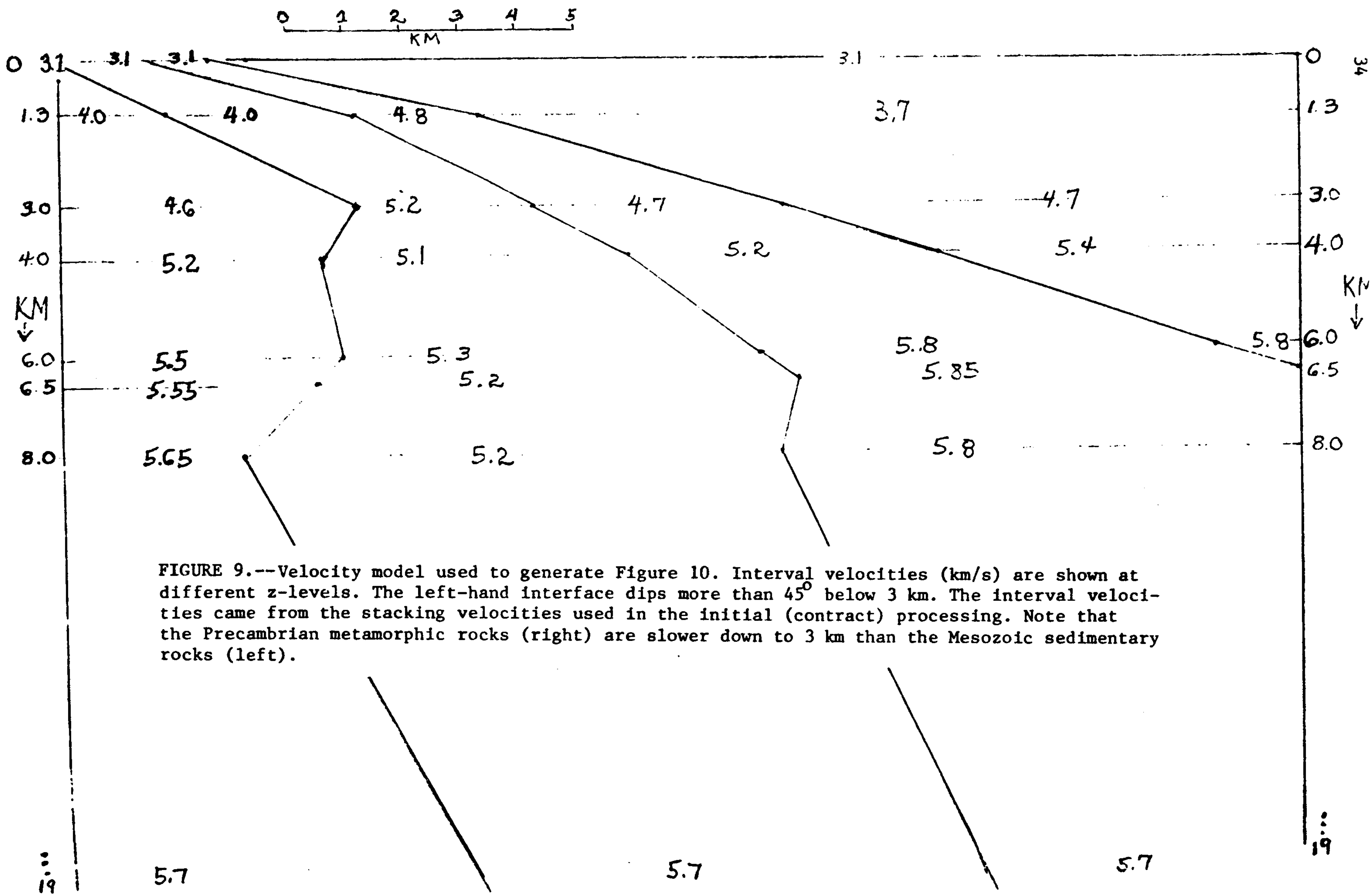

FIGURE 9.--Velocity model used to generate Figure 10. Interval velocities (km/s) are shown at different z-levels. The left-hand interface dips more than 45° below 3 km. The interval velocities came from the stacking velocities used in the initial (contract) processing. Note that the Precambrian metamorphic rocks (right) are slower down to 3 km than the Mesozoic sedimentary rocks (left).

