A Data Oriented Shifting Frame

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This is follow-up work to be appended to Shifting Frames, 16 April 1974. In the earlier work we considered a frame which translated in the x-direction with a time variable velocity. The velocity was based on a stratified medium ray. Now the shifting velocity may change from one ray to another as a function of time so that it is, in reality, somewhat arbitrary.

Surprisingly, by making a few adjustments to the moveout function, we will still be able to achieve a differential equation with a Q_{tt} which vanishes in a stratified medium and a Q_{xt} term which vanishes in a homogeneous medium.

The purpose of this effort is to accommodate a major shortcoming in present day reflection seismic data gathering. Good velocity resolution is achieved by including lots of energy in an angular spectrum surrounding 45 degrees from the vertical, but due to the shortness of the receiver cable (seldom over three kilometers) such angles are generally not available in echoes arriving much later than about 2-1/2 seconds. Thus, our "slant frame" must change its angle if it is to simultaneously utilize the best early data along with the only available later data.

Figure 1 illustrates the transformation we have in mind. The transformation is based on human choice of a line $\tilde{x}(t)$ through the good events in the data. In the transformed domain (x', t') these good events will lie along the vertical line $x' = o$.
It probably turns out that the x-axis can be interpreted as either the

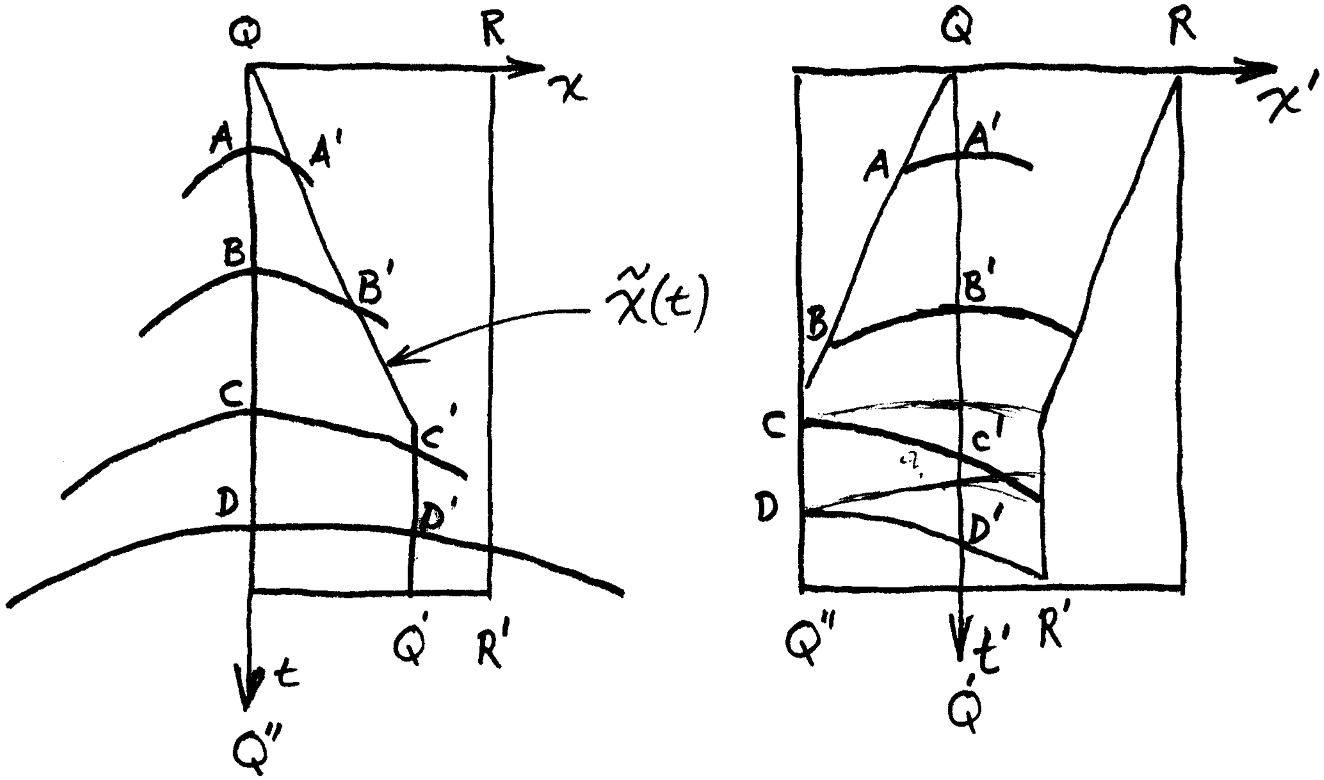


Figure 1. A raw data frame (left) transformed to a data oriented shifting frame (right).

