RELATING THE DIRECT ARRIVAL TO THE SHOT WAVEFORM

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Practical problems encountered in shot waveform estimation are often due to its long duration caused by oscillations of the high-pressure gas bubbles. Unfortunately, the far-field vertically propagating pulse is not monitored during the actual survey. What is available (at least in deep water) is the direct arrival which has traveled a more or less horizontal path. There are several factors which make this waveform different from the desired vertical waveform. The antenna responses of the shot array and recording group array and the free surface reflections are the most important. Any attempt to use the direct arrival as a source monitor must incorporate the effects of these phenomena. In this paper, we define several linear filters to include these effects, and thus present a deconvolution scheme which uses the direct arrival. The objective is to obtain a reflection seismogram which is a collection of much shorter waveforms and therefore a suitable input to more conventional shot waveform estimators. A derivation of some of these filters is possible from an analysis of the recording field geometry and an equation is developed that should allow for reasonable filter estimates to be made from the recorded data themselves.

A typical recording geometry is depicted in Fig. 1, where we define several time series and geometrical parameters. The waveform sent into the subsurface, $B_v(Z)$, will depend upon the characteristics of each air gun, interaction of the individual gas bubbles due to gun spacing, and the depth of the source array. Let $B(Z)$ be the near-field acoustic pulse from one of the guns, assuming that they are all nearly the same and omnidirectional. We define a filter, $S(Z)$, which sums the near-field vertical pulse from each gun, and a filter, $G_s(Z)$, to account for the ghosting effect due to the source depth. Then we can write

$$B_v(Z) = B(Z)\ S(Z)\ G_s(Z). \qquad (1)$$

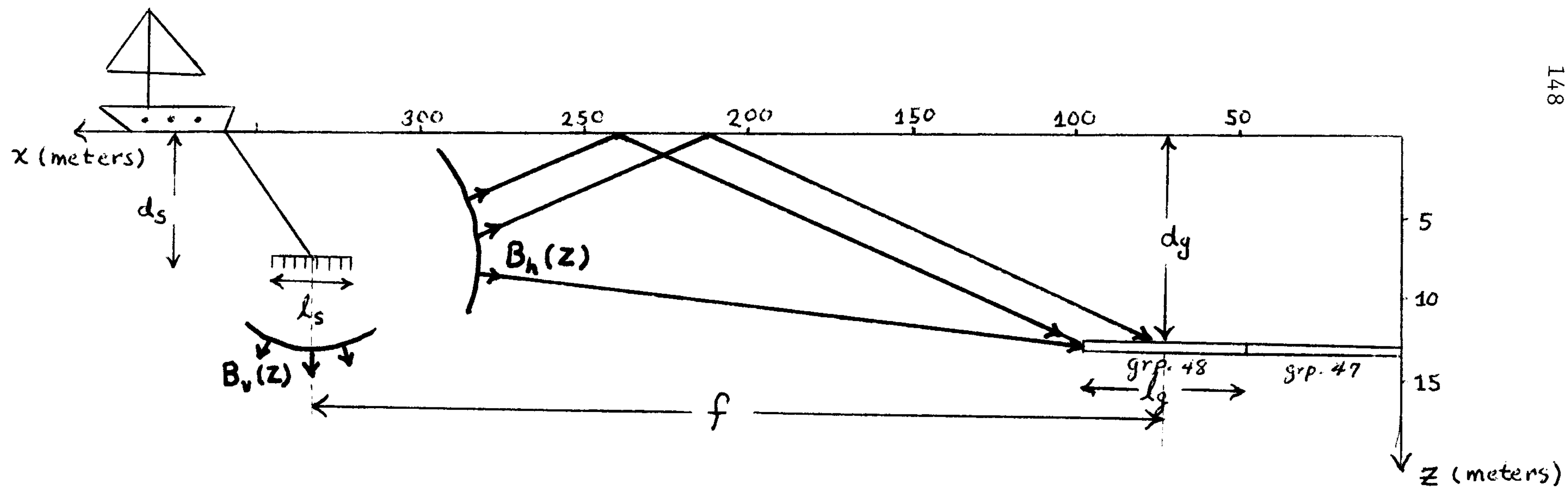


FIGURE 1.—A typical recording geometry. Distance scales are in meters, and there is a 5:1 vertical exaggeration. We define the following Z polynomials and geometrical parameters:

