MIGRATED TIME AND MIGRATED DEPTH SECTIONS

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The terms migrated *depth* and migrated *time* sections are becoming commonplace now in the seismic imaging jargon. By definition, a migrated *depth* section is one where the wavefield is presented as a function of the lateral and depth coordinates and (hopefully) represents a close picture of the earth structure. This is the type of section one would like to have to base drillings on. If the medium is laterally constant, a migrated *depth* section is obtained by simply stretching or compressing the time axis of the CMP section according to the vertical velocity structure. What the interpreter usually looks at, however, are migrated *time* sections which have the nice feature that the timing characteristics of multiple reflections are preserved making the multiples more easily interpretable. Migrated *time* sections are the result of inadequate migration algorithms that do not correctly take into account the true velocity structure of the section being migrated. They are considerably cheaper to produce, but if the medium is laterally inhomogeneous, the result can be grossly different from the true *depth* section as demonstrated by Larner and Hatton, 1977.

Hubral (1977) defines a migrated *time* section using image rays in a ray theoretical approach. We wish to do the same here using a wave theoretical approach. We will do this by examining the 15^o wave equation in two different coordinate systems.

If the earth structure is laterally variable, then the ray path for the fastest travel time from some point (x_D, z_D) to the surface does not necessarily emerge at the surface at x_D, but rather at some point x_m. Hubral notes that this particular ray always emerges vertical to the surface and calls them image rays [Fig. 1a].

