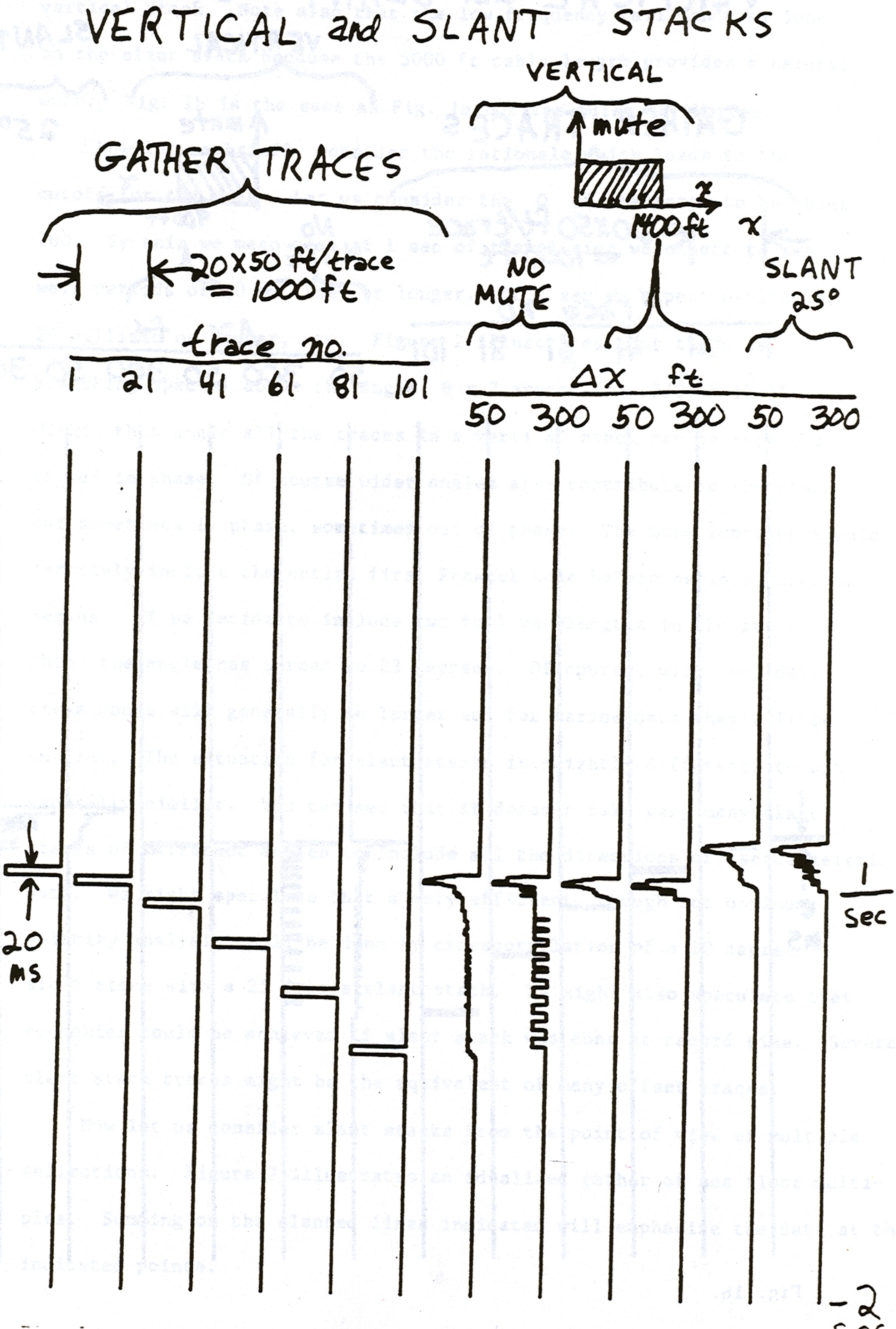
Slant Stacks and Radial Traces

by Jon F. Claerbout

In ordinary geophysical practice we frequently utilize NMO stacks. In an NMO stack seismograms have time axes compressed in such a way as to have all signal events in alignment before stacking. This is to provide best signal enhancement. In our studies of multiple reflections we have had theoretical difficulties with the NMO stack. The strong multiple reflection interference seen in deep water data motivates efforts to accurately predict multiple reflections. Accuracy becomes very important in the prediction of these high amplitude events. Irregularities on the sea floor build up complex diffractions on multiple reflections, especially peglegs. So far, when restricted to NMO stacked data we are unable to predict diffracted multiples from primaries. When stacking into a common geophone is done without moveout correction, then we simulate a downgoing plane wave and diffracted multiples can be accurately predicted and removed by the methods of Don C. Riley (SEP No. 3). We are now making progress on a similar theoretical approach called "slant stacks" which may also allow accurate prediction of diffracted multiples. Who knows, someday we may even know how to do accurate multiple predictions directly on the NMO stacks. Meanwhile, it may be that in some areas of the world vertical stacks or slant stacks may be worthwhile. In a perfectly stratified environment the NMO stack should have a better signal to noise ratio. However, when lateral irregularities become noticeable,"noise" becomes a signal generated phenomena, not a random additive quantity, and then vertical and slant stacks may be superior to NMO stacks because diffractions are predictable. It will be seen that the "radial traces" described by M. Turhan Taner in his paper "Long Period Multiples and Their Suppression" presented at the 1974 SEG meeting are closely related to "slant stacks".

