Slant Stacks and Diffracted Multiples

by Jon F. Claerbout

In considering application of Don C. Riley's diffracted multiple reflection techniques to field data, we encountered several practical barriers. The theory can be applied individually to separate profiles, but this is too cumbersome and costly to inspire present implementation. The theory can also be applied to vertical stacks. To see why this is so, consider the upcoming wave equation

$$U_{tz} = - U_{gg} + c(g,z)\, D_t \qquad (1)$$

where the up and downgoing waves U and D are functions of geophone horizontal coordinate g, geophone depth coordinate z, time t, and the surface shot horizontal coordinate s. Note that (1) could be thought of as a separate problem for each numerical choice of s. Now let us sum equation (1) over s. We can commute the operations of summation and partial differentiation obtaining

$$\partial_{tz} \sum_s U = - \partial_{gg} \sum_s U + c(g,z) \sum_s D \qquad (2)$$

Defining vertically stacked sections by

$$U'(g,t,z) = \sum_s U(g,s,t,z) \qquad (3a)$$

$$D'(g,t,z) = \sum_s D(g,s,t,z) \qquad (3b)$$

Equation (2) now becomes the equation for vertically stacked sections

$$U'_{tz} = - U'_{gg} + c(g,z)\, D'_t \qquad (4)$$

Following Claerbout (SEP September 1974, p. 377) it can be found that if one is willing to neglect offset dependence of reflection coefficient that a section equation like (4) can be found for stacks over offset f. The trouble is that we cannot get an equation for the stack if we do NMO before stacking. To see why, recall for example SEP September 1974, page 76, equation (5)

$$\left((h/d)\,(\partial_y + \partial_h) + \partial_d \right) \partial_r Q = -\frac{1+h^2/d^2}{4}\, d\, (\partial_y + \partial_h)^2 Q \quad (5)$$

In this equation h is the half-offset. Now if we sum (5) over h we will not be able to bring the sum through the coefficients as we did in going from (1) to (2). Riley and I took the attitude that even though we can't prove that (5) should apply to CDP stacked sections, maybe it does, at least approximately. Unfortunately we were unable to establish the validity of such an approach with the data we had. Furthermore, we began to appreciate that diffractions on multiples, unlike diffractions on primaries, can be rather strongly offset dependent. Consequently, we thought seriously of forming a vertical stack (over offset) of some field data. We were immediately faced with the realities of present day data recording, namely, the sizable offset of the recording cable from the source points. This problem is particularly severe since it is the inner traces which are the main contributors to a vertical stack. To estimate the importance of the absence of these traces we took the inner traces and compared, for the data at hand, the lateral shift between the first bounce off the sea floor and the first multiple. This shift was sizable compared to the topography on the sea floor, so we were discouraged about interpolating the zero and small offset traces from the inner traces.

Another approach to the problem can now be suggested. The problem with equations controlling NMO data as (5) is that the coefficients of the equations are offset dependent so that summation over offset cannot be brought through the coefficients. It may now be recalled that the slant frame equations have laterally independent coefficients. But the slant frame transformations achieve much of what is achieved by NMO. This suggests that perhaps a rigorous theory for diffracted multiples can be found in some slant frame. In such a frame the absence of recording inner traces should have little significance. Also, it may yet turn out that some form of CDP stack may be controlled, with a reasonable accuracy, by up and downgoing section equations.