$B_v(Z)$	=	the net vertical waveform sent into the subsurface
$B_h(Z)$	=	the nearly horizontally traveling waveform
d_s	=	depth to the shot array
d_g	=	depth to the cable
f	=	source to near-group offset
l_s	=	length of the shot array
l_g	=	length of the group interval

In the real experiment, an array of guns is used in an effort to produce a net vertical waveform which is shorter than the near-field pulse of each gun. $G_s(Z)$ acts as a differentiator [Eq. (8)] due to the near-perfect free surface reflector. Likewise, we can define a filter, $I_s(Z)$, and write the horizontal waveform as

$$B_h(Z) = B(Z)\ I_s(Z). \tag{2}$$

$I_s(Z)$ can be shown to be an integrator [Eq. (9)] and will depend upon the separation of the guns and the total length of the array.

Neglecting the later arriving vertical waveform, the net wavefield arriving at the streamer cable will be composed of this nearly horizontal shot waveform, $B_h(Z)$, plus various reflected phases from the free surface as shown in Fig. 1. In addition, the recording of a single channel is done over a finite number of phones making up the group. Defining a ghosting filter, $G_a(Z)$, we can write the waveform arriving at the first phone of the near-group as $B_h(Z)\ G_a(Z)$. Let's assume that this waveform is the net waveform traveling down the streamer and recorded by each phone in the group. Then the recorded direct arrival is given by

$$A(Z) = B_h(Z)\ G_a(Z)\ I_g(Z), \tag{3}$$

where $I_g(Z)$ is an integrator due to the recording group. Substitution of (2) into (3) gives

$$A(Z) = B(Z)\ G_a(Z)\ I_s(Z)\ I_g(Z), \tag{4}$$

and using (1) for $B(Z)$ we have

$$A(Z) = \frac{B_v(Z)\ I_s(Z)\ I_g(Z)\ G_a(Z)}{S(Z)\ G_s(Z)}, \tag{5}$$

which relates the recorded direct arrival $A(Z)$ and the vertical shot waveform $B_v(Z)$.

Finally, we wish to write an expression for the colored reflection seismogram $\tilde{R}(Z)$. The ghosting effect due to the cable depth is included by introducing another ghosting filter, $G_g(Z)$. Then,

$$\tilde{R}(Z) = B_v(Z)\ R(Z)\ G_g(Z), \tag{6}$$

where R(Z) is the uncolored or broadband seismogram. Solving for R(Z) and using Eq. (5) to eliminate $B_v(Z)$, we arrive at the desired result,

$$R(Z) = \frac{\tilde{R}(Z)\ I_s(Z)\ I_g(Z)\ G_a(Z)}{A(Z)\ S(Z)\ G_s(Z)\ G_g(Z)} = \frac{\tilde{R}(Z)}{A(Z)}\, O(Z), \tag{7}$$

a deconvolution scheme involving the direct arrival. Now the problem of estimation and deconvolution of a long source waveform becomes one of estimating the operator, O(Z), and applying (7).

Referring again to the recording geometry shown in Fig. 1, we can write analytical expressions for some of the components of the operator O(Z). Expressing the two-way travel time delay due to the source depth ds as $\alpha\,\Delta t$, where α is some number representing units of time delay, and Δt is the sampling interval, the ghosting filter at the shot is

$$G_s(Z) = 1 - Z^{\alpha}. \tag{8}$$

In addition, the summation effect producing the horizontal waveform is given by

$$I_s(Z) = (1 - Z^{\kappa})/(1 - Z^{\delta}), \tag{9}$$

where $\delta\,\Delta t$ is the time delay of each pulse due to the gun separation, and $\kappa\,\Delta t$ is the entire time delay over the length of the array, l_s. The effect of the $1 - Z^{\kappa}$ term is to limit the integration to the finite array length. In writing (9), we have assumed that the gun separation is constant.

