Two Stratified Media Frames

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For refined velocity estimates it may be important to use a coordinate frame based on a stratified model rather than a homogeneous model. The first part of this task is to prepare some tables based on ray tracing in the stratified medium. We'll consider first an emergent angle frame. First we must prepare a table $\tau(p,w)$ which contains the travel time τ of a ray from the earth's surface to a depth w given that the ray is characterized by $p = \sin\theta / v$ (which by Snell's law is independent of position on the ray). Simultaneously for the same rays we can prepare a table of the horizontal displacement $u(p,w)$ between the top of the ray at $z=0$ and the bottom at $z=w$. Having completed this task we may then define the emergent angle frame as shown in figure 1 as the inverse of the coordinate transformation defined below.

$$s(\,p,\,y,\,d,\,r\,) = y - u(\,p,\,d\,) \tag{1a}$$

$$z(\,p,\,y,\,d,\,r\,) = d\;\; 2\,r\,/\,(1+r) \tag{1b}$$

$$g(\,p,\,y,\,d,\,r\,) = y + u(p,d) - u(\,p,\,d\;2r\,/\,(1+r)) \tag{1c}$$

$$t(\,p,\,y,\,d,\,r\,) = 2\,\tau\,(p,d) - \tau\,(\,p,\,d\;2r\,/\,(1+r)) \tag{1d}$$

In a homogeneous medium we have

$$\tau(p,w) = w\,/\,v\,\cos\theta = (\,1 - p^2 v^2\,)^{-1/2}\; w/v \tag{2a}$$

$$u(p,w) = w\,\tan\theta = (\,1 - p^2 v^2\,)^{-1/2}\; w\,p\,v \tag{2b}$$

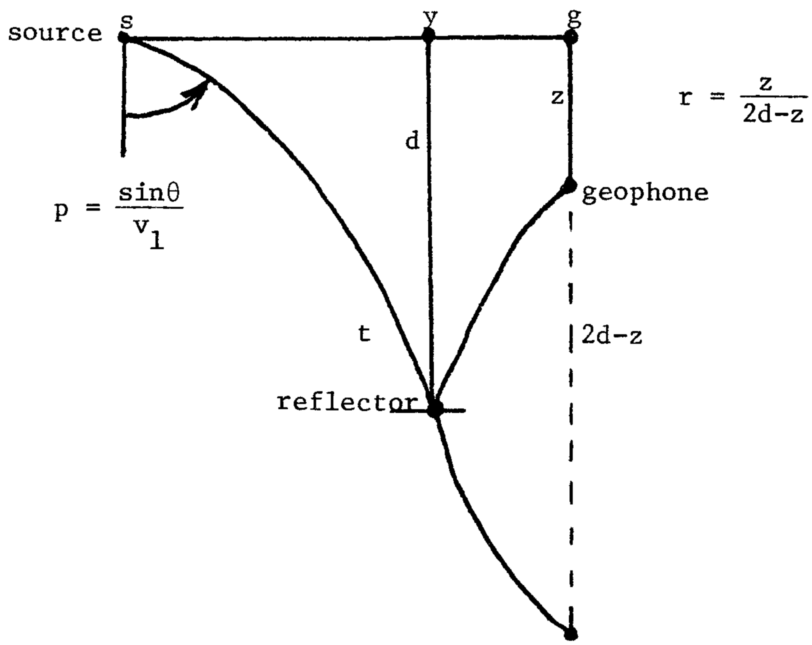

Figure 1. The stratified, emergent angle geometry.

I have not checked it, but I believe (1) with (2) should reduce to our earlier work.

Figure 2 illustrates another frame which replaces the ray parameter p as independent variable by the shot to midpoint offset parameter h .

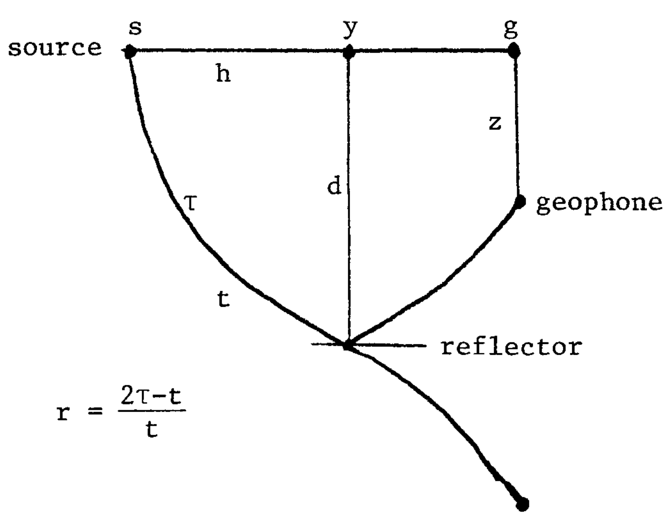

Figure 2. The stratified-offset-frame geometry.

For this frame we need a different set of tables. We need $u(p,\tau)$, $w(p,\tau)$ and the inverses $\tau(u,w)$ and $p(u,w)$. Quantitative definition for this frame is given by the inverse of the transformation

$$s(h, y, d, r) = y - h \tag{3a}$$

$$t(h, y, d, r) = \tau(h,d)\ 2 / (1+r) \tag{3b}$$

$$z(h, y, d, r) = w(\ p(h,d),\ \tau(h,d)\ 2r / (1+r)) \tag{3c}$$

$$g(h, y, d, r) = y + h - u(\ p(h,d),\ \tau(h,d)\ 2r / (1+r)) \tag{3d}$$

In a homogeneous medium we have

$$u(p,\tau) = v\,\tau\,\sin\theta = v^2\,\tau\,p \tag{4a}$$

$$w(p,\tau) = v\,\tau\,\cos\theta = v\,\tau\,(\ 1 - p^2 v^2\ )^{1/2} \tag{4b}$$

and the inverse relations

$$p(u,w) = (u/v)\ (\ u^2 + w^2\ )^{-1/2} \tag{5a}$$

$$\tau(u,v) = (\ u^2 + w^2\ )^{1/2}\ /\ v \tag{5b}$$

I have not checked it, but I believe (3) with (4) and (5) should reduce to the wave equation in "A Shot-Offset Frame for Velocity Estimation". I spent 2 or 3 days trying to analytically invert the Jacobian of the transformation (3) but without success. I believed I would be able to establish the vanishing of the Q_{dd} term and perhaps the Q_{dh} and Q_{dy} terms. If this frame is to be used in practice it may be necessary to compute the coefficients by numerically inverting the 4 by 4 matrix on a sufficiently dense grid and interpolating them onto the mesh used by the P.D.E.

As a practical matter I have not formed an opinion on which of the two transformations is preferable. The advantage of the p frame is that it is probably simpler. I probably would have been able to invert the Jacobian of (1) whereas I could not do so for (3). I also suspect that the Q_{dp} and Q_{dy} terms may vanish in the p frame whereas the corresponding terms may not vanish in the h frame making the Fresnel-like approximations better in the p frame . A trouble with the p-frame is that data tends to be grossly over sampled near $d=0$ which leads to the $1/d$ coefficient in the P.D.E.