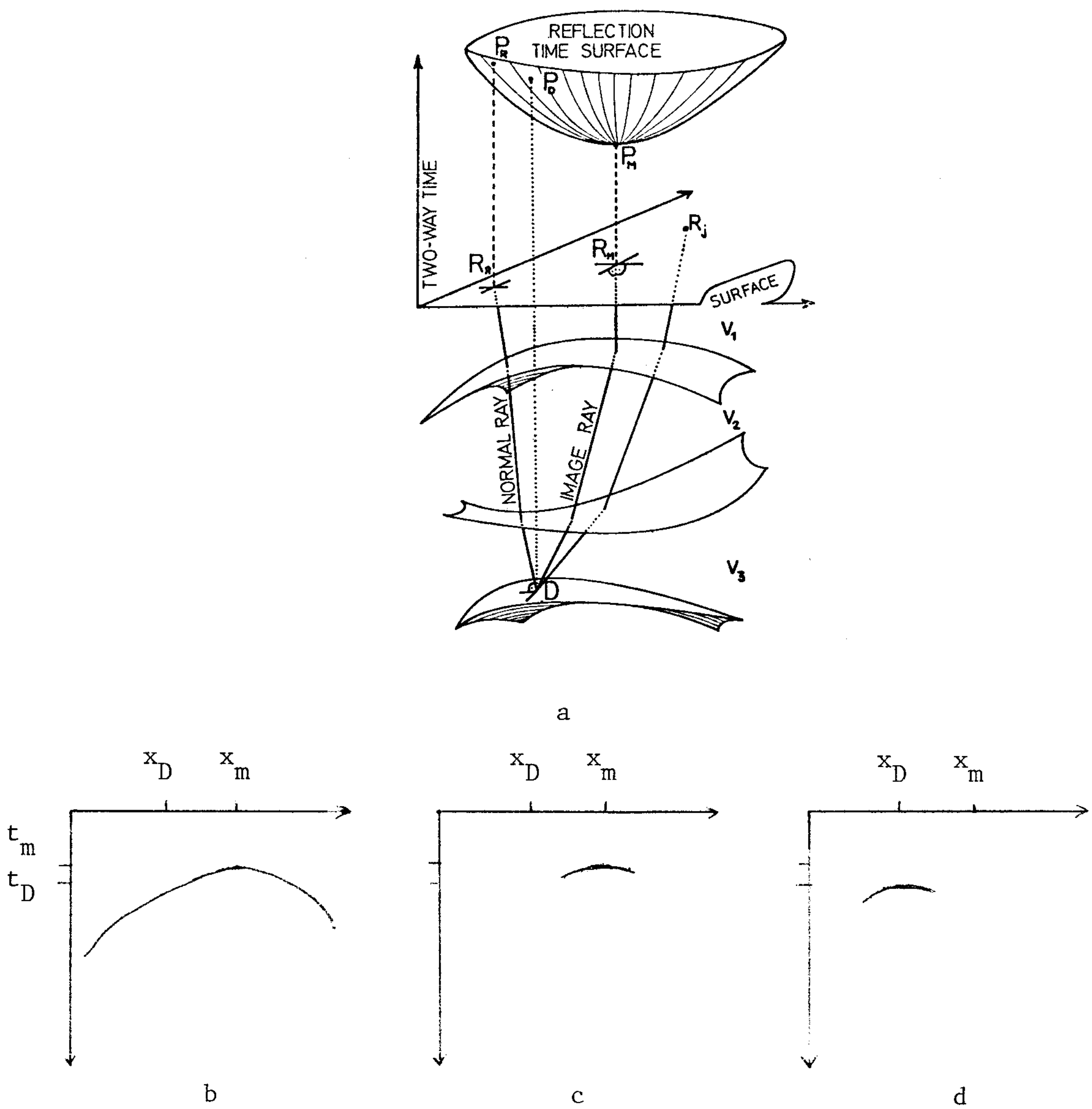

Fig. 1a. Taken from Hubral (1977). Relation between image rays, normal rays and travel time curves. Image rays are normal to the surface of the recording plane and normal rays are normal to the reflecting surface. The fastest ray path from point D (x_D, z_D) to the surface is always along the image ray path. Because of this, the energy from D will tend to migrate under x_m as shown in (b) if the lateral velocity variations are not taken into account in the migration. However, because of the non-hyperbolic shape, the energy from a point scatterer will not completely focus (c). To put the energy in its correct spatial location a mapping must be performed using the image rays (d).

A correct migration of the event in Fig. 1b corresponding to the point at (x_D, z_D) would be located, of course, in a lateral position $x = x_D$. However, to correctly image the point, we would have to use the correct velocity structure of the medium. For summation type programs, this would require some ray tracing as the summation trajectories are nonhyperbolic in laterally varying medium. For finite difference methods, it would require keeping an annoying shifting term in the migration equation. If these requirements are ignored, then the energy in the event in Fig. 1b will not collapse under x_D , but will collapse towards the top at x_m . Hubral defines this as a migrated *time* section and what it represents is just a better focused section. Because the event is non-hyperbolic to begin with, we can't expect the energy to focus very well by summing along hyperbolic paths as seen in Fig. 1c. To get a true *depth* section, a mapping must be done from (x,t) space to (x,z) space. This mapping is accomplished using the image rays and is shown in Fig. 1d.

To see the analog with differential methods, consider the 15^o equation in retarded time coordinates,

$$P_{zt'} = \frac{-\bar{v}(z)}{2} P_{xx} + \varepsilon(x,z) P_{t't'} \qquad (1)$$

where

$$t' = t + \int_0^z \frac{dz}{\bar{v}(z)} , \qquad (2)$$

$$\varepsilon(x,z) = \frac{-\bar{v}(z)}{2} \left[\frac{1}{v^2(x,z)} - \frac{1}{\bar{v}^2(z)} \right] ,$$

and $v(x,z)$ is the true velocity of the medium. The second term on the right hand side of eq. (1) accounts for the lateral velocity variations. For a point scatterer, the effect of this term is to move the hyperbola top laterally so that by the time the hyperbola is collapsed during the downward continuation, it is located in its correct spatial location. Dropping this term and using

$$P_{zt'} = \frac{-\bar{v}(z)}{2} P_{xx} \tag{3}$$

will not do the necessary lateral movement of energy, but will focus energy (within the limits of the 15° equation) and give a migrated *time* section as shown in Fig. 1c. This is not quite the same as the summation type *time* migration as a different rms velocity function can be used at every mid-point, whereas eq. (3) implies the same rms velocity function at every mid-point.