Figure 1a shows some synthetic vertical stacks. For a wavelet pulse I have chosen the rectangle function of 20 ms duration. The rectangle function has more high frequency and more d.c. than realistic seismic data and was chosen in order to exaggerate the various truncation effects. The synthetic gather traces arise from an earth model which consists of a single layer at 1 sec two way travel time. Trace equalization was done before stack. The velocity is taken to be 5 ft/millisec and the trace separation is taken to be 50 ft. The displayed traces are 1, 21, 41, 61, 81 and 101. The interval between displayed traces is 1000 ft. The first vertical stack is done with no mute at a trace spacing of 50 ft. Note that the result resembles a filtered version of the near offset trace (trace number 1 has 25 feet of offset). The filter resembles the half order integral described in SEP 1, page 208. Note the slowly decreasing tail on the wavelet. The low frequency in this tail would of course be filtered out by ordinary frequency filtering. In the next vertical stack we see a series of rectangular pulses in the tail because the 300 ft trace spacing is too great. These pulses are undesirable and could easily be removed with a mute. A good choice for a mute would be some tapering function, but to emphasize truncation effects I chose the rectangle function which cuts off at 1400 ft. (More later about choice of mute cutoff.) The final two stacks were done on a 25 degree slant.


VERTICAL and SLANT STACKS
VERTICAL
mute
x
1400 ft
x
GATHER TRACES
20 x 50 ft/trace = 1000 ft
NO MUTE
SLANT 25°
trace no.
1 21 41 61 81 101
Δx ft
50 300 50 300 50 300
20 ms
1 sec
-2 sec