geophone axis g or the offset axis f . The sharp corner in figure 1 in the line $\tilde{x}(t)$ can probably cause difficulties unless it is smoothed over. What has the transformation achieved? All the good events A' , B' , C' , and D' now are lined up on the vertical t' axis. The events A' and B' now have zero dip on the t' axis because the $\tilde{x}(t)$ curve was chosen to start coinciding with a ray, so the early data behaves just like the old slant frame data. The events C' and D' occurred after the $\tilde{x}(t)$ curve switched from a slanted ray to a vertical ray, and, as we will see, this means that C' and D' will have some dip on the t'-axis . In fact, little moveout correction was done on CC' and DD' ; they were mainly shifted sideways. Luckily, the events CC' and DD' wouldn't require much moveout correction anyway because they occur so late.

A common expedient in seismic data analysis and interpretation is to use a time variable velocity instead of a depth variable velocity. The two are chosen to agree for vertical rays. Suppose, for example, that the event BB' marked an abrupt, severe velocity change. The wavefront BB' is never beneath the interface and is not affected by whatever velocity is found beneath the interface. However, if the data on the left frame of figure 1 is blindly processed with a time variable velocity, then the wavefront BB' will be treated, for a short part of its path, as if it was going through the velocity beneath B . One justification commonly given for time variable velocity is that the error is small because the path over which the error is made is short. Now let us think about the difference between a velocity which depends on t and a velocity which depends on t'. In other words, instead of basing the time variable velocity on a vertical ray, let the velocity be that seen by waves along the trajectory $\tilde{x}(t)$ (which need not be a ray). In the transformed (x', t') frame, all the events of interest A', B', C', and D' are treated with the correct velocity and velocity errors increase slowly off the t' axis. It seems that a t' dependent velocity is much more justifiable than a t dependent velocity.

We now inspect the Q_{xt} term in the stratified media version of the differential equation (4). It is

$$2 \left(x'_t \,/\, \tilde{v}(z)^2 - t'_x \right) Q_{xt} \tag{10}$$

We have chosen the shift function $h(t) = -\tilde{x}(t)$ in equation (1b), but we have not yet chosen the moveout correction function $f(x)$. In (10) this means that the moveout derivative t'_x has not yet been determined, but $x'_t / \tilde{v}(z)^2$ has been. This suggests that we try to pick an $f(x)$ so that the coefficient of the convection term (10) vanishes. This can be done in homogeneous media by

$$t'_x = x'_t \,/\, v^2 \tag{11}$$

In inhomogeneous media we can hope to choose t' to get the coefficient small. We can't extinguish it because $v(z)$ depends on z but t'_x may not depend on z by the hypothesis (1a). If we define a time variable velocity $\bar{\bar{v}}(t)$ to in some way approximate velocities in the medium, say along the trajectory $\tilde{x}(t)$, we can then define t'_x by