If we use the spatially varying earth velocity in the coordinate transformation instead of some lateral average as in eq. (2), i.e.,

$$t' = t + \int_0^z \frac{dz}{v(x,z)} \tag{4}$$

we obtain the equation

$$P_{zt'} = \frac{-v(x,z)}{2} P_{xx} - 2qP_{xt'} - q^2 P_{t't'} \tag{5}$$

where

$$q = \int_0^z \frac{\partial}{\partial x} \left[\frac{1}{v(x,z)} \right] dz$$

and we have neglected transmission terms of the form $aP_{t'}$. Now the migrated *time* section is obtained by dropping the latter two terms and using

$$P_{zt'} = \frac{-v(x,z)}{2} P_{xx} \tag{6}$$

The migrated section using this equation is closer to that obtained using the summation method with a different rms velocity function at each mid-point. If we keep all of the terms in eq. (5), however, we again obtain the migrated *depth* section.

The effect of the last two terms in eq. (5) can be related directly to the image ray concept. Including the $P_{xt'}$ takes care of the lateral shift of the energy (Fig. 2a). This is easily seen by looking at the equation

$$P_{zt'} = -2qP_{xt'}$$

or

$$P_z = -2qP_x$$

which has solutions

$$P = P(2qz-x) \tag{7}$$

if we neglect the gradient of q. Hence, this is just a shifting term in the mid-point direction.

To see that this term moves the energy in the correct direction, consider the sketch in Fig. 3.

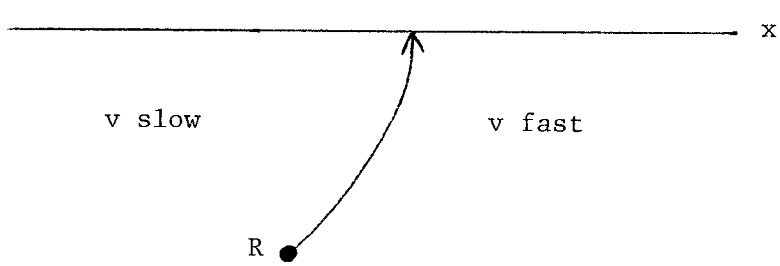

Fig. 3. Image ray to a point R . Velocity increases in the +x direction.

