Noah's Method of Deconvolution by Flooding

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The idea here is that it is mainly the presence of the free surface perfect reflector which causes a practical problem with multiples. We wish to replace the seismogram $A = 1 + 2R$ by the seismogram $-C$ in figure 1.

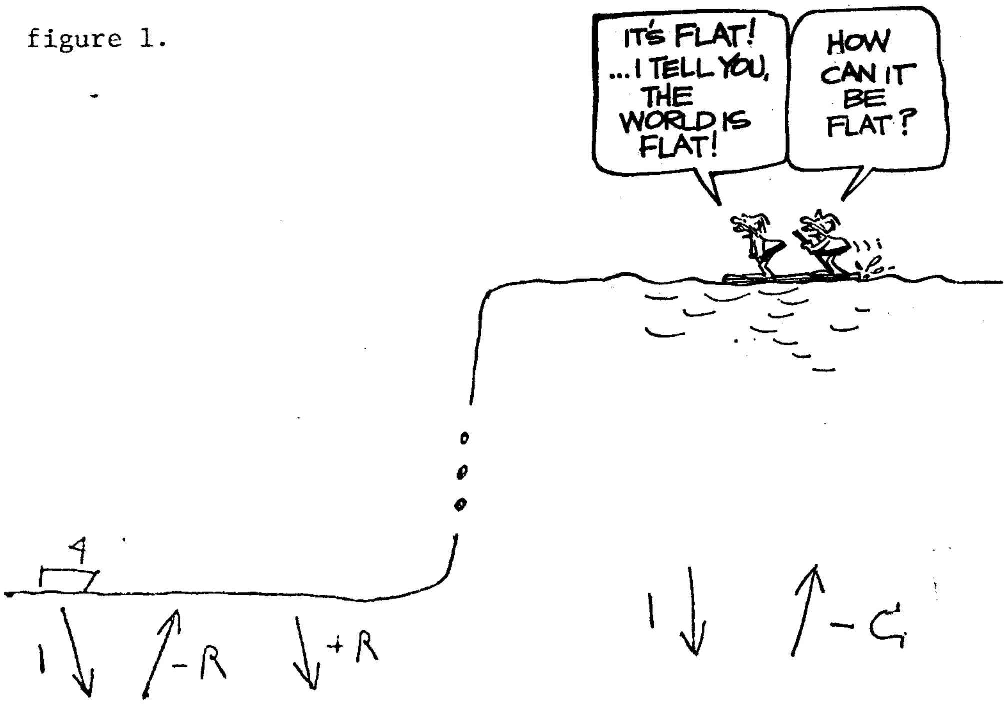

Fig. 1. Our waves on the left, Noah's on the right.

For us the down-going and up-going waves are

$$D = 1 + R$$

$$U = \quad - R$$

and for Noah they are

$$D = 1$$

$$U = - C$$

The admittance A of the earth at ordinary sea level is given by

$$\text{Admittance} = \frac{\text{velocity}}{\text{pressure}} = \frac{D - U}{D + U} = 1 + 2R$$

Noah's seismogram in terms of ours is

$$C = \frac{-U}{D} = \frac{R}{1 + R} = \frac{2R}{2(1+R)} = \frac{A - 1}{A + 1}$$

In terms of z-transforms this is

$$(A(z) + 1)\, C(z) = A(z) - 1$$

Now collect the coefficient of z^t for t greater than zero.

$$c_t + \sum_{k=0}^{t} a_{t-k}\, c_k = a_t$$

If c_t is taken to be unknown but c_{t-1}, c_{t-2}, ... are known then we can get c_t by the recursion

$$(1 + a_0)\, c_t = a_t - \sum_{k=0}^{t-1} a_{t-k}\, c_k \qquad (1)$$

Theoretically $a_0 = 1$ but there is an unknown scale factor, say $2u$ in a_1, a_2, ... a_∞ (among other problems). Thus (1) becomes

$$c_t = u\left(a_t - \sum_{k=0}^{t-1} a_{t-k} c_k \right) \qquad (2)$$

Note that c_t immediately feeds back to the calculation of c_{t+1}. Because of the finite extent of the source wavelet it is proposed that (3) will have the good features of (2) but will be safer.

$$c_t = u \left(a_t - \sum_{k=N_1}^{t-1} a_{t-k} \, c_k \right) \tag{3}$$

Figure 2 indicates possible choice of parameters N_1 , N_2 , and N_3 .