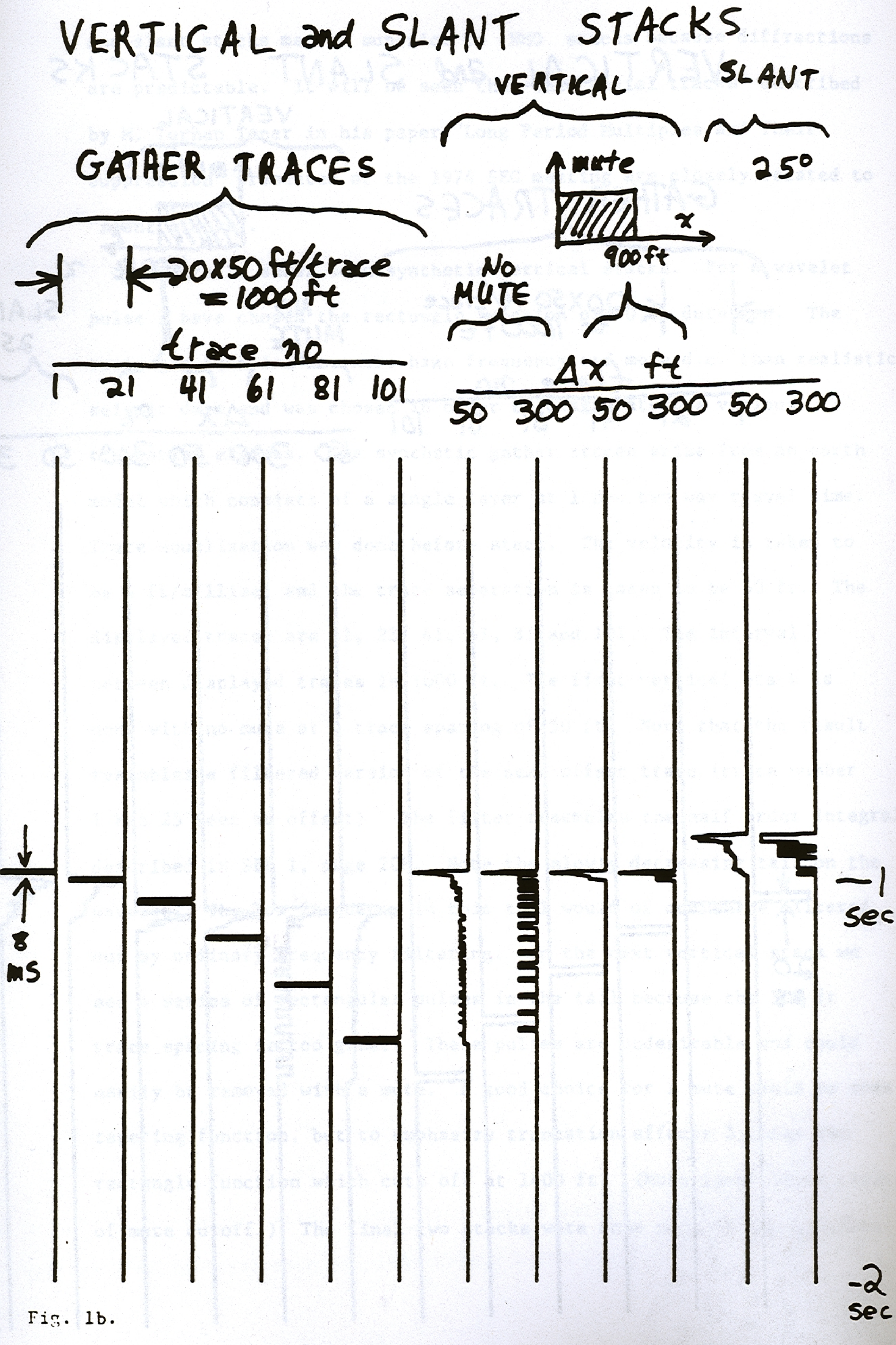

VERTICAL and SLANT STACKS
VERTICAL
SLANT
GATHER TRACES
mute
25°
x
900ft
20x50 ft/trace
=1000ft
No
MUTE
trace no
1 21 41 61 81 101
Δx ft
50 300 50 300 50 300
8
ms
-1
sec
-2
sec

Fig. 1b.

Notice that the arrival is slightly before the 1 sec arrival of the vertical stack. Note also that the low frequency tail isn't as long on the slant stack because the 5000 ft cable length provides a natural mute. Fig. 1b is the same as Fig. 1a but the pulse is shorter.

Now let us briefly consider the rationale which leads to the cutoff for the mute. Let us consider the Q of the earth to be about 100. By this we mean that at 1 sec of travel time we expect to see wave periods of 10 millisec or longer. At 2 sec we expect periods of 20 millisec or longer, etc. Figure 2 illustrates that there is something special about the angle $\theta = 2 \arccos (1 - .5/Q) = 11.5°$. Within this angle all the traces in a vertical stack can be expected to add in phase. Of course wider angles also contribute to the stack, but sometimes in phase, sometimes out of phase. The mute function should certainly include the entire first Fresnel zone before serious tapering begins. If we decide to include two full wavelengths in the stack then the angle has spread to 23 degrees. Of course, with land data these cones will generally be larger and for marine data they will be smaller. The situation for slant stacks is slightly different, though generally similar. You can see that it doesn't take very many slant stacks of different angles to include all the directions of useful seismic data. We might speculate that a very efficient, though not optimum, velocity analysis could be done by crosscorrelation of a 10 degree slant stack with a 25 degree slant stack. We might also speculate that economies could be achieved if slant stack was done at record time. Severa slant stack traces might be the equivalent of many offset traces.

Now let us consider slant stacks from the point of view of multiple reflections. Figure 3 illustrates an idealized gather of sea floor multiples. Summing on the slanted lines indicated will emphasize the data at th indicated points.