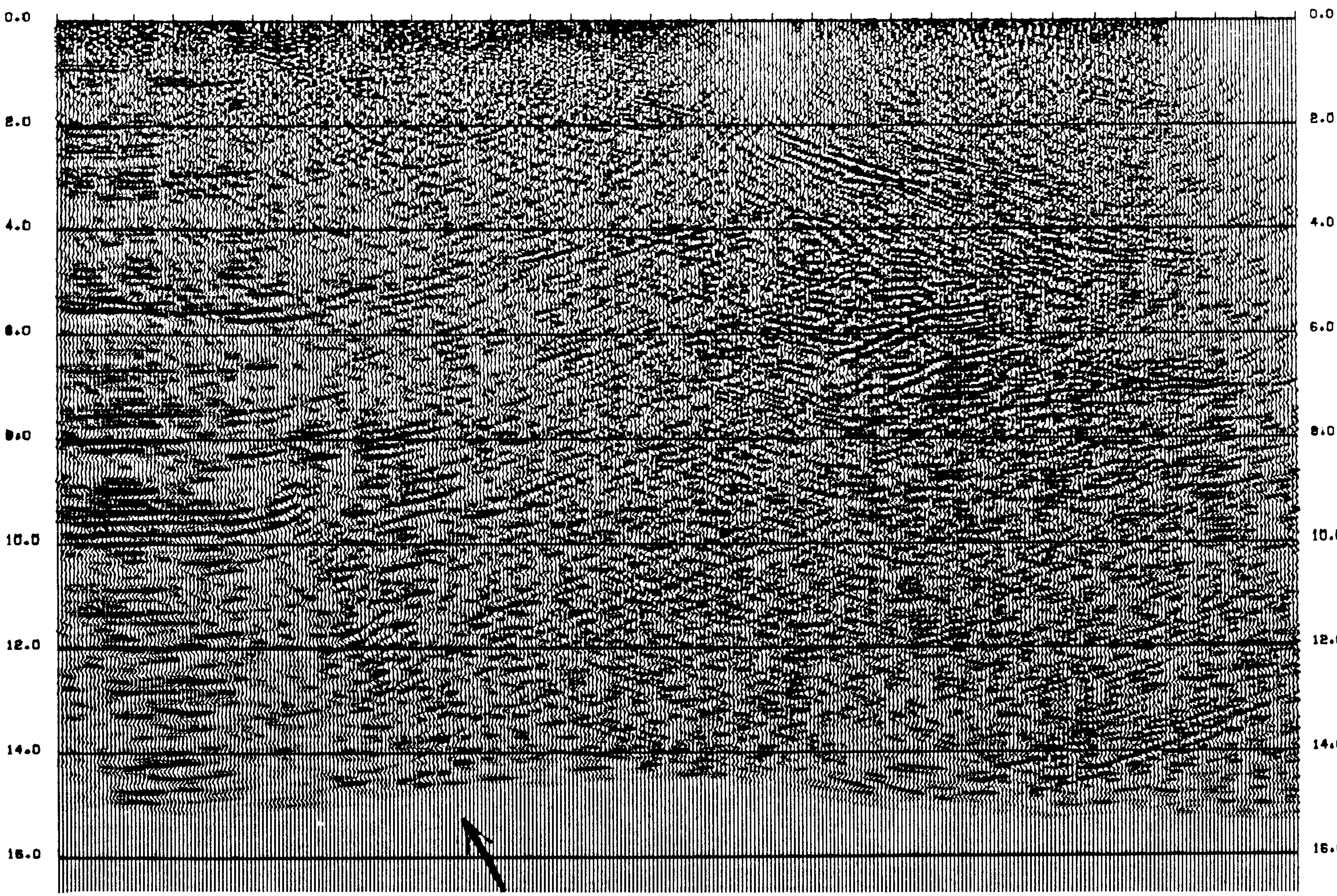

FIGURE 10.--Artifacts in the migration of laterally varying earth dataset due to >45° dipping interface. (Input dataset from Bloxsom and Ottolini's paper, this SEP report.) Viscosity, non-symmetric matrix, and the velocity model in Figure 9 were used. **The arrow points to the artifacts.**

