

Data Interpolation in Pyramid Domain



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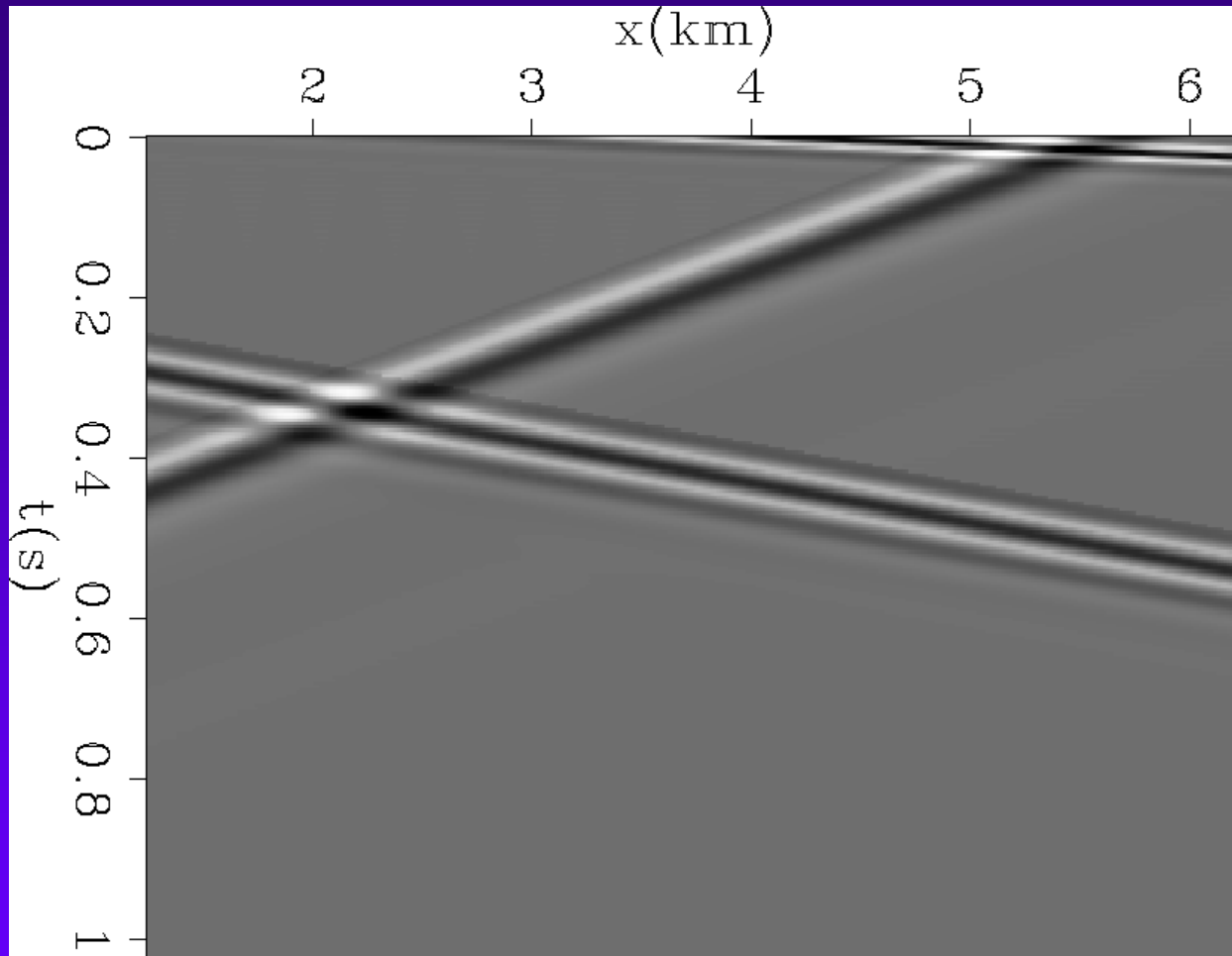
SEP-134, pg 225

