Slant Plane Wave Interpretation Coordinates

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Our purpose in introducing slant plane wave interpretation coordinates is to enable us, in the future, to make a better wave equation analysis of lateral velocity variations (within the spread length) and diffracted multiple reflections. Considerable background information is found in earlier SEP reports.

Plane waves at angle θ to the vertical are generated by firing shots at all points along the x-axis at time $t_0(x) = (x \sin\theta)/v$. Hereafter, the velocity v is taken unity. Let the wave reflect from a layer at depth d and arrive at a receiver located at (g,z) after a travel time of t. Figure 1 shows the geometry and how the upcoming wave data at (g,t) is ideally related to a reflectivity function at (y,d). From the figure we obtain the transformations

$$g(y, d, z, \theta) = y + (d-z)\tan\theta \tag{1a}$$

$$t(y, d, z, \theta) = (y + d\tan\theta)\sin\theta + 2d\cos\theta - z/\cos\theta$$

$$= y\sin\theta + d\,\frac{\cos^2\theta + 1}{\cos\theta} - z/\cos\theta \tag{1b}$$

Since the equations are linear, straight-forward means immediately yield the inverse transform

$$y(g, t, z, \theta) = \frac{\cos^2\theta + 1}{2\cos^2\theta}\,g - \frac{\sin\theta}{2\cos^2\theta}\,t + \frac{z}{2}\tan\theta \tag{2a}$$

$$d(g, t, z, \theta) = -\frac{\sin\theta}{2\cos\theta}\,g + \frac{1}{2\cos\theta}\,t + \frac{z}{2} \tag{2b}$$

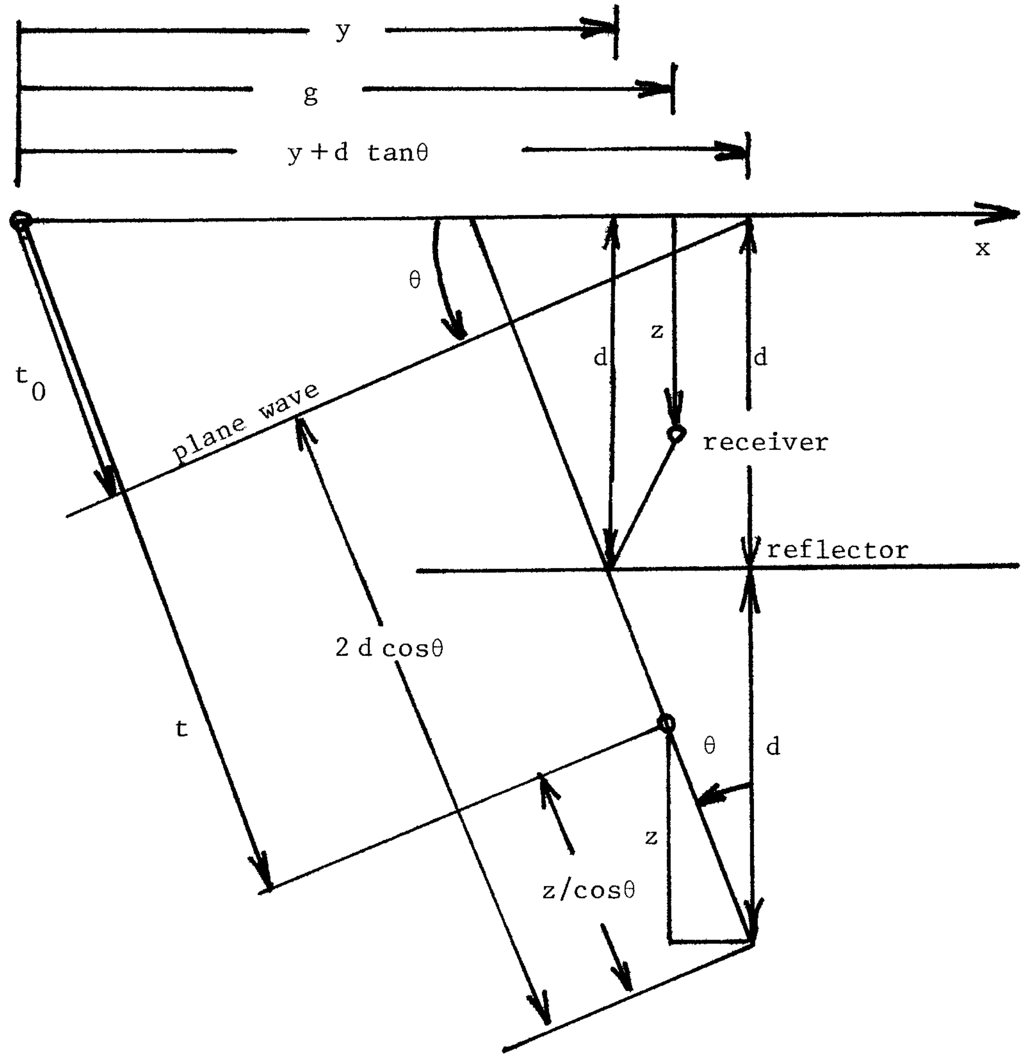

Figure 1. Conversion between observation coordinates (g,t,z) and interpretation coordinates (y,d,z) for a downgoing plane wave initiated by surface sources at times $t_0(x) = x \sin\theta$.

