

Introduction to Hyperbolic Penalty

Jon Claerbout
2009 Fall report

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- What is IRLS ?

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- I knew I had to do better than IRLS.

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- Why?

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- Don't need precisely homogeneous layers $dv/dz=0$.

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- Choose these parameters by percentiles

$$t^2 = z^2 + x^2$$

$$C^2 = r^2 + R^2$$

$$C_i = \sqrt{r_i^2 + R^2} - R$$

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- tends to CG quality convergence

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- Most methods start with a guess and then improve it.
- Convex optimization is the only way to get an answer independent of the starting guess.

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