Chapter 4 Downward Continuation of Sections

Introduction

In the previous chapter we developed equations useful for downward continuing data recorded as a single profile. As such, those equations were valid for wave fields generated by a single source at a fixed location. Here we wish to find equations useful for downward continuing data recorded as a section. That is, we want equations which can downward continue a wave field generated by many sources at varying locations.

An obvious approach might be to separately apply the equations of chapter 3 to the wave fields generated by each source. The main practical disadvantage of this approach is that data are often recorded with receiver cables which are too short to adequately define the initial conditions for the profile continuation equations. Initialization is particularly difficult in the common case where data are recorded with a moving array of a single source and receiver. Because of the high costs of overcoming this data initialization problem, we shall not use the profile approach to downward continue sections. Instead,we will reformulate the problem in a way in which data initialization difficulties are not so severe. In doing this,we shall use an approach like that found in chapter 3. First,we will formally describe the section data display in terms of a coordinate system. Then we will find transformation equations between these data display coordinates and the cartesian coordinates in which the wave equation is usually expressed. Finally, we will find a section continuation equation by transforming the wave equation into the section coordinates.

Coordinate Transformation and Wave Equation

As a first step in finding the section continuation equation, we must define both the field recording coordinates and the data display coordinates. As in the profile chapter we will assume that the reflectors are independent of one horizontal coordinate. Thus, our coordinate systems will describe horizontal distances only along the line of the section. Referring to Figure 4-1 we will fix the recording geometry. We chose to position the sources along the line of the section at locations $s_1, s_2, \ldots, s_n$. The receivers (geophones) will be positioned along the same line at locations $g_1, g_2, \ldots, g_n$. We will use t for reflection travel time and z for depth (+z down). The section data display coordinates will be defined as follows: h is half the surface source-receiver offset; y is the horizontal coordinate of the surface source-receiver midpoint measured from a fixed origin; d is a moveout corrected two way travel time and r is the receiver depth (+r down).

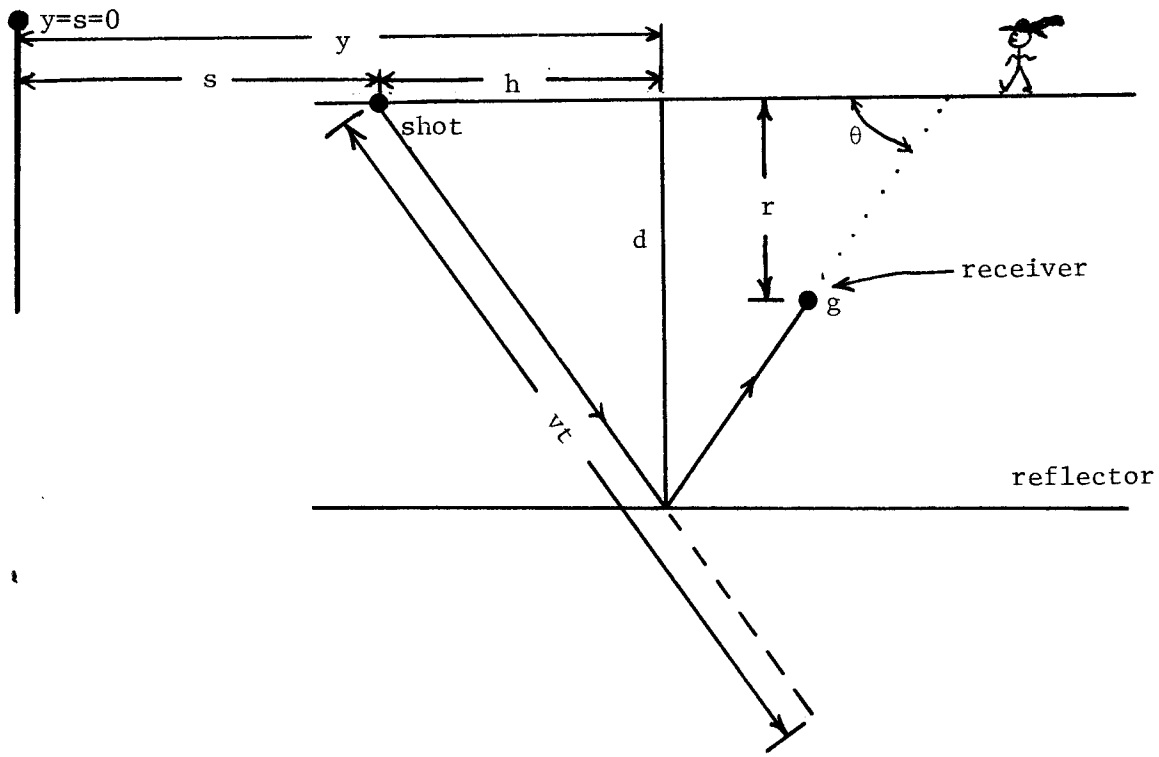

Figure 4-1. Geometry for the offset midpoint coordinate system.

