MAXIMUM ENTROPY SPECTRAL ANALYSIS

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This thesis presents the theoretical development of an unconventional method of estimating power density spectra of stationary time series from partial knowledge of their autocorrelation functions. Conventional methods of estimating a power spectrum from a measured autocorrelation function make the assumption that the correlation function is zero at all lags for which no estimate is available and uses some treatment of the estimated lags to reduce the effect of truncation on the autocorrelation function. The method discussed in this thesis instead retains all of the estimated lags without modification and uses a non-zero estimate for the lags not directly estimated. The particular estimation principle used is that the spectral estimate must be the most random or have the maximum entropy of any power spectrum which is consistent with the measured data. The result of this new analysis technique is that it gives a much higher resolution spectral estimate than is obtained by conventional techniques with very little increase in computing time.

The thesis is conceptually divided into three major parts. The first and largest is concerned wtih single channel spectral analysis from autocorrelation information. The single channel theory has been quite thoroughly developed both in theory and practice. All of the important theorems of the single channel theory are presented and proved, often more than once in different contexts. In addition, the philosophy behind the highly important Burg technique for estimating the second order statistics from samples of a stationary time series is explained and the mathematics presented in a generally useful form. It is argued that the most important description of the second order statistics of a stationary time series is in terms of reflection coefficients (partial correlation coefficients), both from a practical and a theoretical point of view. It is seriously suggested that the old Wiener definition of the power spectrum should be replaced by a new definition involving maximum entropy spectra. An interesting theory of pure line spectra is presented as a limiting case of maximum entropy spectra.

The second major part of the thesis deals with multichannel spectra. Again, all of the important theorems are presented but in less detail than in the single channel case. In general, multichannel spectra do not have the necessary structure to permit the correspondingly rich development as occurs in the single channel case. Because of this, no simple extension of the Burg technique to the multichannel case is possible. A particularly noteworthy proof is the one showing that the multichannel prediction error filter is minimum phase.

The final major portion of the thesis considers the maximum entropy philosophy to be a special case of the general variational principle approach to estimation of functions. A general solution technique is discussed and philosophical comments are presented concerning the consistency and usefulness of measurements.

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