An Offset Squared Transformation

by Jon F. Claerbout

Cecil Green may have been the first author in Geophysics (1938, Vol. 3, No. 4) to write about $x^2 - t^2$ velocity analysis. Here we consider a natural extension of his work, namely, a coordinate frame for the wave equation where velocity residuals show up as dipping events rather than curved events. Following the 28 June work, "A Shot-Offset Frame for Velocity Estimation" we will use the residual variables (h,d) rather than (x,t) . Clearly, resampling the offset parameter for more points at great offset is advantageous from a points-per-wavelength viewpoint, but such resampling of the time coordinate is disadvantageous. Thus, instead of an (h^2,d^2) analysis, we will do an $(h^2/d, d)$ analysis. Define the new parabolic variable as e where

$$e = h^2 / d \tag{1}$$

Figure 1 shows that the data will be sensibly resampled.

Thinking of $P(h,d) = Q(e,d)$ we have

$$P_d = Q_d + e_d Q_e = Q_d - (h^2/d^2) Q_e \tag{2a}$$

$$P_h = e_h Q_h = 2 (h/d) Q_e \tag{2b}$$

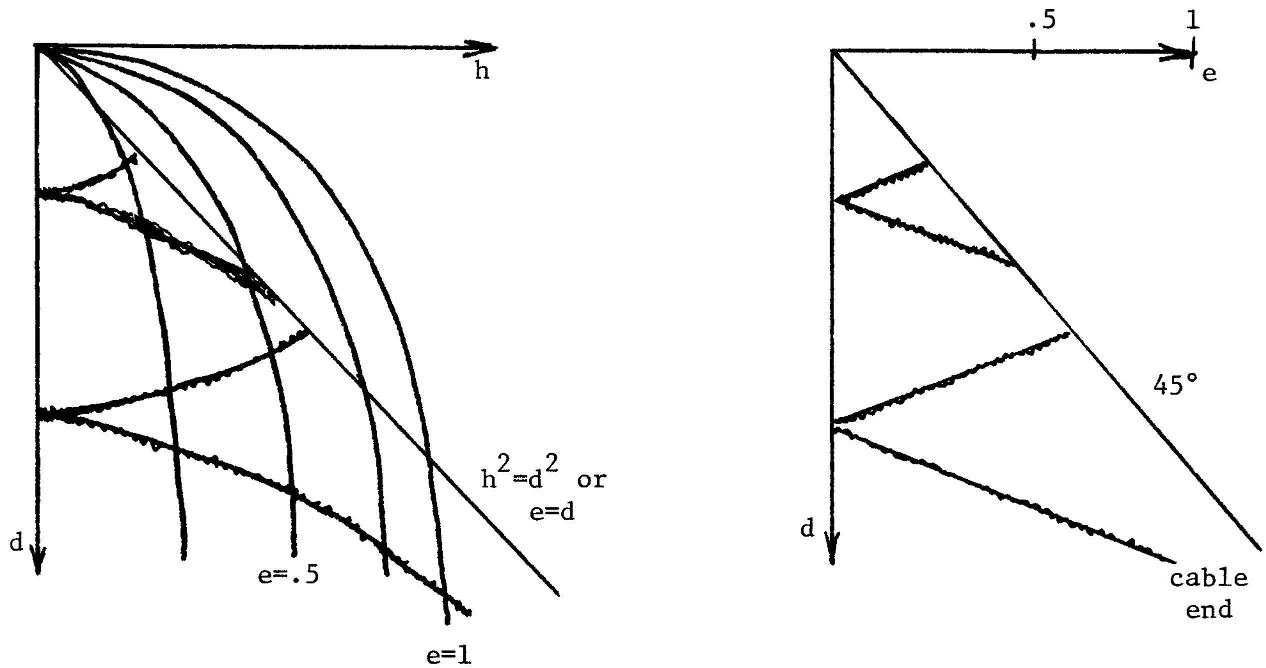

Figure 1. The transformation to a parabolic variable.

Equation (5) of the June 28 work is

$$(\, h/d \,(\, \partial_y + \partial_h \,) + \partial_d \,)\, \partial_r Q = -\frac{1+h^2/d^2}{4}\, d\, (\, \partial_y + \partial_h \,)^2 Q$$

which upon substitution from (2) becomes

$$(\partial_d + h/d\, \partial_y + h^2/d^2 \partial_e) \partial_r Q = -\frac{1+h^2/d^2}{4}\, d\, (\, \partial_y + 2h/d\, \partial_e \,)^2 Q$$

and with the elimination of h by (1) gives the final result

$$(\partial_d + (e/d)^{1/2} \partial_y + e/d\, \partial_e) \partial_r Q = -\frac{(d+e)}{4}\, (\, \partial_y + 2\,(e/d)^{1/2}\, \partial_e \,)^2 Q \quad (3)$$

Before bemoaning the pole in the coefficients, note that for 45° rays we have e=d . Thus, there will be no data anyway at the pole.

In fact, I believe this frame may turn out to be the best yet in terms of the following 3 goals:

(1) Data sampling should be fairly uniform in terms of points per wavelength;

(2) Coefficients of the downward continuation equation should have a modest range of numerical values;

(3) The downward continuation equation should be moderately simple.