Migration of Common Offset Sections

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Earlier we spent much effort in finding wave equations which could be used to downward continue profiles and zero offset sections. Here we shall discuss the application of the downward continuation approach to multi-offset sections (sections of CDP gathers) and non-zero offset sections. As a part of the discussion, we will include some comments on velocity estimation. We believe that the applications of downward continued multi-offset sections to this field are as important as their applications to the fields of migration and reflector mapping.

The equations which can be used for migrating common offset sections are many and varied. Other parts of this report indicate just a few of the kinds of coordinate frames and equations that can be used. The equation one uses depends mainly on the priorities one sets in the formulation of the continuation problem. Among the things one might wish to consider in choosing a frame are accuracy, cost, storage requirements, range of offsets to be treated, range of dips, degree of understanding of the terms in the continuation equation, and ease of explanation to others. It was with these things in mind that we chose to use a frame like the 'h' frame defined in the section titled 'A shot offset frame for velocity estimation' (hereafter called $). The frame used here differs from that frame in that r is defined as the depth of the receivers and in that d is a two way travel time. Specifically we have,

$$h = \frac{(g-s)}{2} \left(1 + z/v(t^2 - (g-s)^2/v^2)^{1/2} \right) \tag{1a}$$

$$y = \frac{(g+s)}{2} + z \frac{(g-s)}{2v} / \left(t^2 - (g-s)^2/v^2 \right)^{1/2} \tag{1b}$$

$$d = \left(t^2 - \frac{(g-s)^2}{v^2} \right)^{1/2} + \frac{z}{v} \tag{1c}$$

$$r = z \tag{1d}$$

Figure 1 gives a geometric view of the coordinate system.

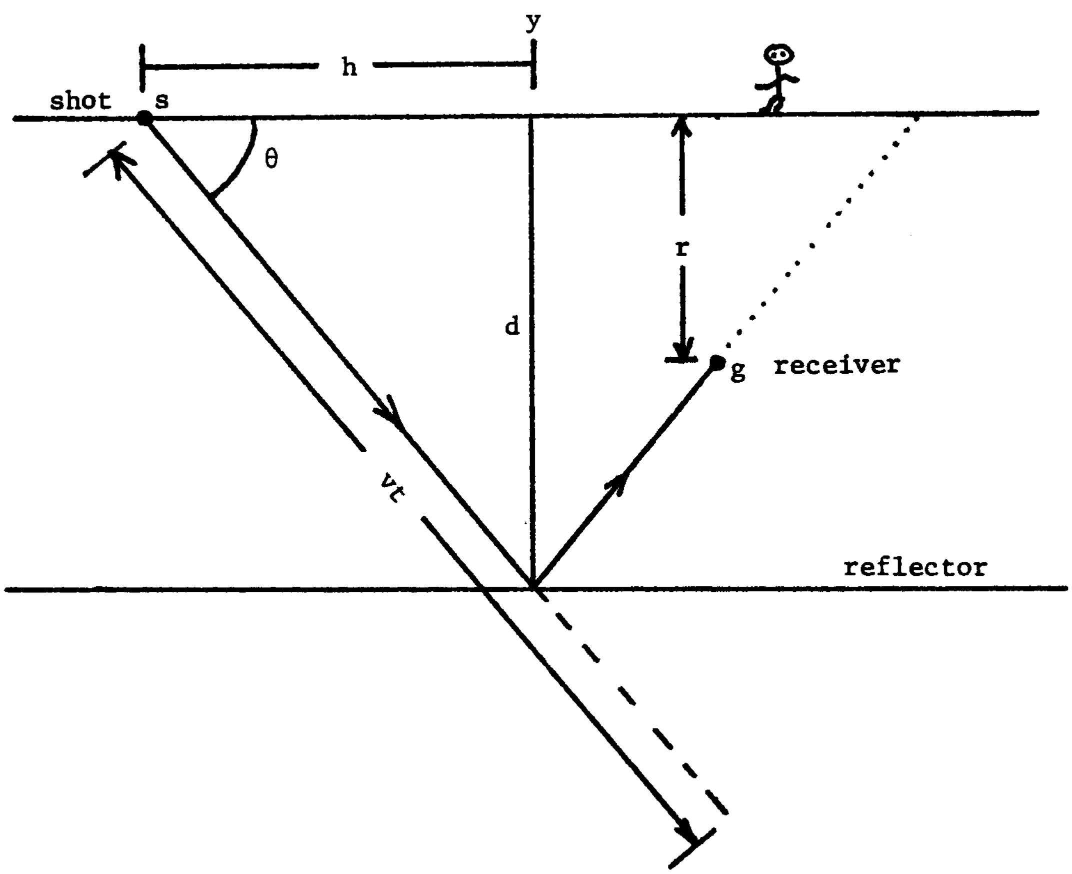

Figure 1. Geometry and coordinate system for continuation of section data.