$$t'_x = x'_t \,/\, \bar{\bar{v}}(t)^2 \tag{12}$$

With this definition (10) becomes

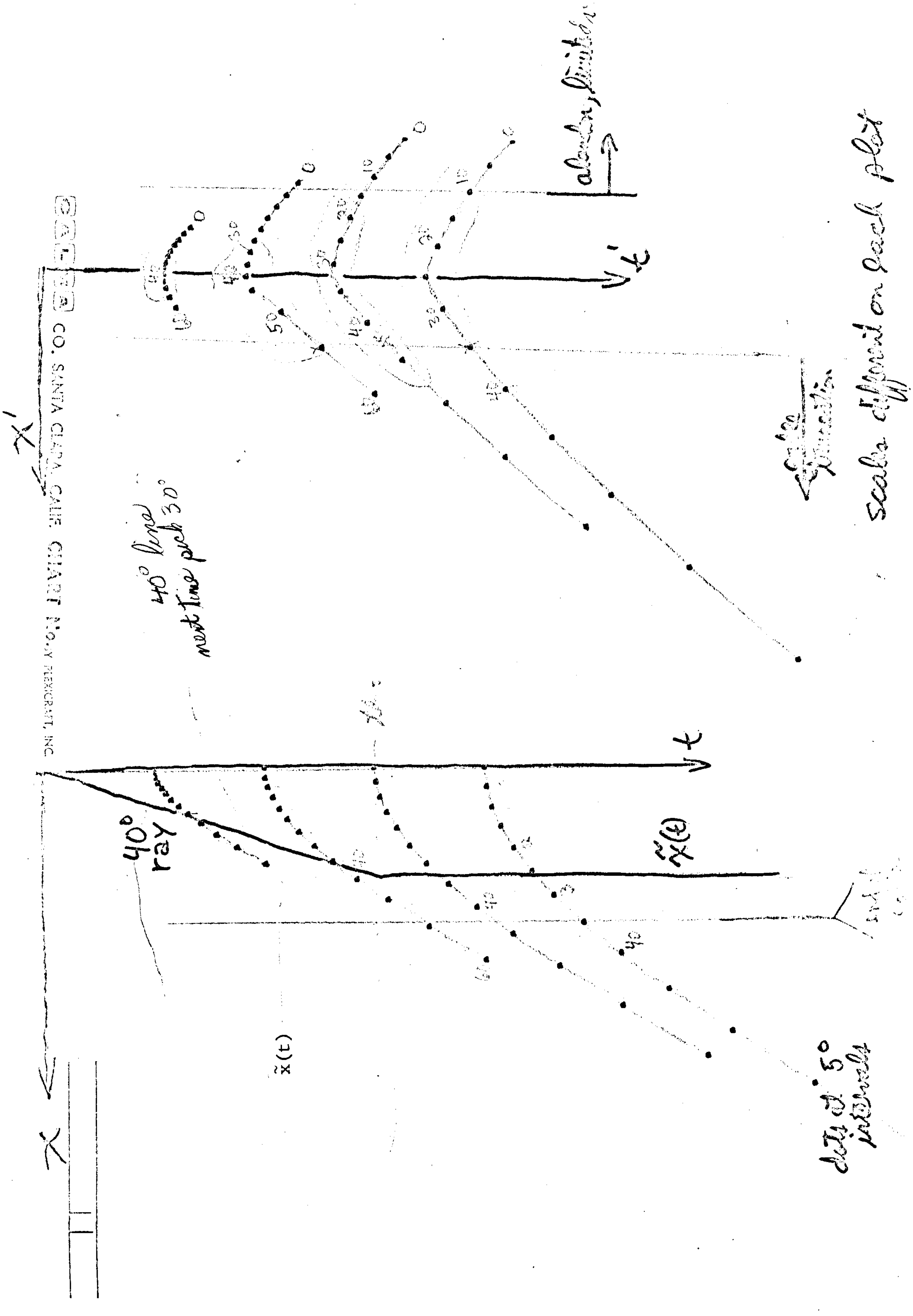
$$2\ x_t' \ (\ 1 / \tilde{v}(z)^2 - 1 / \bar{\bar{v}}(t)^2\ )\ Q_{xt} \qquad (13)$$

Now to get the $Q_{t't'}$ term to vanish, we will clearly need to satisfy the eikonal-like equation (5) . This can be done by using (5) to <u>define</u> $t_z'^{\,2}$.

At last we have completed definition of all the coefficient functions required to migrate with equation (9) which now takes the form

$$Q_{zt} = \frac{1}{2t_z'} \left[\left(\frac{\tilde{x}_t^2}{v(z)^2} - 1 \right) Q_{xx} + 2\ x_t' \left(\frac{1}{\tilde{v}(z)^2} - \frac{1}{\bar{\bar{v}}(t)^2} \right) Q_{xt} \right] \qquad (14)$$

Now what have we achieved? Equation (14) distinguishes between space variable velocity and time variable velocity. The purpose of introducing the time variable velocity was to get the sideways convection term Q_{xt} small. This term now vanishes in a homogeneous medium. Two advantages of a small Q_{xt} term are (1) there is less pressure against the Fresnel-like approximation (Hey, let's call it the <u>paraxial</u> approximation) and (2) there is no extra pressure on the numerical choice of Δx , Δt , or Δz .


X'
t'
t
X
40° ray
x̃(t)
x̃(t)
40° line
next time pick 30°
Scales different on each plot
data at 5° intervals