The equations which describe the transformation from the field recording coordinates (s, g, t, z) to the section coordinates (y, h, d, r) are given by:

$$h = \frac{(g-s)}{2} + z\left(\frac{g-s}{2v}\right) / \left(t^2 - \frac{(g-s)^2}{v^2}\right)^{1/2} \tag{4-1a}$$

$$y = \frac{(g+s)}{2} + z\left(\frac{g-s}{2v}\right) / \left(t^2 - \frac{(g-s)^2}{v^2}\right)^{1/2} \tag{4-1b}$$

$$d = \left(t^2 - \frac{(g-s)^2}{v^2}\right)^{1/2} + \frac{z}{v} \tag{4-1c}$$

$$r = z \quad . \tag{4-1d}$$

We have used v as the constant moveout correction velocity. As in the profile frame we have included a depth dependence so that h , y , and d are constant along a plane layer reflection ray path. There is no coordinate describing the shot depth since we are explicitly requiring that sources remain at the earth's surface, $z=0$.

Equations 4-1 are of course invertible. Their inverse is given by

$$g = y + h - \frac{2hr}{vd} \tag{4-2a}$$

$$s = y - h \tag{4-2b}$$

$$t = \left(d^2 + 4h^2 / v^2\right)^{1/2} \frac{\left(d - \frac{r}{v}\right)}{d} \tag{4-2c}$$

$$z = r \tag{4-2d}$$

Each wave field generated by each individual source must separately satisfy the wave equation. Thus, instead of one wave equation, the section problem requires n wave equations if there are n sources. In terms of the field coordinates the equations which govern the wave field are

$$P_{gg} + P_{zz} - \frac{1}{\tilde{v}^2} P_{tt} = \delta(s_i - g, t, z) \quad i = 1, n \qquad (4\text{-}3)$$

As in chapter 3 we are using a 2-D scalar wave equation. We are also using subscripts to denote derivatives, $\tilde{v}$ to denote the variable wave velocity and the delta function to denote the sources. As a convenience, we will assume that there are a continuum of sources located along the line of the section. If we do this, equation (4-3) can be considered a single equation in four variables (g, s, t and z) . Since we are not interested in describing wave fields at the source locations, we can drop the delta function from (4-3). Doing this we are left with the equation to be transformed into the section coordinates

$$P_{gg} + P_{zz} - \frac{1}{\tilde{v}^2} P_{tt} = 0 \qquad (4\text{-}4)$$

Continuation Equation

Having equations (4-1), (4-2) and (4-4) we can find the continuation equation for $Q(y,h,d,r)$, the wave field in the section coordinate system. In doing this we will adopt the high frequency assumption and the small dip assumption discussed in chapter 3. Using these assumptions and

the chain rule, the continuation equation for Q is

$$
\begin{aligned}
&\left(h_g^2 + h_z^2 - \frac{1}{\tilde{v}^2}\, h_t^2 \right) Q_{hh} + \left(y_g^2 + y_z^2 - \frac{1}{\tilde{v}^2}\, y_t^2 \right) Q_{yy} \\
&+ Q_{dd}\left(d_g^2 + d_z^2 - \frac{1}{\tilde{v}^2}\, d_t^2 \right) + 2\left(h_g y_g + h_z y_z - \frac{1}{\tilde{v}^2}\, h_t y_t \right) Q_{hy} \\
&+ 2\left(h_g d_g + h_z d_z - \frac{1}{\tilde{v}^2}\, h_t d_t \right) Q_{hd} + 2\left(h_g r_g + h_z r_z - \frac{1}{\tilde{v}^2}\, h_t r_t \right) Q_{hr} \\
&+ 2\left(y_g d_g + y_z d_z - \frac{1}{\tilde{v}^2}\, y_t d_t \right) Q_{yd} + 2\left(y_g r_g + y_z r_z - \frac{1}{\tilde{v}^2}\, y_t r_t \right) Q_{yr} \\
&+ 2\left(d_g r_g + d_z r_z - \frac{1}{\tilde{v}^2}\, d_t r_t \right) Q_{dr} = 0
\end{aligned}
\qquad (4\text{-}5)
$$

where h_z , h_g , h_t etc. denote partial derivatives of the section coordinates.