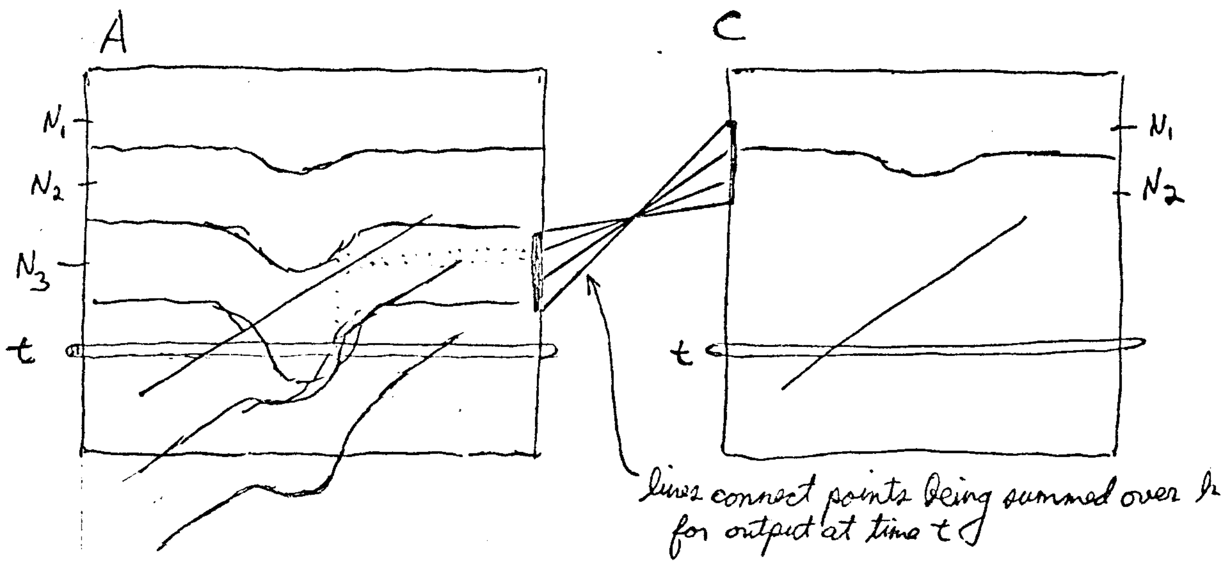

Fig. 2. Our seismogram on the left and Noah's on the right.

Some economy can be achieved if it is desirable to eliminate only sea floor multiples and sea floor peg-legs by limiting the sum in (3) to a maximum index of N_2

$$c_t = u \left(a_t - \sum_{k=N_1}^{N_2} a_{t-k} \, c_k \right) \tag{4}$$

The propagation of the unknown u during the computation goes as indicated in (5)

$$c_t = u \left(a_t - \sum_{k=N_1}^{N_2} a_{t-k} \, c_k \right) \tag{5}$$

Because of this propagation of u and our belief that u can be estimated from the relative strength of the primary and first multiple we get u by the minimization

$$\min_u \sum_{t=N_2}^{N_3} \left(a_t - u \sum_{k=N_1}^{N_2} a_{t-k}\, a_k \right)^2 \tag{6}$$

To understand (6) define y_t, the convolution of primary on itself by

$$y_t = \sum_{k=N_1}^{N_2} a_{t-k}\, a_k \tag{7}$$

The minimization (6) is trying to extinguish the multiple by means of the primary convolved on itself. This works perfectly if the gate contains no new structure primaries. Notice that u becomes 1 if the primary has a magnitude equal the reflection coefficient. Finally, let us admit that u is really a waveform, not a scalar. It should be the inverse of the waveform actually transmitted into the earth. Figure 3 shows why u_t is taken to be an anti-causal filter.

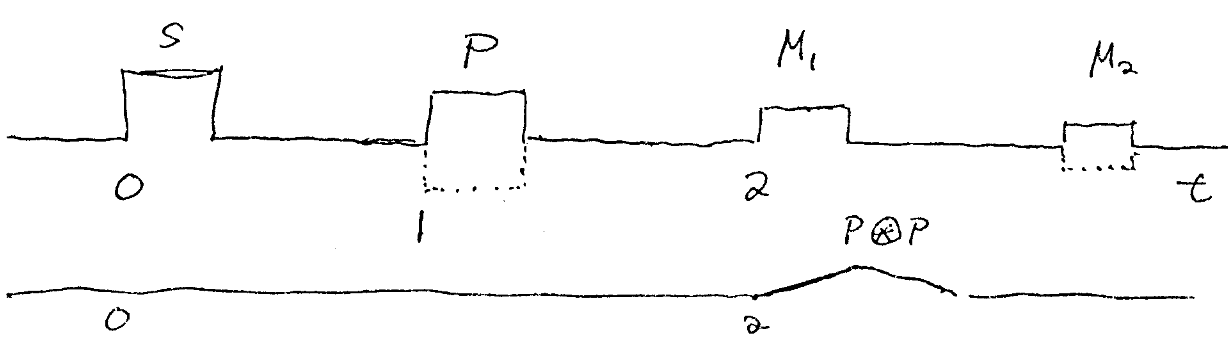

Fig. 3. Since $P \otimes P$ comes later than M_1 we use an anti-causal filter to push it to an earlier time.

Thus (6) with (7) becomes

$$\min_{u} \sum_{t=N_2}^{N_3} \left(a_t - \sum_{k=0}^{-(N_1 \text{ or less})} u_k \; y_{t-k} \right)^2 \qquad (8)$$

Once u_k has been estimated we generalize (4) to a convolution with u_k. Then (4) is run out to large values of time in a completely deterministic fashion. No reflection coefficients are estimated. In generalizing this to diffracting waves a rough guess is that letting z be the outward diffraction operator then (4) becomes

$$c_t = \underbrace{z^{-t}}_{A} \underbrace{\left(a_t - \sum_{k=N_1}^{N_2} \underbrace{z^{k/2} \underbrace{\left(c_k \left(\underbrace{z^{k/2} \, a_{t-k}}_{B} \right) \right)}_{C}}_{D} \right)}_{E} \qquad (9)$$

where the lettered operations have the following interpretation

A: migration of primaries
B: diffraction of downgoing N^{th} multiple
C: reflection at depth k with reflection coefficient c_k
D: diffraction of upcoming wave to give $N + 1^{st}$ multiple
E: cancellation of multiples but not primaries

A more precise statement of (6) with more gating possibility is

$$\min \sum_{t=N_3}^{N_4} \left(a_t - u \sum_{k=N_1}^{\min(t-1,N_2)} a_{t-k} \, a_k \right)^2 \qquad (10)$$

Defining the gate $N_4 - N_3$ to be larger than before we include the possibility of estimating u by fitting the N^{th} multiple to the $N + 1^{st}$. Expanding the $N_2 - N_1$ gate includes the fitting of the first multiple self convolved to the third multiple.

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