The reflection from the free surface of the nearly horizontal waveform will be delayed in time according to the depths of both shot and phone arrays and the offset. Letting that time delay be $\varepsilon\,\Delta t$, where ε will in general be $\ll 1$ since $f \gg$ ds or dg, this ghosting effect is given by

$$G_a(Z) = 1 - Z^{\varepsilon}. \tag{10}$$

The integration effect of the group will be analogous to that at the source array and is given by

$$I_g(Z) = (1 - Z^{\beta})/(1 - Z^{\tau}), \tag{11}$$

where $\tau\,\Delta t$ represents the time delay between the successive phones in the group and $\beta\,\Delta t$ the entire time delay over the group interval l_g. Again, we have assumed that the phones are equally spaced over the group. Assuming near vertical incidence of the upcoming waves and letting $\gamma\,\Delta t$ be the two-way time delay due to the cable depth, we can express the ghosting filter at the cable as

$$G_g(Z) = 1 - Z^{\gamma}. \tag{12}$$

Recalling the operator given in (7),

$$O(Z) = \frac{I_s(Z)\; I_g(Z)\; G_a(Z)}{S(Z)\; G_s(Z)\; G_g(Z)}, \tag{13}$$

and using the filter estimates above, we obtain an estimate of the operator given by

$$\hat{O}(Z) = \frac{(1 - Z^{\kappa})(1 - Z^{\beta})(1 - Z^{\varepsilon})}{(1 - Z^{\delta})(1 - Z^{\tau})(1 - Z^{\alpha})(1 - Z^{\gamma})}. \tag{14}$$

In general, we cannot estimate the complicated effects included in the filter $S(Z)$ from the shot array configuration. The interaction of the gas bubbles to produce a net downgoing waveform is very complicated and most likely a nonlinear process. Moreover, commonly used shot patterns employ several different sized guns, which are not equally spaced, and, oftentimes, not fired simultaneously. Even the individual phones in a group are usually not spaced equally over the entire group length, although this effect is probably negligible. However, we can think of applying a deconvolution using the direct arrival and our estimate of the operator,

$$R(Z) = \frac{\tilde{R}(Z)}{A(Z)}\,\hat{O}(Z), \tag{15}$$

resulting in a seismogram that is a collection of much shorter waveforms. A time domain formulation of (15) will not be stable since the denominator of the operator is not causally invertible. And since the positions of the "holes" in the spectra of these ghosting filters will be highly sensitive to the appropriate time delays, a frequency domain application will most likely involve divisions by very small numbers.

We note that most of the components of the operator given by (13) are short in time, although the operator itself is not. If we write the numerator of the operator,

$$N(Z) = I_s(Z)\ I_g(Z)\ G_a(Z), \tag{16}$$

and the denominator,

$$D(Z) = S(Z)\ G_s(Z)\ G_g(Z), \tag{17}$$

we might be able to estimate these expressions from the data, as in the case of a short source waveform. Referring to Fig. 2, we can write analogous expressions for the gated primaries and multiple including the important ghosting effect at the streamer:

$$P_1(Z) = B_v(Z)\ G_g(Z) \cdot C_1\ , \tag{18a}$$

$$P_2(Z) = B_v(Z)\ G_g(Z) \cdot C_2\ , \tag{18b}$$

$$-M_1(Z) = B_v(Z)\ G_g(Z) \cdot C_1 \cdot C_2\ . \tag{18c}$$

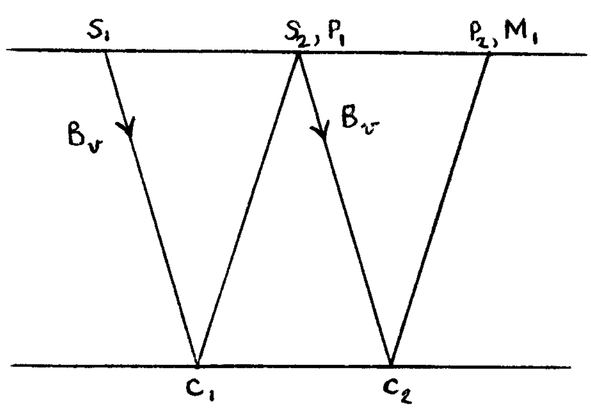


FIGURE 2.—S_1 and S_2 are two adjacent shot points; p_1 is the primary corresponding to the first one and p_2 to the second; m_1 is the first multiple, corresponding to the first shot. The waveform of both sources is considered the same. [Taken from Estevez, SEP-8, p. 204.]

Combining Eqs. (18) yields

$$P_1(Z)\ P_2(Z) \ = -B_v(Z)\ M_1(Z)\ G_g(Z). \tag{19}$$

Substitution of Eq. (5) for $B_v(Z)$ in terms of $A(Z)$, the recorded direct arrival, gives

$$P_1(Z)\ P_2(Z)\ N(Z) \ = -A(Z)\ M_1(Z)\ D(Z). \tag{20}$$

This equation is the basis for the estimation of the numerator and denominator of the operator, $O(Z)$, directly from the data.