We shall need expressions for the partial derivatives in equation (4-5). Thankfully, there is an immediate simplification. The definitions of y and h differ only by the sign of s in the first terms of (4-1a) and (4-1b) . Thus, we have

$$
\frac{\partial y}{\partial (g,t,z)} = \frac{\partial h}{\partial (g,t,z)} \qquad (4\text{-}6)
$$

The expressions for the remaining derivatives are

$$
\frac{\partial r}{\partial (t,z,g)} = 0 \ , \ 1 \ , \ 0 \qquad (4\text{-}7a)
$$

$$
\frac{\partial d}{\partial (t,z,g)} = \alpha \ , \ \frac{1}{v} \ , \ \frac{-2h}{v^2 d} \qquad (4\text{-}7b)
$$

$$
\frac{\partial y}{\partial (t,z,g)} = \frac{-rh}{vd\,\beta}\alpha \ , \ \frac{h}{dv} \ , \ \frac{1}{2} + \frac{r}{2v\beta} + \frac{2rh^2}{v^3 d^2 \beta} \qquad (4\text{-}7c)
$$

$$
\text{where } \alpha = \left(1 + \frac{4h^2}{v^2 d^2} \right)^{1/2} \text{ and } \beta = (d - r/v) \qquad (4\text{-}8)
$$

Using (4-6), (4-7) and (4-8) equation (4-5) becomes

$$\frac{2}{v} Q_{dr} + \frac{2h}{vd} (Q_{yr} + Q_{hr}) + \frac{\alpha^2}{\beta 2} \{ \frac{d^2}{4} + \frac{r^2 h^2}{v^4 d^2} \gamma \} (Q_{yy} + 2Q_{yh} + Q_{hh})$$

$$- \frac{2hr\alpha^2\gamma}{v^3 d \beta} (Q_{dh} + Q_{dy}) + \frac{\alpha^2}{v^2} \gamma Q_{dd} = 0 \qquad (4\text{-}9)$$

where α and β are as defined in (4-8) and

$$\gamma = (1 - \frac{v^2}{\tilde{v}^2}) \qquad (4\text{-}10)$$

If the moveout velocity is the same as the velocity in the wave equation, then $\gamma = 0$ and equation (4-9) becomes

$$\frac{2}{v} Q_{dr} + \frac{2h}{vd} (Q_{yr} + Q_{rr}) + \frac{d^2\alpha^2}{4\beta^2} (Q_{yy} + 2 Q_{yh} + Q_{hh}) = 0 \qquad (4\text{-}11)$$

Substitution for α and β and rearrangement gives

$$Q_{dr} + \frac{h}{d} (Q_{yr} + Q_{hr}) = - (\frac{d}{d-r/v})^2 \frac{v}{8} (1 + \frac{4h^2}{v^2 d^2}) (Q_{yy} + 2Q_{yh} + Q_{hh}) \qquad (4\text{-}12)$$

Equation (4-12) governs the downward continuation of multi-offset sections (sections of CMP gathers). Although we could devise a scheme for numerically integrating (4-12) and use it to perform downward continuation, we would rather work with a simpler equation if possible. Much of the complexity of (4-12) is due to the Q_{hr}, Q_{yh} and Q_{hh} terms. The presence of these terms in the continuation equation seems to indicate that common offset sections cannot be migrated separately. This implied coupling not only increases the computer time and storage required to migrate a single common offset section, it also precludes the possibility of migrating data recorded with a moving array of a single source and receiver. In an effort to simplify (4-12) we will examine these and other terms to see if they are important in describing the

$$\tau = t_0 - \frac{2h^2}{\tilde{v}^2 t_0} \sin^2\phi \qquad (4\text{-}17)$$

Taking a derivative with respect to h we get

$$\tau_h = \frac{\partial\tau}{\partial h} = -\frac{4h}{\tilde{v}^2 t_0} \sin^2\phi \qquad (4\text{-}18)$$

Simple geometry shows that for a zero offset section we have

$$\frac{\partial\tau}{\partial y} = \tau_y = \frac{2\sin\phi}{\tilde{v}} \qquad (4\text{-}19)$$

If we disregard the fact that the apparent dip of a reflector is a weak function of offset, and use a plane wave assumption we have

$$\frac{Q_h}{Q_y} \approx \frac{\tau_h}{\tau_y} \qquad (4\text{-}20)$$

Substituting from (4-18) and (4-19), equation (4-20) becomes

$$\frac{Q_h}{Q_y} \approx -\frac{2h\sin\phi}{\tilde{v}\, t_0} = -\sin\phi\tan\theta \ , \qquad (4\text{-}21)$$

where θ is as defined in Figure 4-1.

We can use (4-21) as a rough guide to the importance of the Q_h terms in equation (4-12). Table 4-1 shows equation (4-21) evaluated for particular values of θ and ϕ. From the table we can conclude that Q_{hh} is neglectable compared to Q_{yy} for many commonly encountered dips and offsets. (The calculations in the figures of chapter 5 indicate that the errors implied by the Table 4-1 are probably larger than the errors which actually occur.) The table also indicates that the terms in (4-12) containing first order h derivatives cannot be neglected relative to terms containing first order y derivatives if either dip or offset is not small.