If we make the standard assumptions (parabolic 15° approximation, constant velocity etc.) we get a continuation equation of the form

$$Q_{dr} + \frac{h}{d}\,(\,\partial_y + \partial_h\,)Q_r \;=\; -\frac{v}{8}\left(\frac{d}{d-r/v}\right)^2\left(1+\frac{4h^2}{v^2 d^2}\right)(\,\partial_y+\partial_h\,)^2\,Q \tag{2}$$

Equation (2) has essentially the same form as equation (5) of section ($). The main difference is that the coefficients in (2) tend to vary more than those in ($ 5) because of the differing definition of r .

At first glance, equation (2) appears dishearteningly complex. The presence of the Q_{hh} , Q_{hr} and Q_{yh} terms seems to indicate that common offset sections cannot be migrated independently. Ongoing work indicates that the situation is not as bad as it appears, in that, for dips less than 15°, both the implied coupling and its effects are small. This can be explained by noting that (2) governs properly moveout corrected data. Thus, non-zero values of the coupling terms (those dependent on Q_h and Q_{hh}) are due solely to the minor effects of structurally caused residual moveout. We also find, both with sections and with profiles, that directional derivative terms like Q_{yr} and Q_{hr} are small. For data recorded over a point scatterer, their inclusion in the continuation equation causes a minor asymmetry in the focus of the migrated data. We believe that this asymmetry is a result of the fact that only receivers and not sources are downward continued by (2). In any event, wave fields continued without Q_{yr} and Q_{hr} are more consistent with the usual concept of migration than those continued with an equation including these terms. In essence then, we find that large offset (> 45°) sections can be migrated with good accuracy by the use of the equation

$$Q_{dr} = -\frac{v}{8} \frac{d^2}{(d-r/v)^2} \left(1 + \frac{4h^2}{v^2 d^2} \right) Q_{yy} \tag{3}$$

When we first arrived at equations like (3) we assumed that deletion of the Q_h terms would result in the loss of the ability to model the interaction of dip in the y direction with curvature in the h direction. Specifically, we thought that (3) would not model the dip dependence of stacking velocity described by Levin (1971). Surprisingly, this is not true. For all the models tried (dip less than 15°) equation (3) accurately models the 'Levin effect'.

The cover shows a calculation done with equation (3). Figure 2 is a line drawing which describes the axes on the front cover. Both frames illustrate a set of moveout corrected common depth point gathers recorded over a random distribution of point scatterers. Scatterer density decreases from left to right. Any curvature seen on the top frame is due to the 'Levin effect'. Curvature is concave upward since the apparent velocity of reflections from non-layered structures is always greater than the true velocity. The steepest dips correspond to a velocity error of about 3%. Errors would have been greater if larger dips were allowed in the model. Notice that residual moveout increases for mid-points on the right. This occurs because the decreased scatterer density on the right results in an increase of apparent dip.

The bottom frame shows the same set gathers after migration. In this frame the data is independent of offset. Migration has removed the effects of velocity diffusion and structurally caused residual moveout. This calculation indicates that velocity can be accurately estimated in areas where the earth possesses little continuity. The key step in the estimation process is the removal of 'Levin effect' residual moveout by a downward continuation operation.

The comments we have made about equation (3) are strictly true only when velocity is constant. If the wave velocity differs significantly from the frame velocity, there will be much residual moveout and the coupling terms like Q_h will not be small. More important is the fact that when velocity is variable, neither equation (2) or (3) can be used to find the correct wave fields at depth. Recall that we