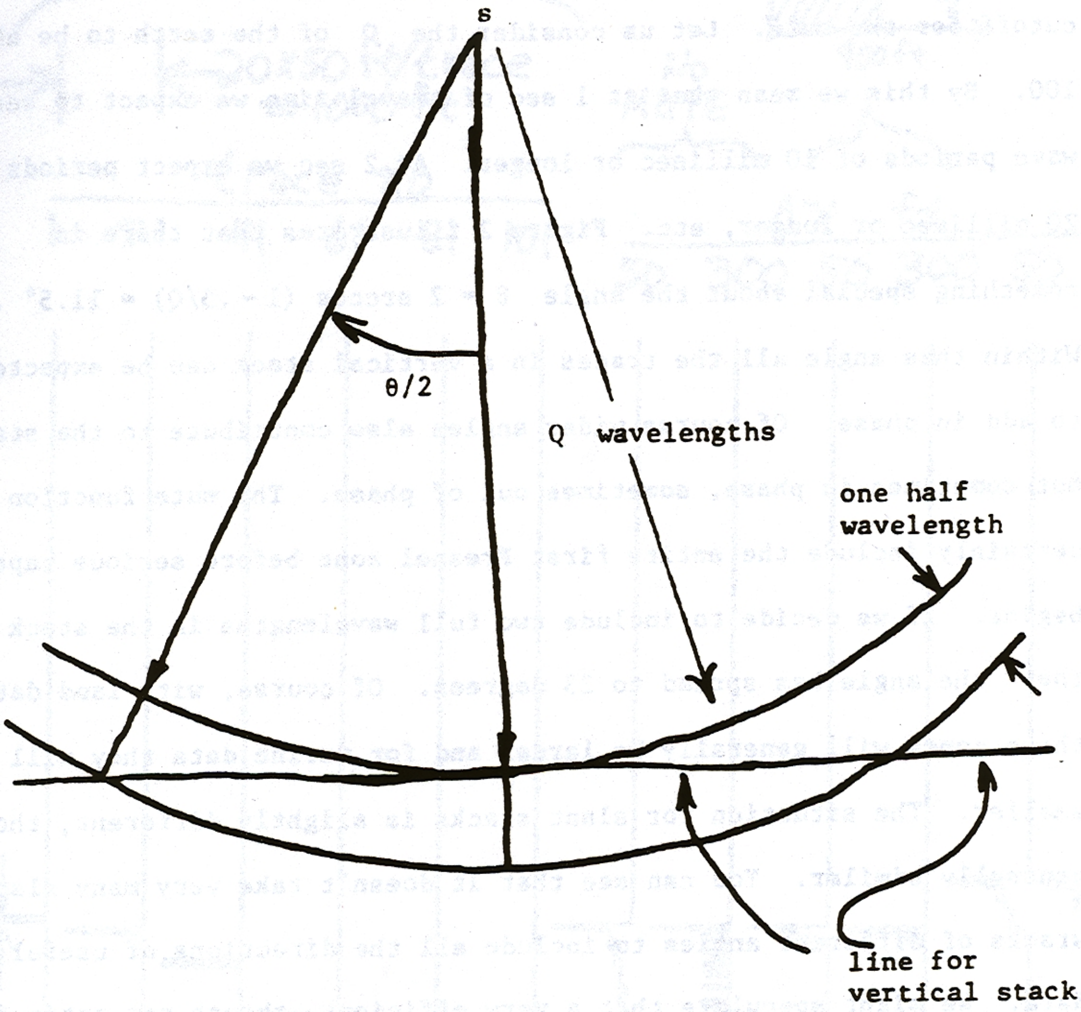


Fig. 2. Definition of Fresnel Zone.