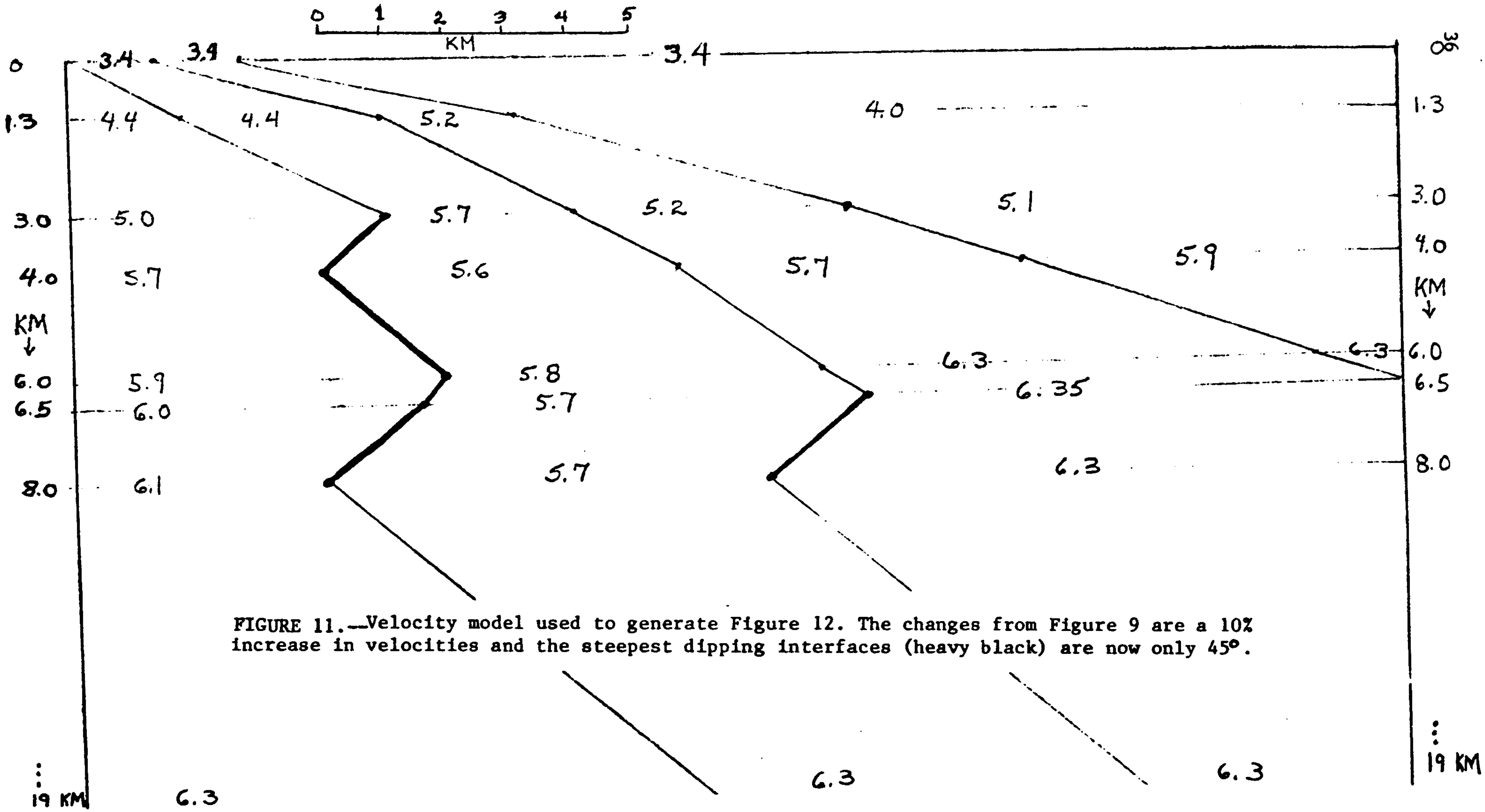

FIGURE 11.—Velocity model used to generate Figure 12. The changes from Figure 9 are a 10% increase in velocities and the steepest dipping interfaces (heavy black) are now only 45°.