There are several ways one might approach solving Eq. (20) for the desired filters. Utilizing convolution matrices for $(p_1 * p_2)$ and $(a * m_1)$ we can write (20) as either

$$\begin{bmatrix} p_1 * p_2 \\ \\ \\ \end{bmatrix} \begin{bmatrix} n \end{bmatrix} = \begin{bmatrix} a*m_1*d \\ \\ \\ \end{bmatrix}, \tag{21a}$$

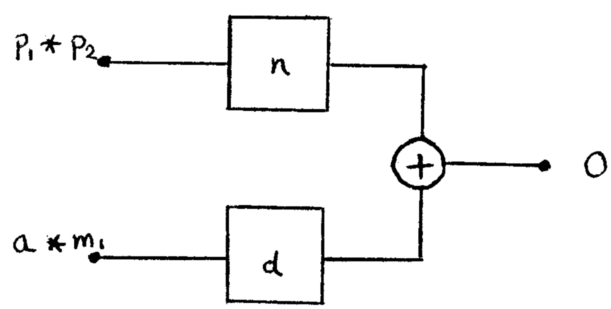
or

$$\begin{bmatrix} \\ a * m_1 \\ \\ \end{bmatrix} \begin{bmatrix} d \end{bmatrix} = \begin{bmatrix} \\ p_1*p_2*n \\ \\ \end{bmatrix}. \tag{21b}$$

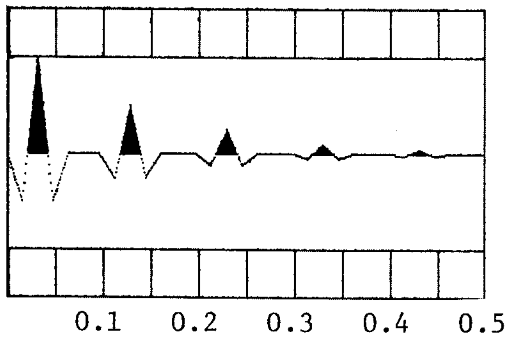
Both (21a) and (21b) represent overdetermined sets of equations for the unknown filter coefficients of n or d. Given an initial guess for d, we first solve (21a) for n. Then with that estimate for n, we solve (21b) for d and proceed in an iterative manner, with the hope that the solutions will converge to those estimates that best minimize, in a least-squared sense, the error, $E_i = (p_1*p_2*n)_i + (a*m_1*d)_i$.

A synthetic model was created to test the validity of this approach. Vertical propagation was assumed, so that $c_1 = c_2$ and $p_1 = p_2$ in Fig. 2 and Eqs. (20) and (21). The numerator of the operator was taken to be $N(Z) = 1 - Z^8$ and the denominator $D(Z) = (1-Z^3)(1-Z^5)$. Figure 3 shows the bubble pulse, the recorded direct arrival, the gated primary, and the gated multiple. Also shown are p*p*n and m*a*d which should sum to zero. The norm of the error in the model was 0.699e-09. The result of solving Eqs. (21) iteratively are shown in Fig. 4. An initial guess of $D(Z) = (1-Z^3)(1-Z^5)$ (the true d) was used. Through five iterations, the estimates of n and d are diverging from the true solutions. The norm of the error in Eq. (20) after the fifth iteration was 0.299e-04. Other starting values of d were chosen with essentially the same results. Once errors are introduced (either from the initial guess or just the approximate nature of the least-squares solution) they tend to accumulate as the iteration proceeds, pushing the estimates further from the desired solutions.

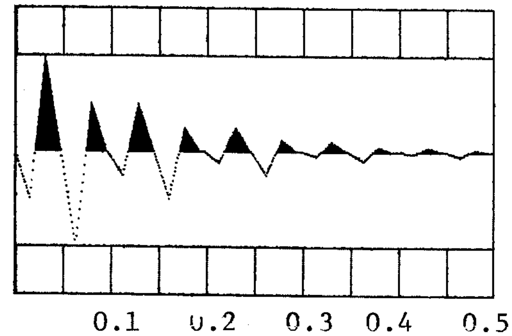
Alternatively, we could view the solution of Eq. (20) as a two-channel least-squares problem as shown below:



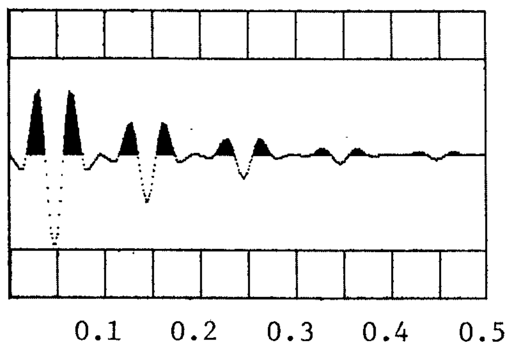
where the output should be zero. Although this formulation seems promising, we have no positive results to report at this time.