SEP meeting, May 2008

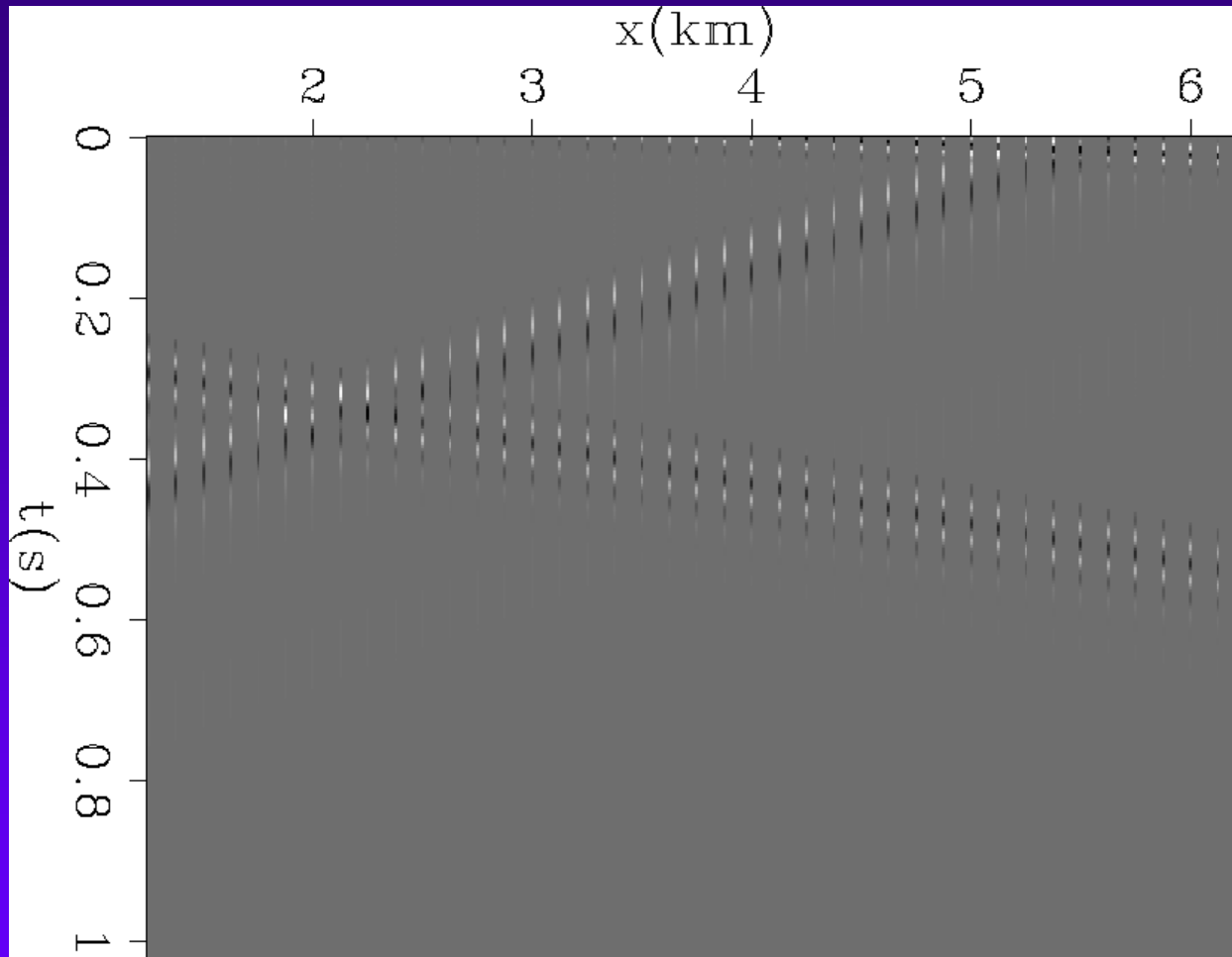
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Why in the pyramid domain?