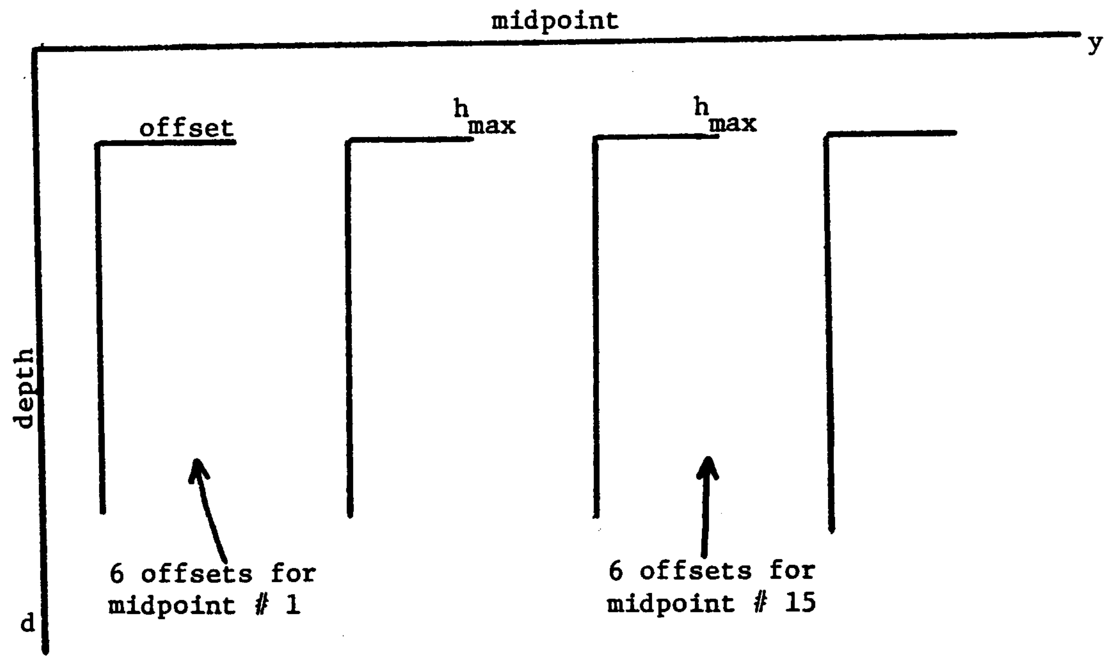

Figure 2. Axes for the cover figure. Each common depth point gather has 6 offsets. Offset ranges from 600' to 3600. Emergence angles range from 4° to 25°. Time ranges from 2.5 to 3.5 seconds. Water velocity was used in the migration.

assumed constant velocity in arriving at (2). If we had assumed a constant moveout velocity, v , and a variable velocity in the wave equation , $\tilde{v}$, instead of equation (2) we would have gotten

$$Q_{dr} + \frac{h}{d}(\partial_y + \partial_h)Q_r = -\frac{v\,\alpha^2}{2\beta^2}\left\{\frac{d^2}{4} + \frac{r^2h^2\gamma}{v^4d^2}\right\}(\partial_y + \partial_h)^2\,Q \qquad (4)$$

$$+\frac{hr\alpha^2\gamma}{v^2 d\beta}(\partial_y + \partial_h)Q_d - \frac{\alpha^2}{2v}\gamma\,Q_{dd} = 0$$

where

$$\alpha^2 = 1 + 4h^2/v^2d^2 \qquad \beta = d - r/v$$

$$\gamma = 1 - \frac{v^2}{\tilde{v}^2} \qquad (5)$$

In general, if velocity is variable, all terms in equation (4) must be used to obtain correctly downward continued wave fields.

Earlier in this report we indicated the desirability of downward continuation before velocity estimation. As this procedure almost guarantees that data will be migrated with incorrect velocities, it is important to know what errors result from the use of an erroneous migration velocity. We will get an idea of what to expect in this situation by examining the γ dependent terms of equation (4). As a result of studying these terms, we also determine the assumptions under which simpler versions of (4) can be used in the important case where velocity is known but is not constant.

As a starting point, assume that the velocity structure is layered and that the reflectors have no dip. In this case, $Q_y = 0$ and, neglecting directional derivative terms, equation (4) becomes,

$$Q_{dr} = - \frac{v\alpha^2}{2\beta^2} \left(\frac{d^2}{4} + \frac{r^2 h^2 r}{v^4 d^2} \gamma \right) Q_{hh} + \frac{hr\alpha^2 r}{v^2 d\beta} \gamma Q_{dh} - \frac{\gamma}{2v} \left(1 + \frac{4h^2}{v^2 d^2}\right) Q_{dd} \qquad (6)$$

Equation (6) looks complicated but what it does is not. The major interesting effect of downward continuation with (6) is the removal of half the surface residual moveout from the data (only half because the sources remain at the surface). It should be understood that we consider the travel time of the zero offset arrival to be part of the residual moveout.