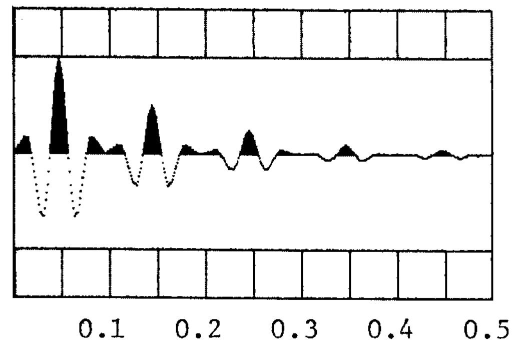
(a) the bubble pulse B(Z)



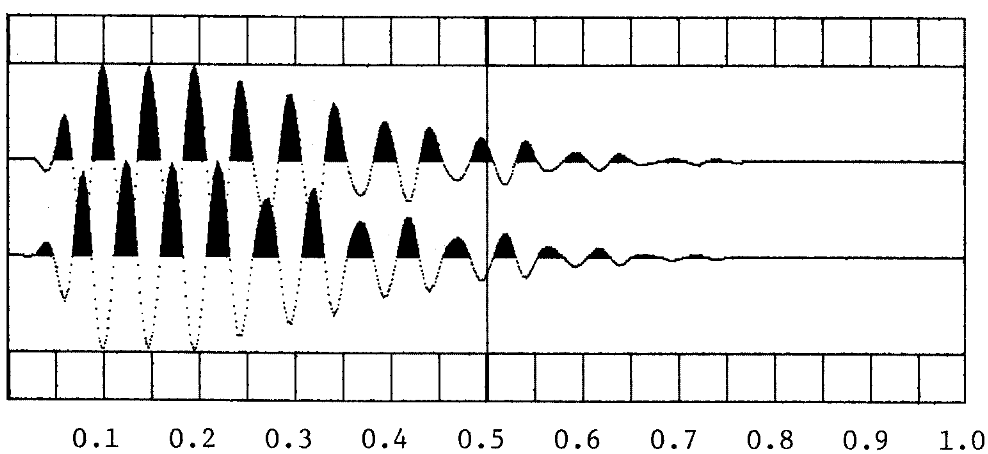
(b) the direct arrival
A(Z) = B(Z) N(Z)