The wave equation (with unit velocity) is

$$P_{gg} + P_{zz} = P_{tt} \tag{3}$$

The expression of the wave field in interpretation coordinates (y,d) is

$$P(g, t, z, \theta) = Q(y, d, z, \theta) \tag{4}$$

In interpretation coordinates the wave equation is

$$[(y_g \, \partial_y + d_g \, \partial_d)^2 + (\partial_z + y_z \, \partial_y + d_z \, \partial_d)^2 - (y_t \, \partial_y + d_t \, \partial_d)^2] \, Q = 0 \tag{5}$$

We may compute all of the coefficients

$$(y_g^2 + y_z^2 - y_t^2) \, Q_{yy} = Q_{yy} / \cos^2\theta \tag{6a}$$

$$(d_g^2 + d_z^2 - d_t^2) \, Q_{dd} = 0 \tag{6b}$$

$$2(y_g \, d_g + y_z \, d_z - y_t \, d_t) \, Q_{yd} = 0 \tag{6c}$$

$$2 \, y_z \, Q_{yz} = \tan\theta \; Q_{yz} \tag{6d}$$

$$2 \, d_z \, Q_{dz} = Q_{dz} \tag{6e}$$

$$Q_{zz} \approx 0 \tag{6f}$$

Adding all of (6) together and making the parabolic approximation $Q_{zz} \approx 0$ we obtain

$$(\partial_d + \tan\theta \, \partial_y) \, \partial_z \, Q + \frac{1}{\cos^2\theta} \, Q_{yy} = 0 \tag{7}$$

For fairly steep propagation angles $(\theta = 60°)$ we can still drop the Q_{yz} term if we wish to study earth models with gentle dips

$Q_d >> Q_y$. This approximation was already made in assuming $Q_{zz} \simeq 0$. Thus, we may commonly expect to work with the upcoming wave equation

$$Q_{zd} = - \frac{1}{\cos^2\theta} Q_{yy} \tag{8}$$

Since we actually record data for point sources rather than plane wave sources it is worthwhile examining how field data may be converted from recording coordinates to interpretation coordinates at $z=0$. First of all, observe the slant stack description in SEP-5 on pages 3-7. Instead of a sum of all the geophones for a given shot as in SEP-5, imagine the reciprocal, a sum of all shots into a given geophone. Such a slant sum would create the data of a plane wave source as indicated here in Figure 1. Note how events on the slant sum of SEP-5, page 3 arrive earlier than the vertical sum because the sum has been placed at a time which corresponds to where the slant summation line crosses the zero-offset trace. Say that the time axis on the slant sum of SEP-5 is called the t' axis. Let us relate this t' to Figure 1. Clearly, at $z=0$,

$$\begin{aligned} t' &= t - t_0 \\ &= 2\,d\,\cos\theta \end{aligned}$$

Hence, the conversion between slant stacks of each common geophone group and interpretation coordinates at $z=0$ is

$$t' = 2\,d\,\cos\theta \tag{9a}$$

$$g = y + d\tan\theta \tag{9b}$$

which easily inverts to

$$d = t' / 2 \cos\theta \qquad (10a)$$

$$y = g - t' \sin\theta / 2 \cos^2\theta \qquad (10b)$$

Equation (9) or (10) may be used in a program to convert common geophone slant stacks to interpretation coordinates of a downgoing plane wave propagating with a component along the positive x-axis. Since the sources (at least the main contributors) are to the left of the geophone in Figure 1, this means that the plane wave is propagating in a direction opposite to the direction in which the ship moved during the survey. If we replace the geophone coordinate in (9) and (10) by the shot coordinate and do the vertical stacks over geophone for a common shot, then we simulate the experiment in which the plane wave propagates in the same direction that the ship goes. Forward and backward plane wave stacks should resemble each other when layers are flat. With dipping beds and diffractions they will look different from one another. Since equation (8) does not distinguish plus θ from minus θ as does equation (7), we may conjecture that for modest dips the forward and backward slant plane wave stacks will be quite similar to each other, departures being evident only for more steeply dipping events.