A Revertible Shifting Frame

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The slant frame transformation is linear and therefore analytically invertible. Raul Estevez inverted it in our March, 1974 report. The shifting frames are non-linear, so the inversion is an iterative process. A slight variation on the shifting frame makes it invertible, even though it is still a rather general non-linear transformation. We define the revertible shifting frame by:

$$t' = t + f(x) + g(z) \tag{1a}$$

$$x' = x + h(t + f(x)) \tag{1b}$$

$$z' = z \tag{1c}$$

where the functions $f(x)$, $g(z)$, and $h(t)$ have not yet been prescribed. The difference between this and the shifting frame of 16 April, lies in equation (1b). In other words, our lateral shift function h will now be activated by moveout corrected time $t + f(x)$ rather than by raw time t . The reversion of (1) is

$$z = z' \tag{2a}$$

$$x = x' - h(t' - g(z')) \tag{2b}$$

$$\begin{aligned} t &= t' - f(x) - g(z) \\ &= t' - f(x' - h(t' - g(z'))) - g(z') \end{aligned} \tag{2c}$$

We note that two unfamiliar derivatives are

$$x'_x = 1 + h' f_x \tag{3a}$$

$$x'_t = h' \tag{3b}$$

We'll use explicitly the fact that $t'_t = 1$, $x'_z = 0$, $z'_z = 1$, $z'_x = 0$, and $z'_t = 0$. The usual procedures lead to

$$P_t = Q_{t'}\, t'_t + Q_{x'}\, x'_t + Q_{z'}\, z'_t$$

$$= (\partial_{t'} + x'_t\, \partial_{x'})\, Q \tag{4a}$$

$$P_x = Q_{t'}\, t'_x + Q_{x'}\, x'_x + Q_{z'}\, z'_x$$

$$= (t'_x\, \partial_{t'} + x'_x\, \partial_{x'})\, Q \tag{4b}$$

$$P_z = Q_{t'}\, t'_z + Q_{x'}\, x'_z + Q_{z'}\, z'_z$$

$$= (t'_z\, \partial_{t'} + \partial_z)\, Q \tag{4c}$$

and

$$(x'_x)^2\, Q_{xx} + 2\, x'_x\, t'_x\, Q_{xt} + (t'_x)^2\, Q_{tt} + Q_{zz} + 2\, t'_z\, Q_{zt} + (t'_z)^2\, Q_{tt} =$$

$$= \frac{1}{\tilde{v}^2} \left((x'_t)^2\, Q_{xx} + 2\, x'_t\, Q_{xt} + Q_{tt} \right) \tag{5}$$

The convection terms we are interested in squashing are the coefficients of Q_{tt} and Q_{xt}. They can be totally extinguished in a homogeneous medium. Further study should enable us to see how to choose f, g, and h to keep these coefficients small in stratified media.

$$Q_{xt}: \quad 0 \approx 2\, (x'_t / v^2 - x'_x\, t'_x)$$

$$0 \approx 2\, (h' / v(x,z)^2 - (1 + h' f_x)\, f_x)$$

$$Q_{tt}: \quad 0 \approx 1 / v(x,z)^2 - f_x^2 - g_z^2$$