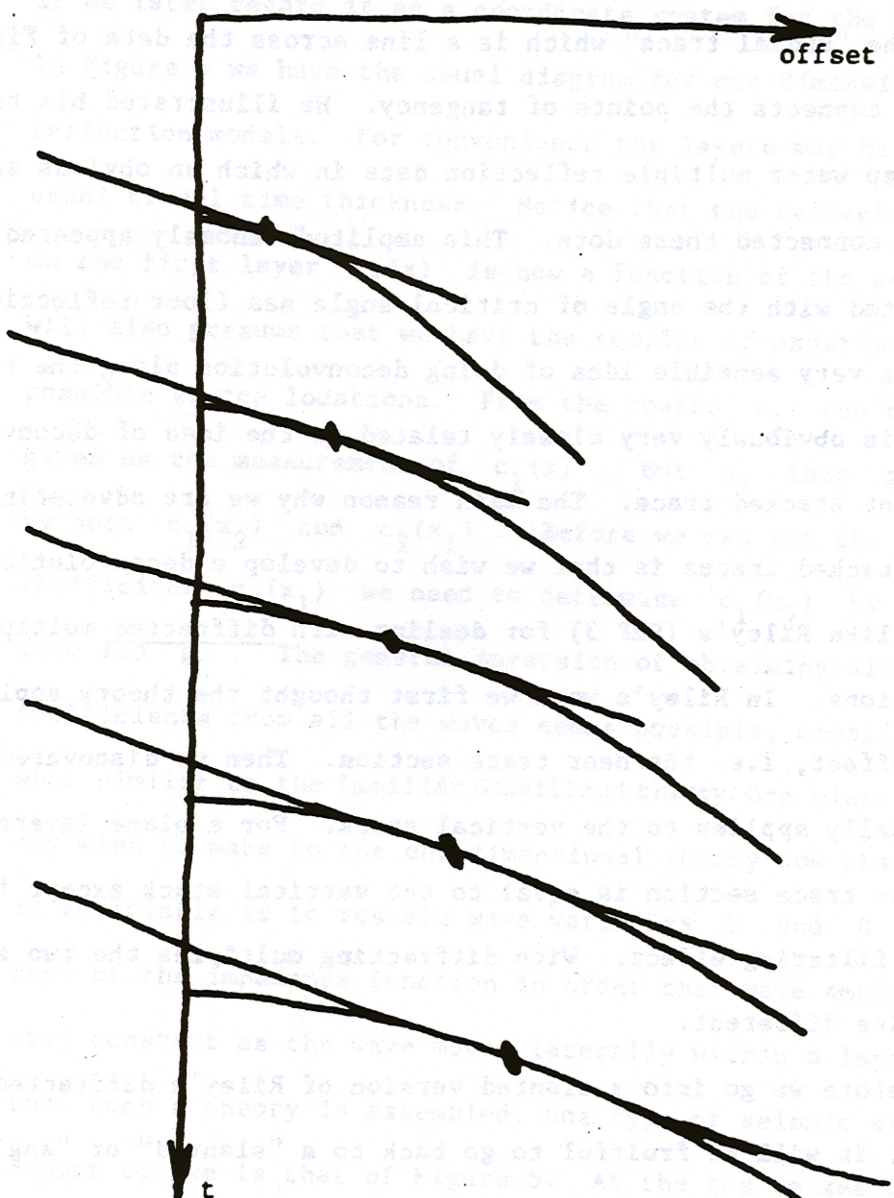


Fig. 3. A family of hyperbolics with tangent lines for vertical stack indicating angle of slant stack. For a slant stack the moveout function is linear. Maximum contribution comes at the point of tangency but large contribution continues within the entire first Fresnel zone (about 12°). The slant stack will be similar to Taner's "radial trace" which is the data along the line which connects the points of tangency.

of tangency. In his SEG talk Taner suggested doing deconvolution along the "radial trace" which is a line across the data of Figure 3 which connects the points of tangency. He illustrated his talk with some deep water multiple reflection data in which an obvious amplitude anomaly connected these dots. This amplitude anomaly appeared to be associated with the angle of critical angle sea floor reflection. Tanner's very sensible idea of doing deconvolution along the radial traces is obviously very closely related to the idea of deconvolving the slant stacked trace. The main reason why we are advocating the slant stacked traces is that we wish to develop a deconvolution theory like Riley's (SEP 3) for dealing with <u>diffracted</u> multiple reflections. In Riley's work we first thought the theory applied to small offset, i.e. the near trace section. Then we discovered that it actually applies to the vertical stack. For a plane layered earth the near trace section is equal to the vertical stack except for the slight filtering effect. With diffracting multiples the two situations are quite different.

