HOW TO MEASURE rms VELOCITY WITH A PENCIL AND A STRAIGHTEDGE

Jon F. Claerbout

Root-mean-square velocity determination will be presented in a form suitable for generalization to laterally inhomogeneous media. Luckily, it also happens to be a nice way to understand conventional velocity estimation.

Consider first a medium with a constant velocity v containing a single horizontal planar reflector at depth z. Let s and g refer to horizontal coordinates of the surface shot and geophone. Two-way travel time t of the echoes will be given by

$$v^2 t^2 = (g-s)^2 + (2z)^2 \ . \tag{1}$$

Differentiating through with respect to g at constant z we get

$$2v^2 t \left(\frac{\partial t}{\partial g}\right)_z = 2(g-s) \ , \tag{2}$$

which we can solve for the velocity

$$v = \left[\frac{g-s}{t} \left(\frac{\partial t}{\partial g}\right)_z^{-1} \right]^{1/2} . \tag{3}$$

It will be seen that everything on the right-hand side of (3) is readily measurable on a data profile. (See Fig. 1.) The velocity v as determined from the data will be

$$v = \frac{F}{[T(T-T')]^{1/2}} \ . \tag{4}$$

The same equation can be used for a common midpoint gather in which case there is considerably less sensitivity to possible dip of the reflector. The equation can also be used on diffractions from point scatterers as seen on seismic sections provided that the value for F is taken to be *twice* the distance from the midpoint to the top of the diffractor.

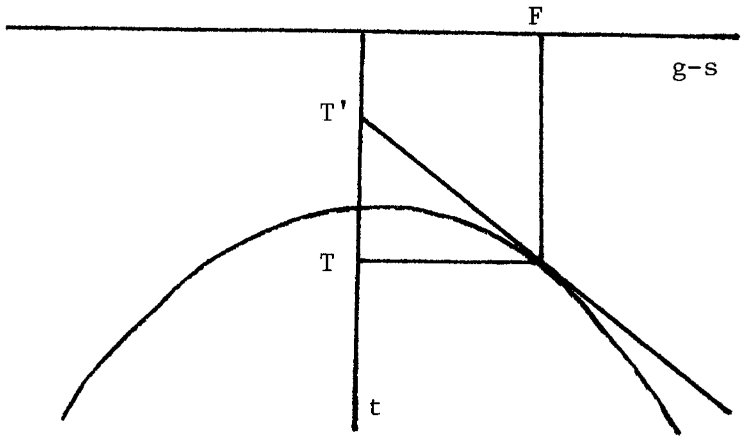

FIGURE 1.—To determine material velocity from hyperbolic arrival time data, first find a spot where the data are of good quality and then draw a tangent to the hyperbolic arrival times. Read from the axes the location of the tangency point $F = g - s$, T, and the slope p of the tangency line $(T - T')/F$. The rms velocity is given in Eq. (4).

This measurement procedure can obviously be used even if the earth does not have a constant velocity and the travel times are not strictly hyperbolic. We will now see that the measured velocity is the root mean square (rms) of the velocity as seen along the ray path. First refer to Fig. 2 to realize that the slope measurement on the data actually measures the Snell's Law parameter $p = (\sin\theta)/v$ of the ray where the straight line is tangent to the hyperbola:

$$p = \frac{\sin\theta}{v} = \left(\frac{\partial t}{\partial g}\right)_z = \frac{T - T'}{F} . \qquad (5)$$

Ordinarily we think of a stratified media velocity $v = v(z) \neq v(x,z)$ as a function of the spatial coordinates. A straightforward computation enables us to transform $v(z)$ to the velocity $v'(p,t)$ of the tip of a ray at time t of Snell's parameter p. To find how far a ray has gone in the horizontal direction, say F, in time T we may integrate the horizontal component of velocity $v'\sin\theta$ with respect to time:

$$F = \int_0^T v'(p,t) \sin\theta \, dt , \qquad (6)$$

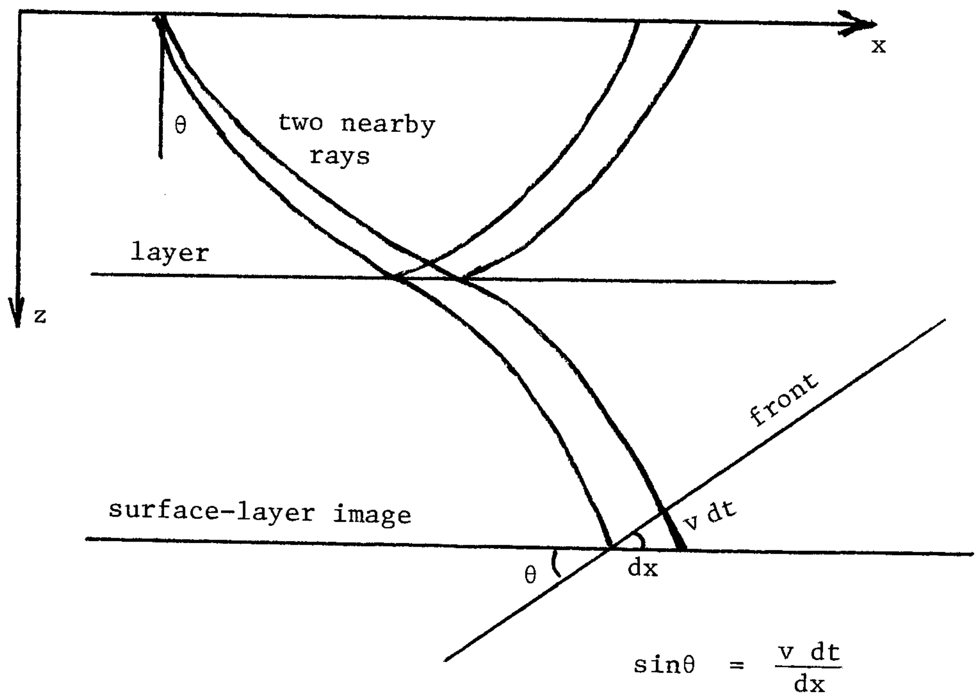

FIGURE 2.—Illustration that at a fixed z (the earth's surface or its image) for two nearby rays, the slope dt/dx of the travel-time curve is the Snell's Law parameter $p = (\sin\theta)/v$ which remains constant $[p \neq p(z)]$ on a ray.

We eliminate angles by use of Snell's parameter,

$$F = p \int_0^T v'(p,t)^2 \, dt \; . \tag{7}$$

The velocity measured by (4) can be interpreted theoretically by substitution of (5) and (7) into (4). First insert (5),

$$v = (F/pT)^{1/2} ; \tag{8}$$

then insert (7), obtaining

$$v = \left[\frac{1}{T} \int_0^T v'(p,t)^2 \, dt \right]^{1/2} . \tag{9}$$

This says that the measured velocity, often called v_{rms} is the root mean square of the velocity seen along the ray path. Notice that by this method we have never made the "small-angle approximation" or "straight-ray approximation" that clutter up some other derivations in the literature.

A slanted coordinate frame can facilitate the velocity measurement. Start by choosing a value of p and replotting the data with a linear moveout of p. In other words, transform the observed upcoming wave $u(g-s,t)$ to $u''(g''-s,t'')$ where

$$g'' = g , \tag{10a}$$

$$t'' = t - p(g-s) . \tag{10b}$$

Now the graphical problem of picking the tangency point of a line to a hyperboloid in (g,t) space has been transformed to the graphical problem of locating tops of hyperboloids in (g'',t'') space.

The accuracy of the velocity estimate can be envisioned as being associated with the accuracy problem of locating the top of a hyperboloid. The equation relating velocity to the tops is found by using (10b) to define $T'' = T - pF$ and eliminating T from (8):

$$v_{rms}^{2} = \frac{F}{p(T''+pF)} = \frac{1}{p}\frac{(F/T'')}{1+p(F/T'')} . \tag{11}$$

We see that the slope F/T'' determines the velocity.

With dipping layers and diffractors the velocity estimation requires also a downward continuation step. First you transform *common shot gathers* to a slant frame and downward continue to the reflectors. Then velocity analysis can be done by finding hyperbola tops on *common geophone gathers*. The reason why this works will be explained in a later paper.