3-D Wave Migration

by Jon F. Claerbout

Let us consider reflection seismic data $P(t, x, y, z=0)$ recorded on the two dimensional surface of the earth. All of the concepts and most of the techniques of 2-D wave migration will be applicable. Because of the burden of the extra dimension we will specialize to slow lateral velocity variation which means that we can migrate CDP stacks. Thus, we are thinking about an equation like

$$P_{zt} = P_{xx} + P_{yy} \tag{1}$$

A sensible approach to this problem is to use the splitting method which means that at alternate steps in z we use the two equations

$$P_{zt} = 2\,P_{xx} \tag{2a}$$

$$P_{zt} = 2\,P_{yy} \tag{2b}$$

Thus, we see that the cost of 3-D migration is twice the cost of 2-D migration of all of the seismic lines. This cost may be compared to the cost of conventional migration as follows. First, we currently believe the cost of conventional 2-D migration to be about equal to wave equation 2-D migration. The costs are the product of three factors. These are NT , the number of time points; NX , the number of midpoints; and NZ which for the wave equation technique is the number of depth points or for the conventional technique, NZ is proportional to the number of points along the semicircle or hyperbola. In 3-D work the semicircle or hyperbola opens up to another dimension, so it seems to me that for conventional migration the cost goes as $NT \cdot NX \cdot NY \cdot NZ^2$, whereas, for wave equation migration the cost goes as $2 \cdot NT \cdot NX \cdot NY \cdot NZ$.

The question arises as to whether either technique can work with undersampled data. I don't believe either technique can work with undersampled data inputs. There does, however, appear to be an undersampling advantage of conventional 3-D migration. The outputs of migration can be undersampled in x or y to avoid display of an unmanageable number of seismic lines. This reduces the cost of conventional migration by a factor NALIAS , but it doesn't help wave equation migration. Thus, it seems that wave equation migration has a cost advantage factor of about NZ / NALIAS .

Looking to higher order accuracy where the equations look like

$$(\partial_t + \frac{\partial^t_{xx} + \partial^t_{yy}}{4}) \partial_z P = \frac{1}{2} (P_{xx} + P_{yy})$$

we recognize that splitting does not work. Luckily, the 2-D explicit technique of Starius (SEP-2, page 114) will remain explicit in three dimensions where δ_{xx} is replaced by $\delta_{xx} + \delta_{yy}$.