Before we go into a slanted version of Riley's diffracted multiple theory, it will be fruitful to go back to a "slanted" or "angle trace" one dimensional theory for multiple reflections. Let us consider a highly improbable earth model in which velocity is constant but impedance is variable. The impedance of any layer may be a function of the lateral coordinate, but it will vary slowly in comparison with wavelengths of interest. It is worthwhile studying the mathematical implications of this earth model. The model may be rather improbable as it stands, but it may be useful for data processing, particularly if we later regard it as a coordinate system for the wave equation. In Figure 4 we have the usual diagram for one dimensional multiple reflection models. For convenience the layers may be taken as having equal travel time thickness. Notice that the reflection coefficient on the first layer $c_1(x)$ is now a function of the x-coordinate. We will also presume that we have the results of experiments with all possible source locations. From the source s_1 the receiver g_1 gives us the measurement of $c_1(x)$. But s_1 into g_2 is affected by both $c_1(x_2)$ and $c_2(x_1)$. Before we can get the deeper reflection coefficient $c_2(x_1)$ we need to determine $c_1(x_2)$ by means of s_2 into its g_1. The general inversion of obtaining all the reflection coefficients from all the waves seems possible, complicated, and somewhat similar to the familiar Goupillaud theory. One minor modification we may wish to make to the one dimensional theory now that the impedance is x-variable is to rescale wave variables U and D by the square root of the impedance function in order that wave amplitudes should stay constant as the wave moves laterally within a layer. Supposing that such a theory is assembled, one type of seismic section we would expect to see is that of Figure 5. At the top we see two possibile peg-leg paths, but as the shot-receiver midpoint is moved left one path will be lost before the other. Thus, a major departure from one dimensional vertical incidence theory is the separation of the two types of pegleg events. It seems quite possible that a one dimensional Noah type processor on radial traces or on a slant stack might be useful for deep water multiples in many areas of the world.

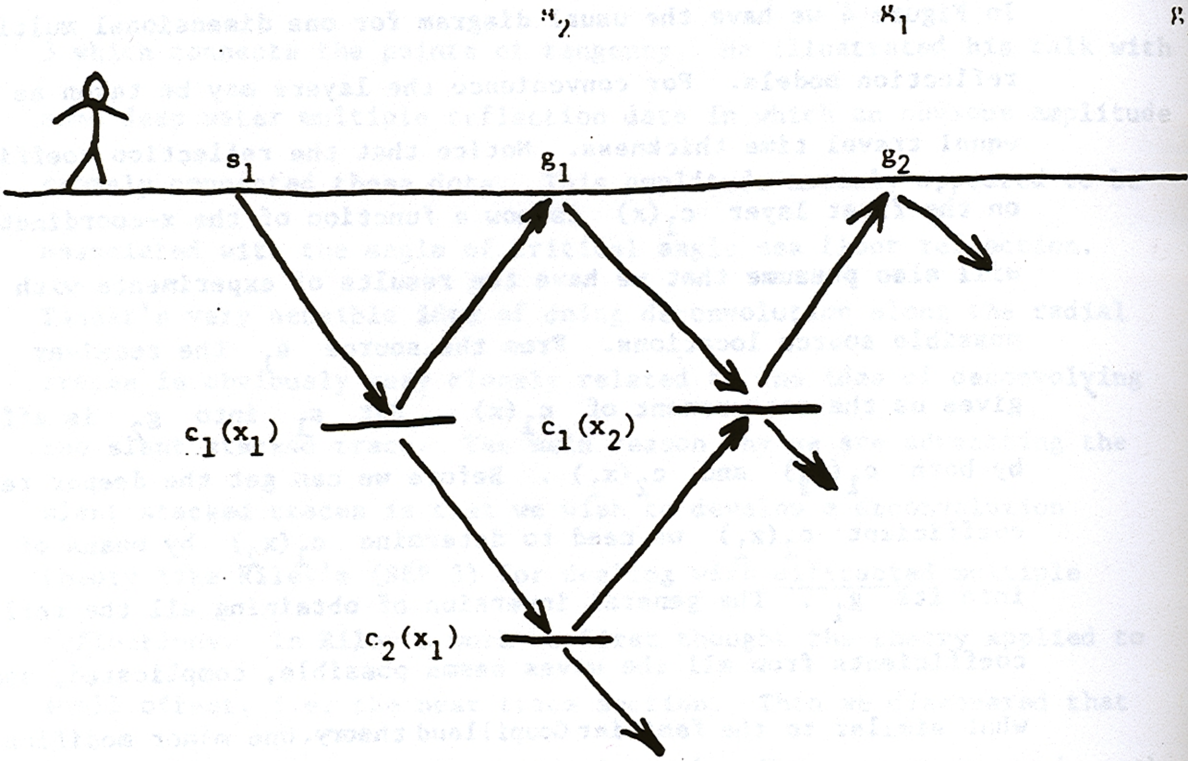


Fig. 4. A model with equal thickness layers of slowly laterally variable impedance and constant velocity. From the source s_1 the receiver g_1 gives us the measurement of $c_1(x)$. But s_1 into g_2 is affected by both $c_1(x_2)$ and $c_2(x_1)$. Before we can get the deeper reflection coefficient $c_2(x_1)$ we need to determine $c_1(x_2)$ by means of s_2 into its g_1 . The general inversion of obtaining all the reflection coefficients from all the waves seems possible, complicated, and somewhat similar to the familiar Goupillaud theory.