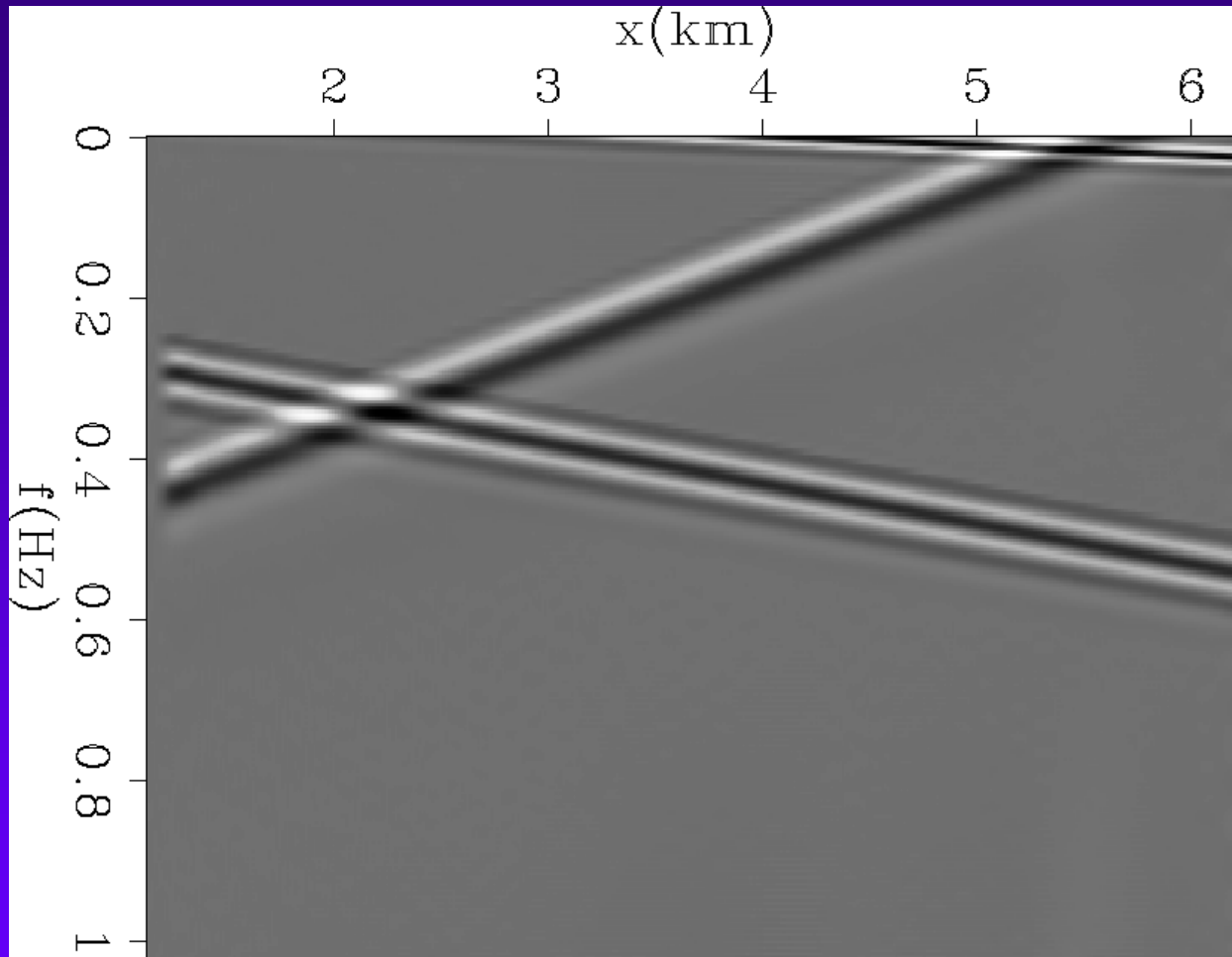
3 plane waves



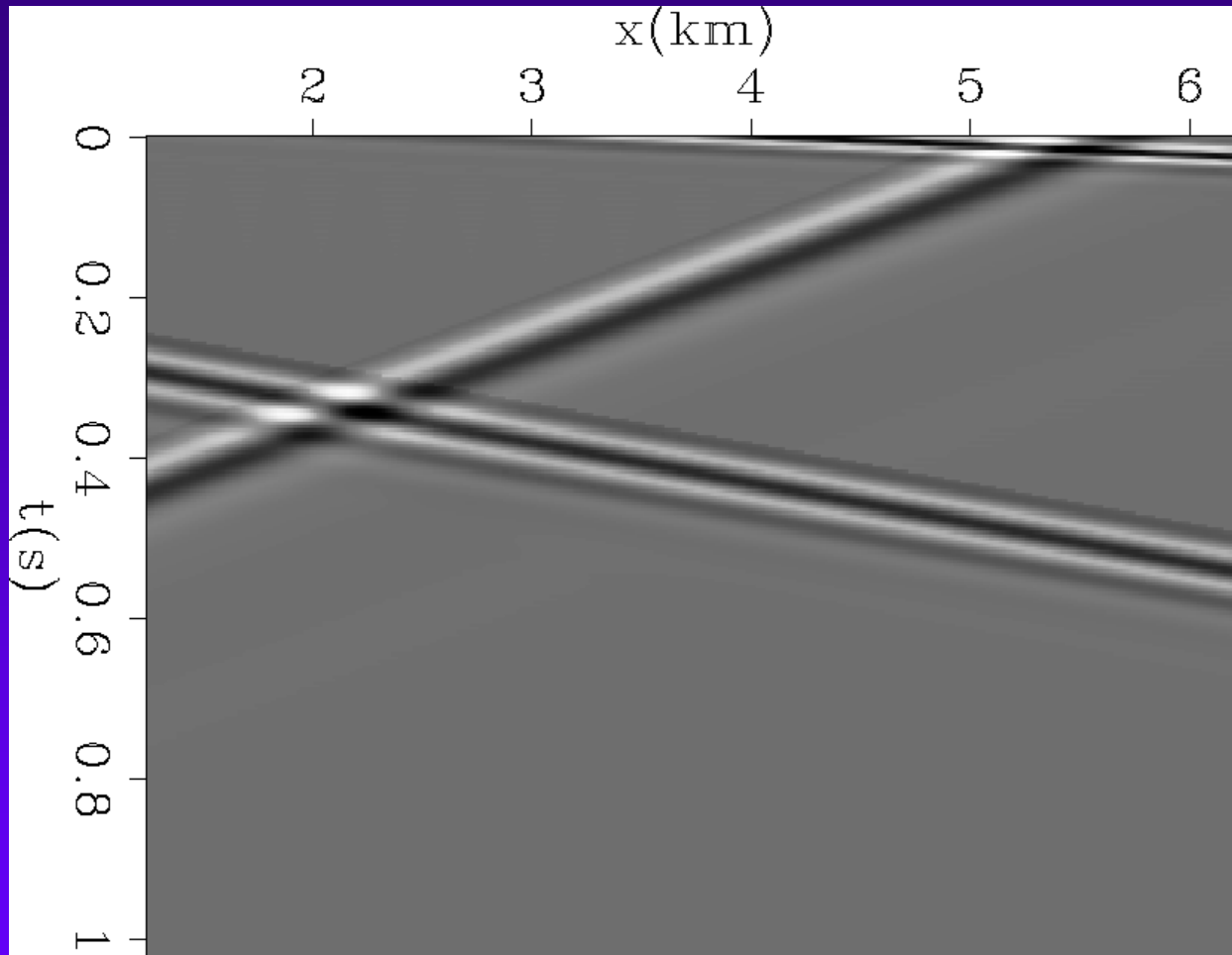
sub-sampled plane waves



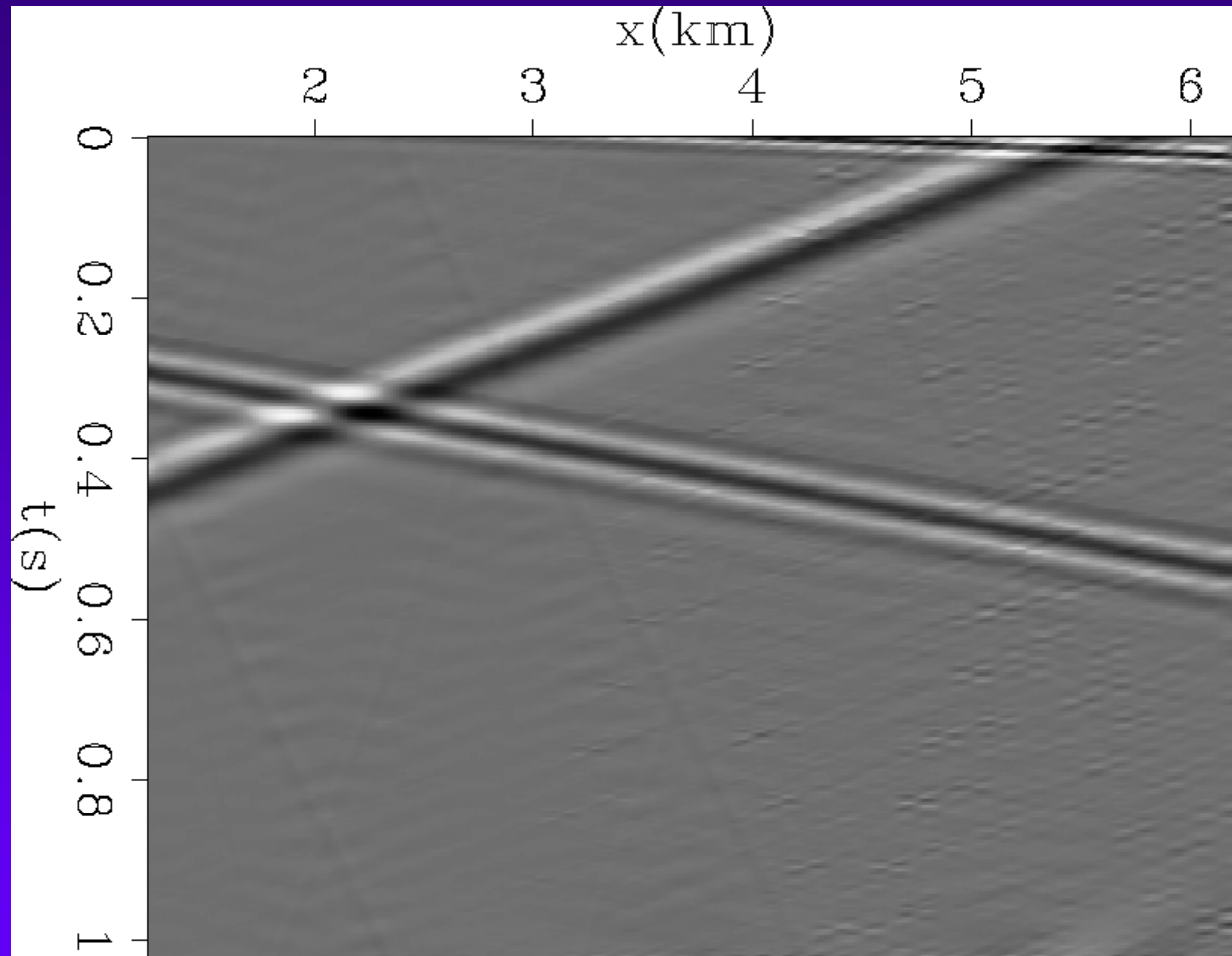