θ	ϕ	$\left\lvert\frac{Q_h}{Q_y}\right\rvert$	$\left\lvert\frac{Q_{hh}}{Q_{yy}}\right\rvert$	measured values of $\left\lvert\frac{Q_{hh}}{Q_{yy}}\right\rvert$
30°	30°	.29	.08	.09
45°	15°	.26	.07	.09
30°	15°	.15	.02	.03
15°	15°	.07	.005	.01
		A	B	C

TABLE 4-1. Ratios of terms in equation (4-12) for various dips and emergence angles. Columns A and B are based on equation (4-21). Column C is the result of numerical experiments on surface synthetic data. The precision of column C is about $\pm$ 20%. Because correctly migrated data are independent of offset, the average value of these ratios during migration should be about half the size shown in the table.

To get a handle on the function of the non-neglectable Q_{hr}, Q_{yr} and Q_{yh} terms, we will investigate their effect on the migration of synthetic data that would be recorded over a point scatterer located at $y=y_0$. Such surface data are symmetric about $y=y_0$ and $h=0$. Consider first the directional derivative terms Q_{yr} and Q_{hr}. Inequality (4-13) implies that the Q_{yr} term should be more important than the Q_{hr} term. On the basis of our study of profiles in chapter 3, we expect that the main result of the inclusion of Q_{yr} in a continuation equation will be the introduction of a minor asymmetry in the focus of migrated scatterer data.

The migrated data of Figure 4-2 were constructed with an equation of the form

$$Q_{dr} + \frac{h}{d} Q_{yr} = -\frac{v}{8} \left(1 + \frac{4h^2}{v^2 d^2}\right) \left(\frac{d}{d - r/v}\right)^2 Q_{yy} \qquad (4\text{-}22)$$

As expected they show a small amount of skewing due to the presence of Q_{yr} in (4-22). As in chapter 3 we interpret this skewing to be the result of source-receiver directivity effects.

Next consider the Q_{yh} term. Because the dip (in y) of the point scatterer hyperbolic changes sign at $y=y_0$, Q_y must also change sign at this location. Equation (4-14) shows that apparent moveout velocity is independent of the sign of dip. Thus, the residual moveout of the point scatterer data and Q_h must be symmetric about $y=y_0$. Because of this we can conclude that Q_{yh} will change sign at $y=y_0$ and thus, must cause some skewing of downward continued scatterer data. Figure 4-3 shows data migrated with and without the Q_{yh} and Q_{yr} terms of (4-12). As expected, the inclusion of these terms in the continuation equation results in some asymmetry in the migrated data. Note, however, that the position of the focus is apparently unchanged by their presence and that the data continued without Q_{yh} and Q_{yr} appear properly migrated.

Although Table 1 indicates that the Q_{yr}, Q_{hr} and Q_{yh} terms cannot be deleted from (4-12) on the basis of size, Figures 4-2 and 4-3 indicate that the effects of their deletion are not large. The same figures also indicate that migrated data generated with equations including these terms appear to exhibit some skewing or

asymmetry which one would not usually expect in earth models. Unfortunately, our present level of understanding and observational experience are not sufficient to allow us to make definitive statements about the exact role of these terms in wave equation migration. Perhaps they arise solely because we downward continue receivers but not sources. (Ideally we would like to downward continue both sources and receivers simultaneously.) Possibly, better migrations could be achieved with equations containing Q_{yr} , Q_{hy} and Q_{yh} if we were to use a reflector mapping principle that accommodated source-receiver directivity effects.

Regrettably, in this thesis we must leave the question of the role of Q_{yh} Q_{yr} and Q_{hr} unanswered. Fortunately, we can still make use of the equations we have derived, since all our calculations show that, for models fitting the assumptions made in chapters 1 and 3, the neglection of these terms, at worst, causes only travel time changes which are very much smaller than a wave period. Because errors of this size are virtually undetectable on field data, we can neglect them leaving an equation of the form:

$$Q_{dr} = -\frac{v}{8}\left(1+\frac{4h^2}{v^2d^2}\right)\left(\frac{d}{d-r/v}\right)^2 Q_{yy} \quad . \qquad (4\text{-}23)$$

One might assume that because of the deletion of Q_h terms, equations like (4-23) cannot model the interaction of dip in the y direction with curvature in the h direction. Specifically, one might think that (4-23) does not model the dip dependence of apparent moveout velocity described in chapters 2 and 5. Surprisingly, this is not true. Many of the figures in chapter 5 demonstrate that equation (4-23) accurately models this phenomena. In fact, equation (4-23) is the continuation operator we shall use in chapter 5 to remove structural effects from the data prior to velocity estimation.