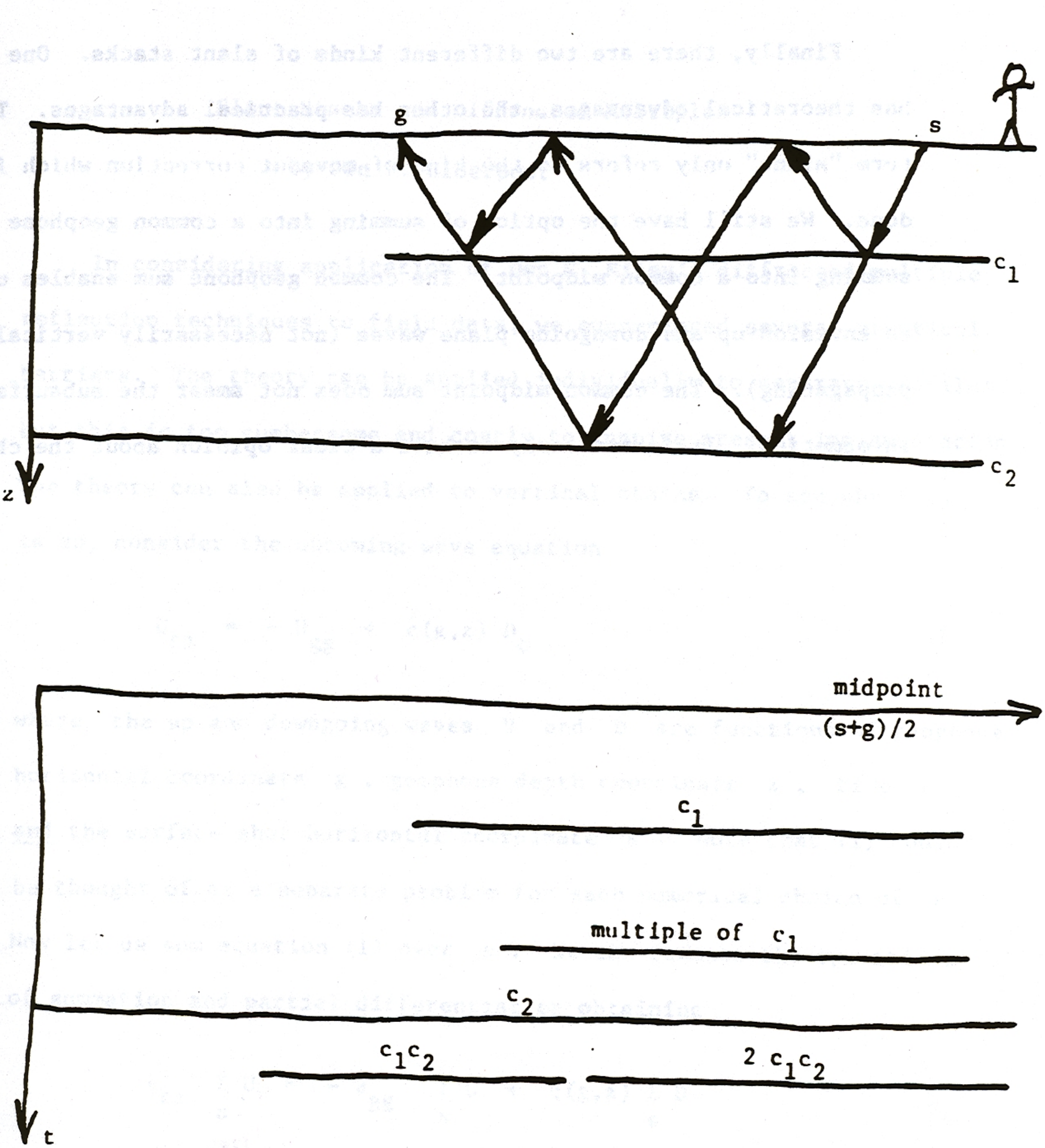


Fig. 5. Peglegs from a terminating shallow reflector.