f-x PEF interpolation result



3 plane waves



Pyramid domain interpolation result



**f-x PEF interpolation need to estimate
more than *100* PEF coefficients**

VS

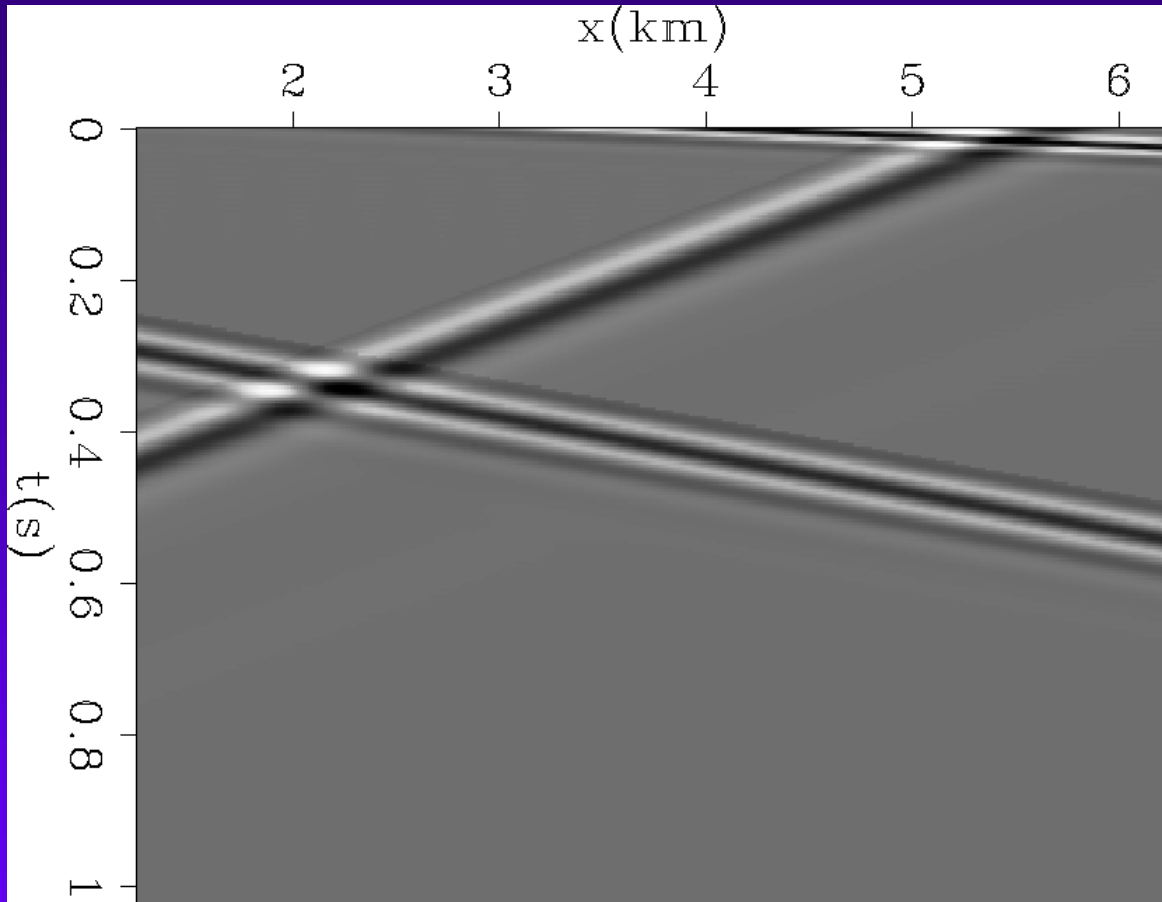
**Pyramid domain interpolation need to
estimate less than *10* PEF coefficients**