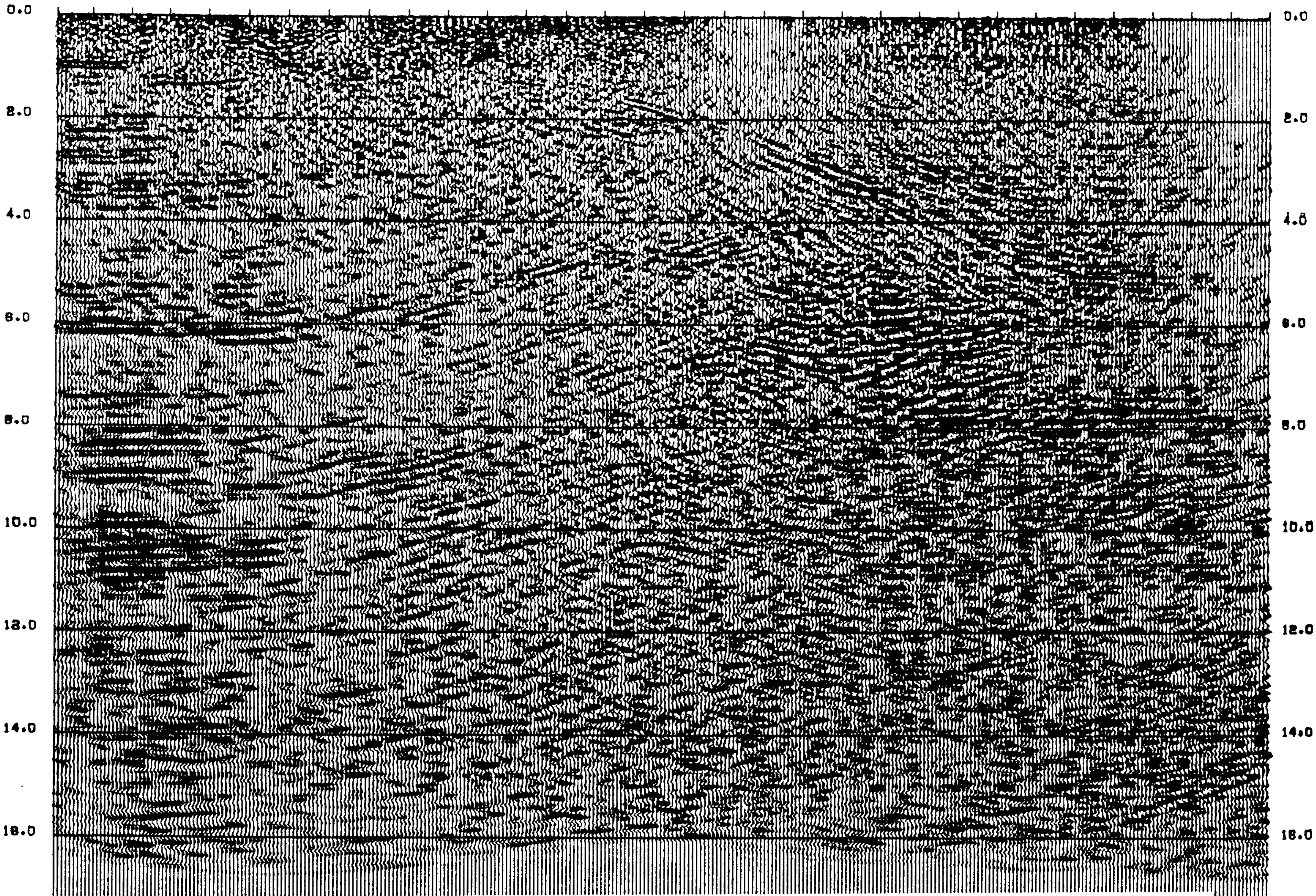

FIGURE 12.--Better migration (apparently no artifacts) due to 45° maximum dip of interface. (Input laterally varying earth dataset from Bloxsom and Ottolini's paper, this SEP report.) Viscosity, symmetric matrix, and the velocity model in Figure 11 were used.