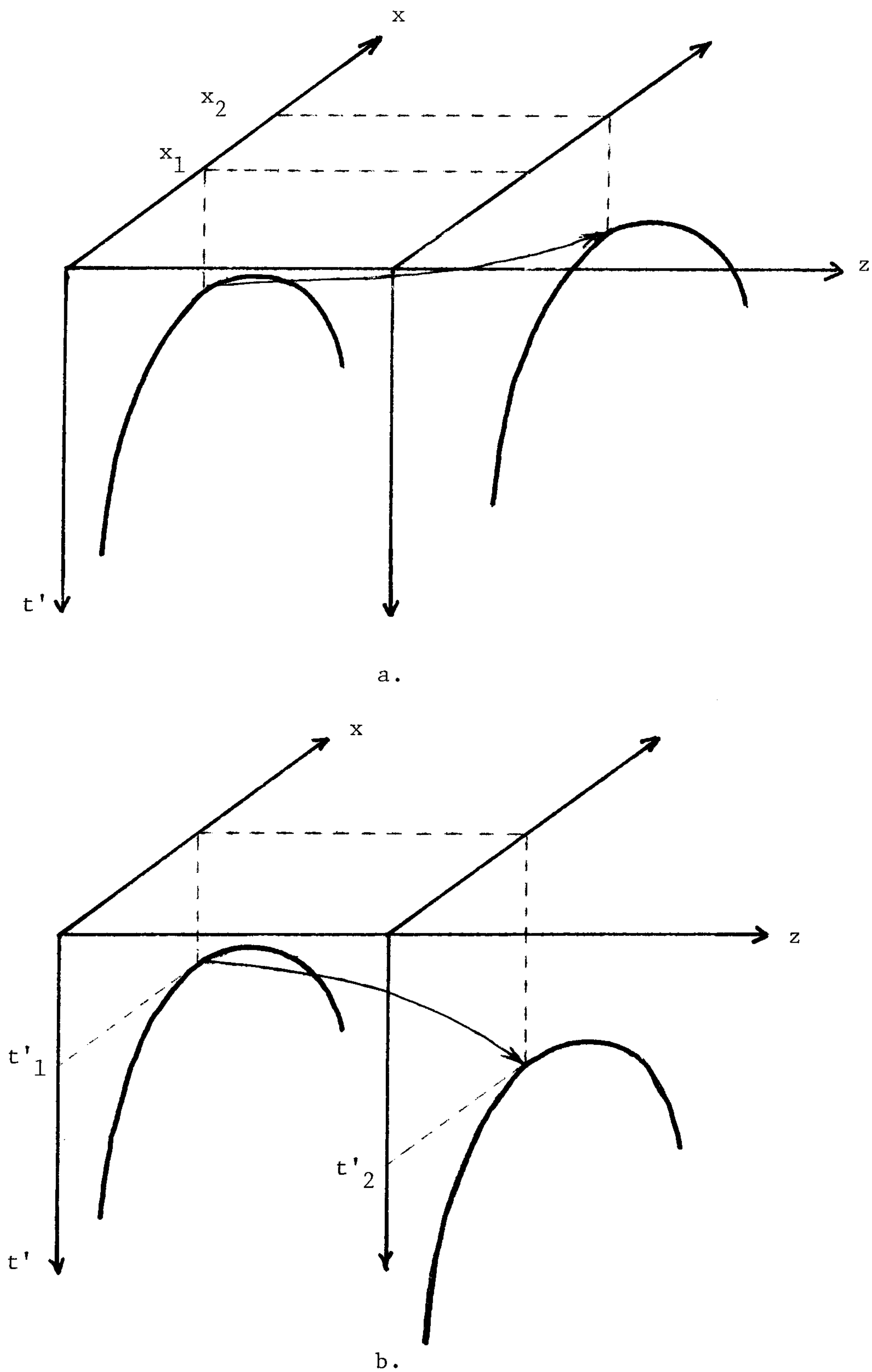

Fig. 2. Effect of $P_{xt'}$ (a) and effect of $P_{t't'}$ (b) in eq. (5) during downward continuation.

The velocity gradient is positive and so the image ray to a point R bends as shown. q is the depth integrated horizontal gradient of the slowness (1/v) and is thus negative. Therefore, applying eq. (7) will move the energy in the -x direction with increasing depth as the receivers are downward continued, as it should.

Similarly, the effect of the $P_{t't'}$ term in eq. (5) is to shift the data temporally during the downward continuation. Note that the coefficient of this term is always negative ($-q^2$). Hence, the effect of including this term is to shift the wavefield down (+t') with increasing depth. This accounts for the fact that the energy does not necessarily migrate to the hyperbola tops, but to some point later in time as seen in Fig. 2b.

Thus, to get a migrated *depth* section, either eq. (3) or (6) may be used followed by an image ray mapping procedure from (x,t') to (x,z) space or eq. (1) or (5) may be used directly. Eq. (1) has two advantages over eq. (5). First, the velocity gradient does not have to be computed. Second, the stopping condition (t=0)

$$t' = \int_0^z \frac{dz}{\bar{v}(z)}$$

is much simpler to code than

$$t' = \int_0^z \frac{dz}{v(x,z)}$$

References

[1] Hubral, P. (1977), "Time Migration - Some Ray Theoretical Aspects," *Geophysical Prospecting*, 25, pp. 738-745

[2] Larner, K. and Hatton, L., (1977) "Depth Migration of Imaged Time Sections," Paper presented at the 47th Annual SEG meeting, Calgary.