Outline

- 1 Introduction
- 2 Data interpolation in pyramid domain
- 3 Future work

- 1 Introduction
 - a Pyramid domain
 - b Pyramid transform
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t-x data

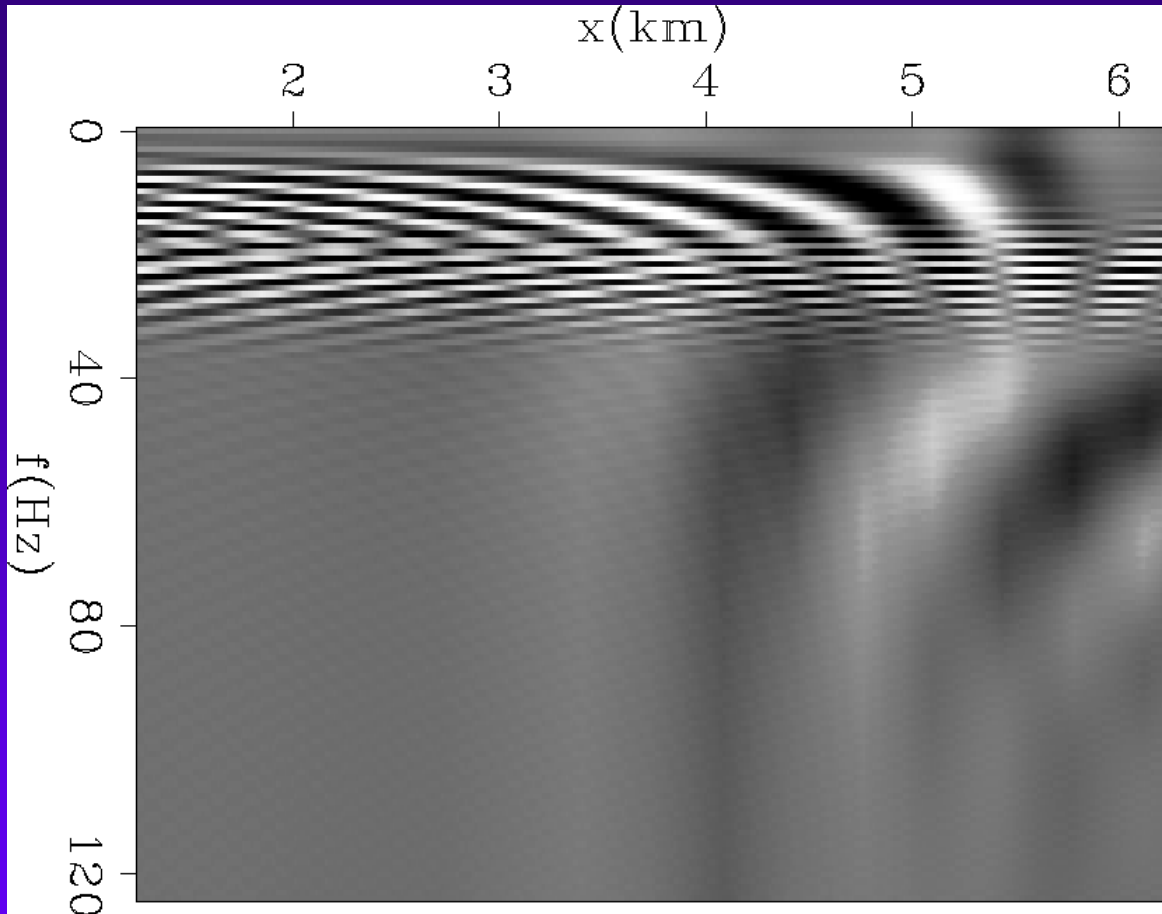


real part

t-x data



f-x data