The answer to the question of whether the terms on the right side of (6) should be used in a continuation equation, depends on both the complexity of the known velocity structure and on how the downward continued data is to be used. As velocity becomes increasingly y dependent, it becomes more and more difficult to justify neglecting these terms. However, when γ is independent of y and γ is not large, the Q_{dd} term in equation (6) performs virtually all the shift required to remove residual moveout. In this case, it is easy to justify neglecting the other terms in (6).

If downward continued data is to be used as part of a velocity estimation scheme, it may not be desirable to model any of the effects governed by equation (6). This is because there is a great practical advantage to be gained by insuring that the residual moveout of the continued data is independent of receiver depth. If this is done, most velocity estimation programs designed for use with surface data can be applied to downward continued data without modification. It is for this reason, that in velocity estimation applications we neglect the h dependent terms in (6) and use a continuation equation no more complicated than

$$Q_{dr} = -\frac{v}{2}\frac{\alpha^2}{\beta^2}\left(\frac{d^2}{4} + \frac{r^2h^2}{v^4d^2}\gamma\right)(Q_{yy} + 2Q_{hy}) + \frac{hr\alpha^2\gamma}{v^2d\,\beta}Q_{dy} - \frac{\gamma}{2v}Q_{dd} \tag{7}$$

To avoid confusion we note that, even in velocity estimation applications, γ is not of necessity completely unknown. For instance we may know that, instead of being constant, velocity increases with depth. In this case we might attempt to get more accurate continued wave fields by continuing the data with an equation like (7) instead of one like (3).

By choosing to ignore the h dependent terms on the right side of (6) we have lost the ability to accurately model situations where velocity varies horizontally. More precisely we have made the assumption that velocity does not vary significantly over distances comparable to a cable length. Since this is the implicit assumption made when velocity is estimated on the basis of correlations of data along hyperbolic paths, we feel that it is justifiable here.

The last term of equation (4) we consider is the horizontal shifting term, Q_{yd} . If $\gamma \neq 0$ correctly downward continued data will have residual moveout (including an incorrect zero offset travel time) because we only downward continue receivers. A look at the definition of y in (1b) shows that y depends not only on the source receiver positions (g,s,z) but also on v and the t . When $\gamma \neq 0$ this dependence along with the residual moveout causes each offset of correctly migrated data corresponding to the same reflection point to appear at a different location on the y axis. If we wish to use common reflection point gathers, this effect necessitates much bookkeeping in order to keep track of the positions (y,h) which correspond to the same reflection point. Fortunately this effect can be eliminated by deleting the Q_{yd} term. After deleting the Q_{yd} term equation (7) is

$$Q_{dr} = -\frac{v}{2}\frac{\alpha^2}{\beta^2}\left(\frac{d^2}{4}+\frac{r^2h^2}{v^4d^2}\gamma\right)\left(Q_{yy}+2Q_{yh}\right)-\frac{\gamma}{2r}Q_{dd} \qquad (8)$$

Now that we have equation (8) we can get an idea of how large velocity uncertainty must be before downward continuation begins to cause degradation of velocity estimates. If γ is such that equation (8) differs markedly from equation (3) we should expect some problems to develop with the downward continuation. Consider first the γ dependent part of the Q_{yy} coefficient. For reasonable values of h and γ , it is small and can be neglected. (For $\theta = 45°$ and $v = 1.2\,\tilde{v}$ its average value is .03.) Next consider the Q_{dd} shifting term. Since it shifts all offsets equally, it by itself can't degrade velocity estimates. In most cases the most important term in either (3) or (7) is Q_{yy} . If $\gamma \neq 0$ the use of (3) instead of (7) will cause the data to be either over or under migrated. This will mean that some structurally caused residual moveout will remain in the downward continued data. Since at least some of this residual will be removed by downward continuation, velocity estimates should be better after continuation. The only remaining term to be discussed is Q_{hy} . Numerical experiments indicate that it is not important for velocity errors up to 10%. However, since it may have a direct effect on residual moveout it is my belief that it is the limiting factor in this approach. If γ or the dip is large enough so that Q_{yh} is not small, then velocity estimates based on downward continued data may be erroneous.

Reference

Levin, F. K., 1971, Apparent velocity from dipping interface reflections: Geophysics, v. 32, no. 5, p. 789-800.