(c) gated primary
P(Z) = B(Z) D(Z) · C

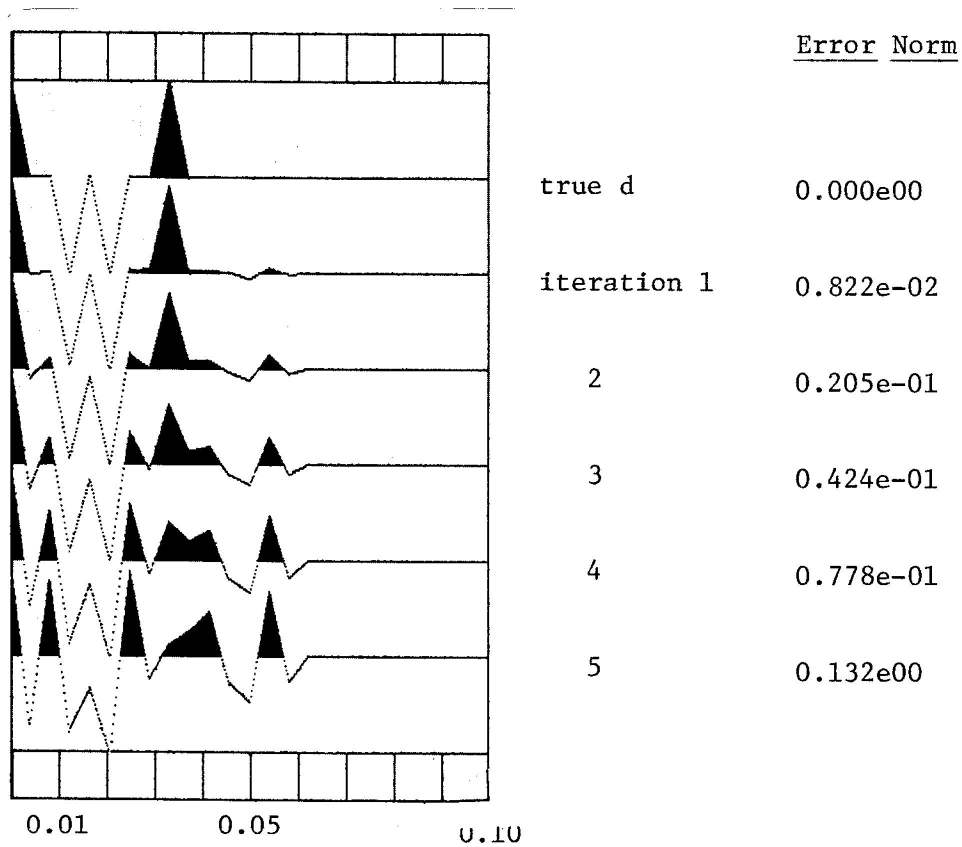


(d) gated multiple
M(Z) = − B(Z) D(Z) · C^2

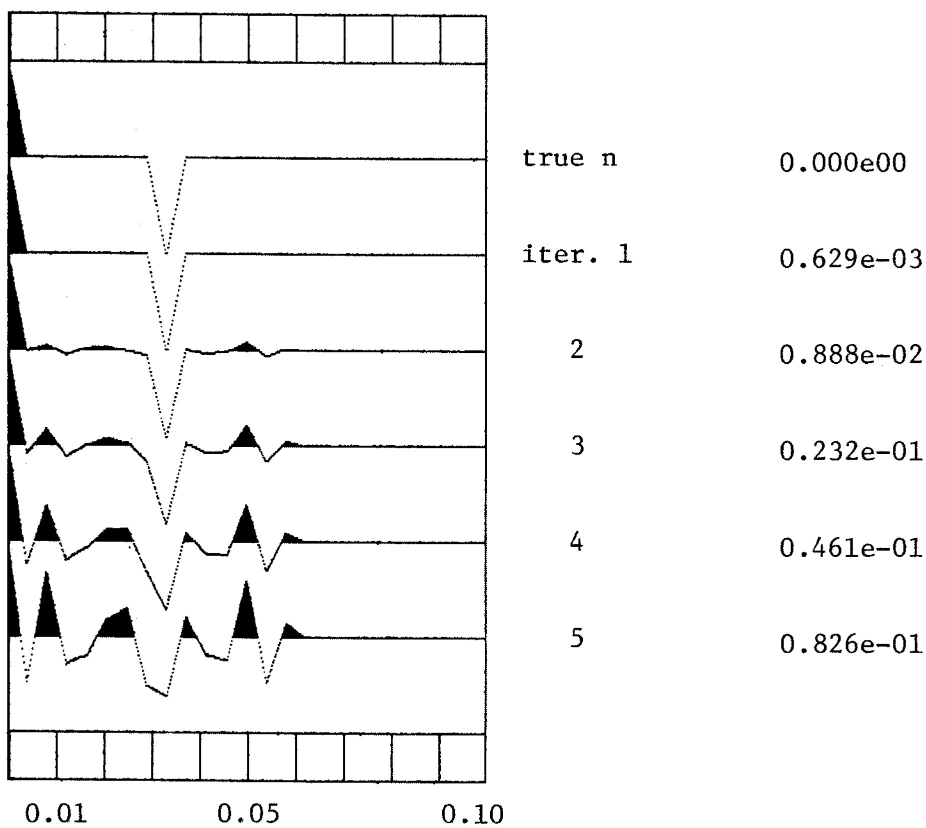


(e) top trace is P(Z) P(Z) N(Z) and bottom trace is M(Z) A(Z) D(Z). The two traces should sum to zero, as given by Eq. (20).

FIGURE 3.—Synthetic traces.



(a) True d and computed d for five iterations of Eqs. (21). Filter length 20-pts long (25 pts plotted). Error norm = $\frac{1}{n}\{\sum_i |d_i^t - d_i^c|^2\}^{1/2}$ where d^t = true d, d^c = computed d, n = 20.



(b) True n and computed n for 5 iterations of Eqs. (21). Filter length is 20 pts (25 pts plotted). Error norm computed as above.

FIGURE 4.