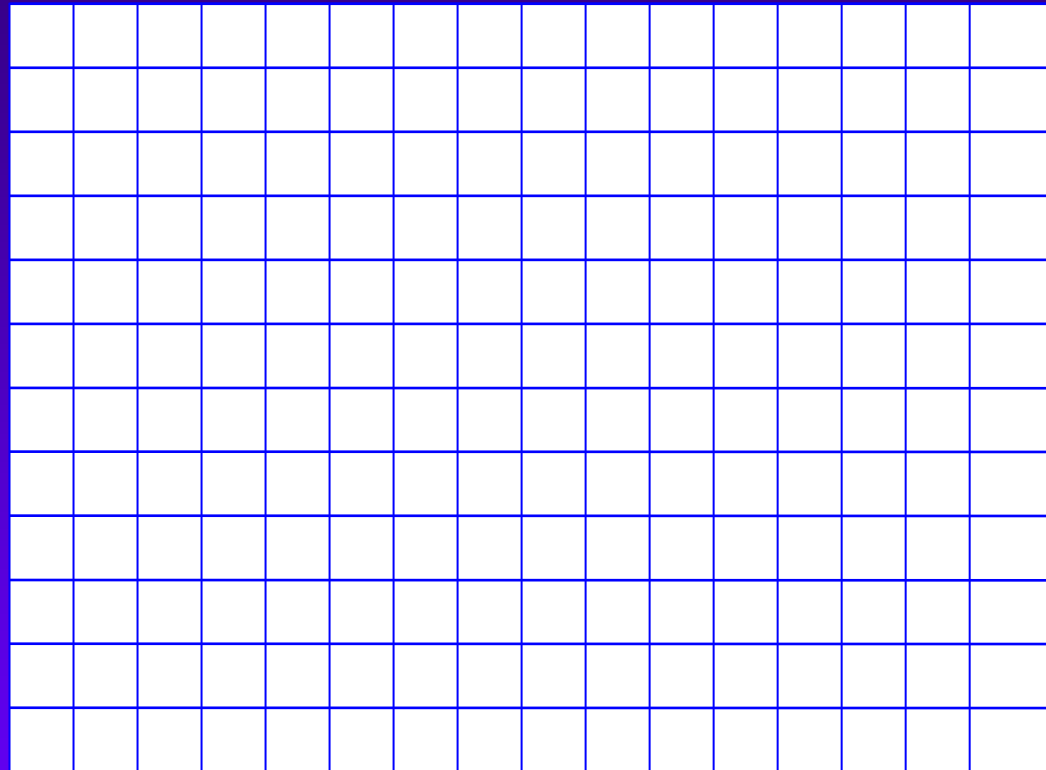
real part

t-x data



f-x data

constant Δx



real part

t-x data

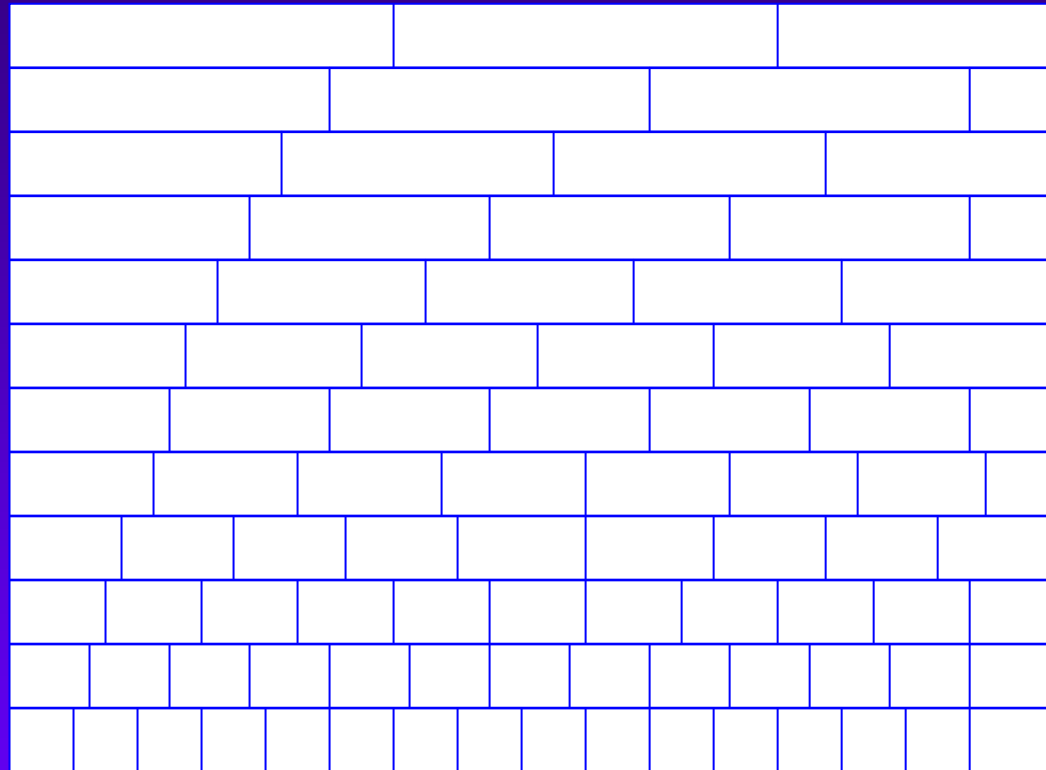


f-x data

constant Δx



$$\Delta x(f) \propto \frac{1}{f}$$



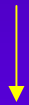
real part

t-x data



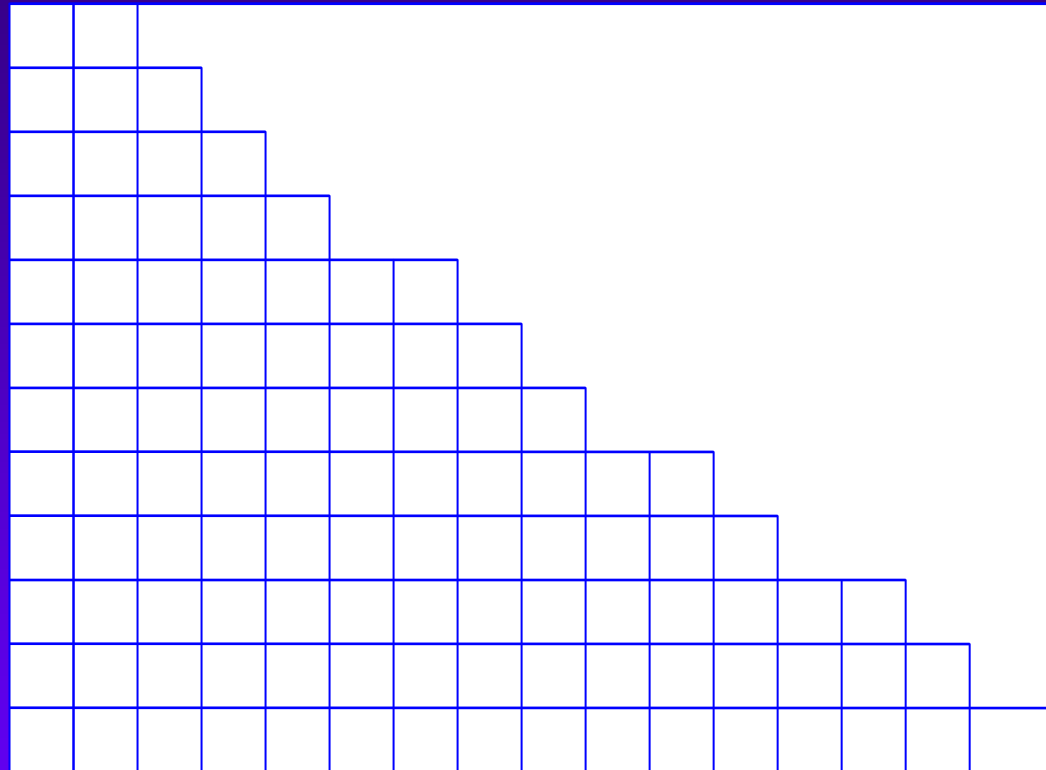