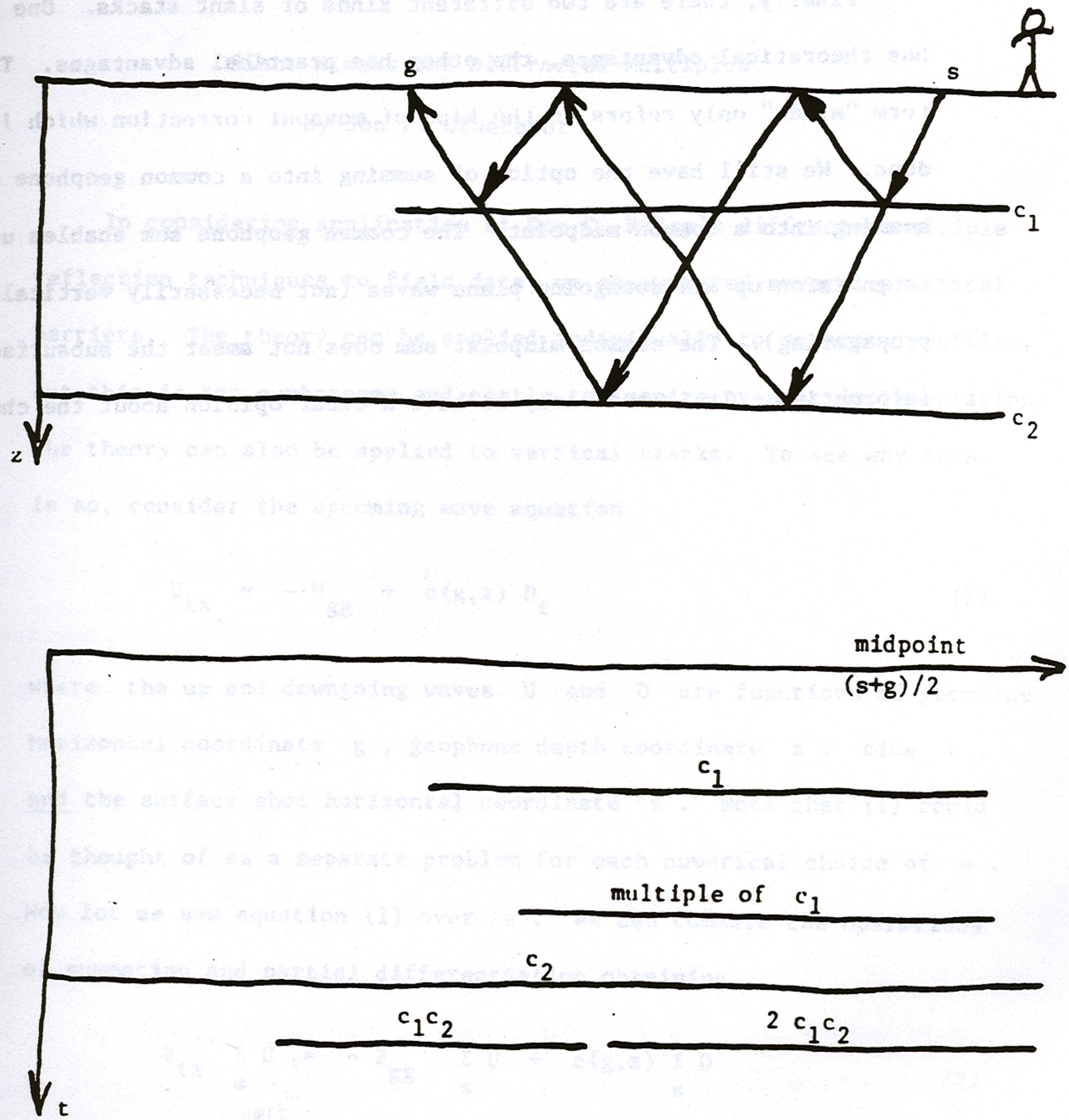


Fig. 5. Peglegs from a terminating shallow reflector.

Finally, there are two different kinds of slant stacks. One has theoretical advantages, the other has practical advantages. The term "slant" only refers to the kind of moveout correction which is done. We still have the option of summing into a common geophone or summing into a common midpoint. The common geophone sum enables us to envision up and downgoing plane waves (not necessarily vertically propagating). The common midpoint sum does not smear the subsurface information. It is too early to have a clear opinion about the choice.