f-x data

constant Δx



$$\Delta x(f) \propto \frac{1}{f}$$

stretch to constant sampling



real part

t-x data



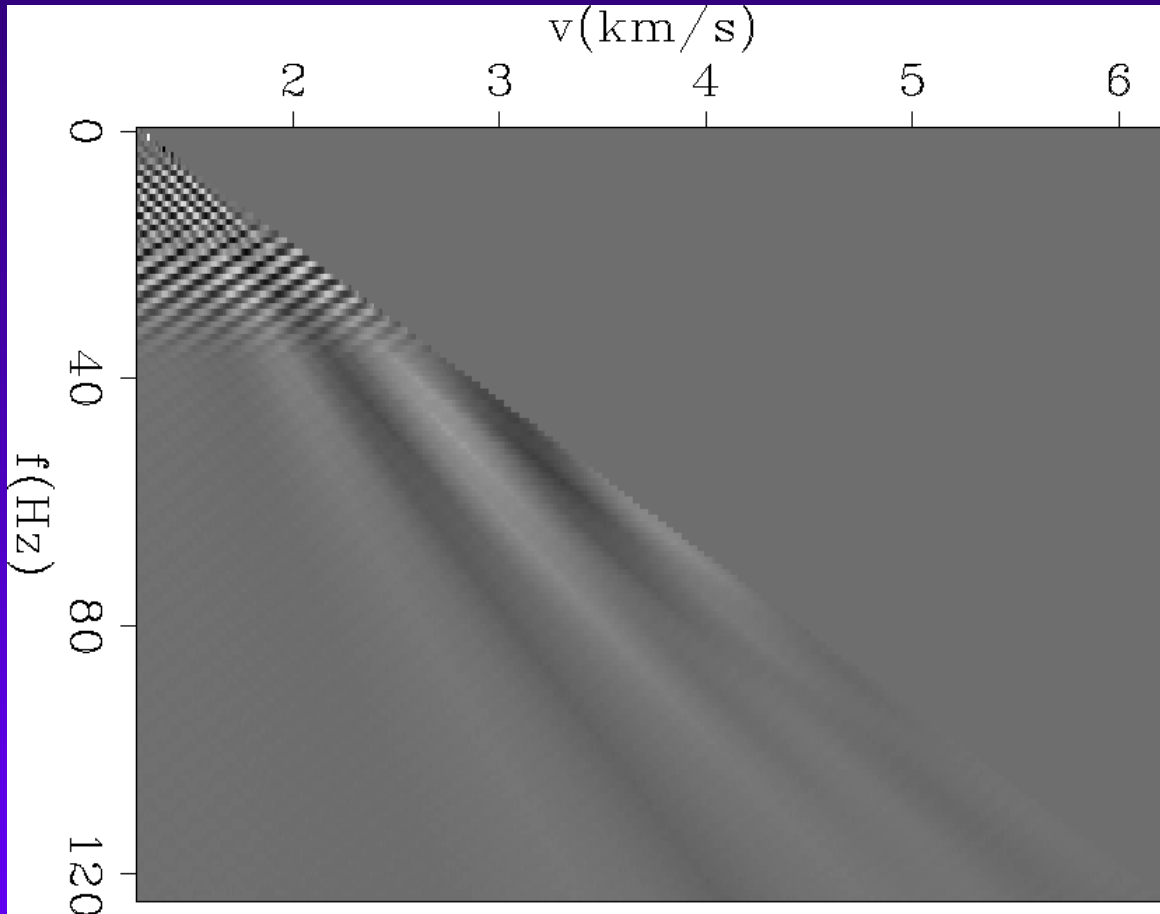
f-x data

constant Δx

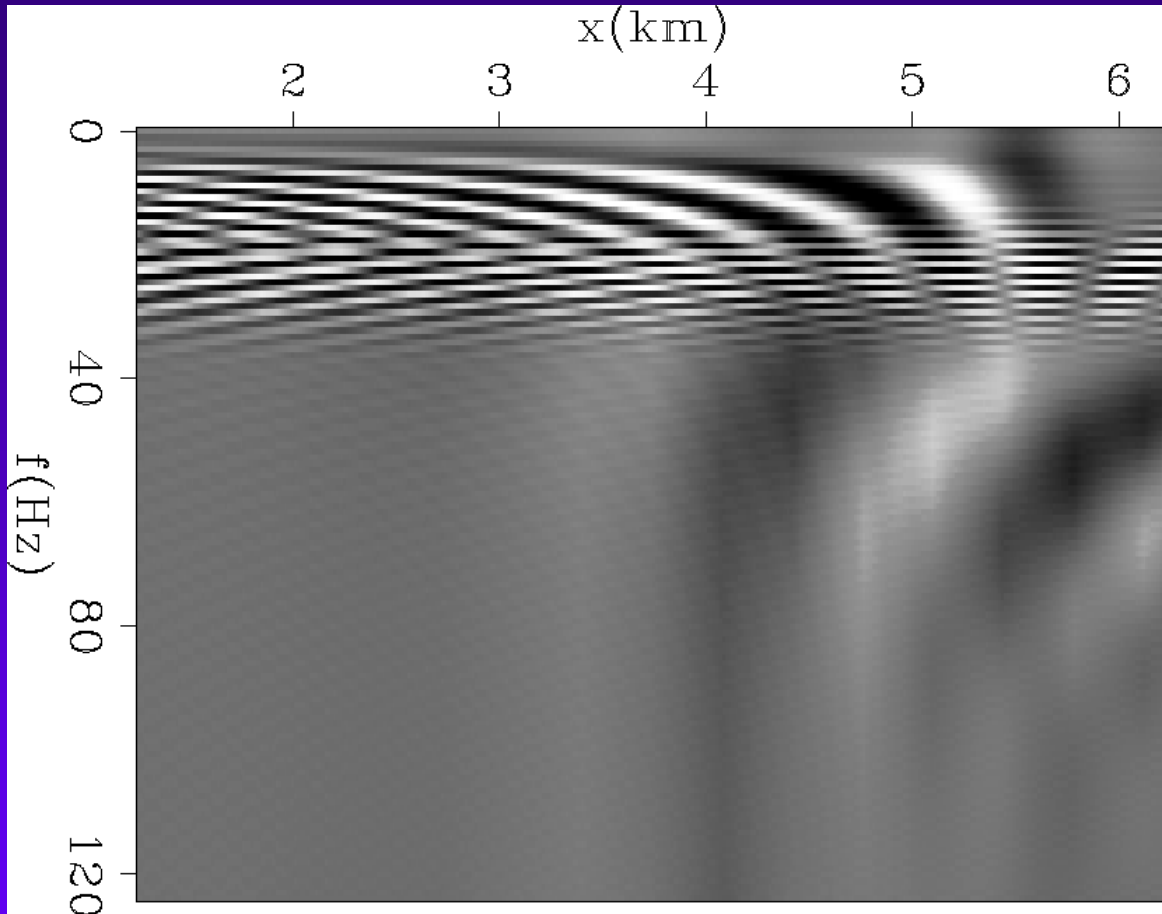
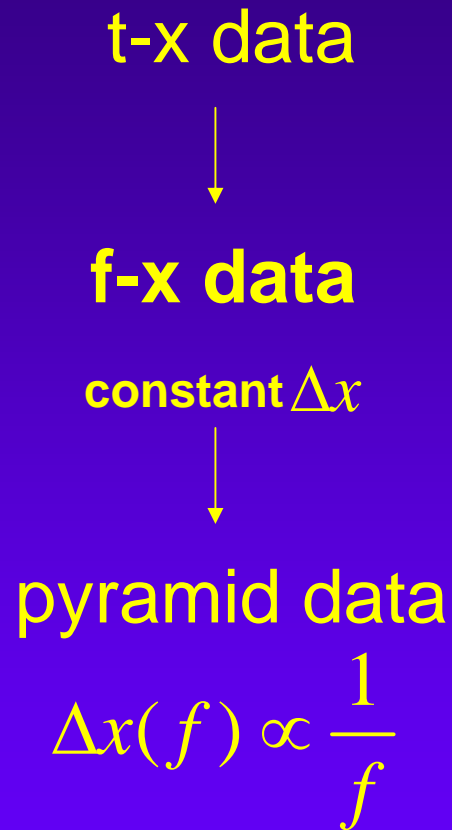


pyramid data

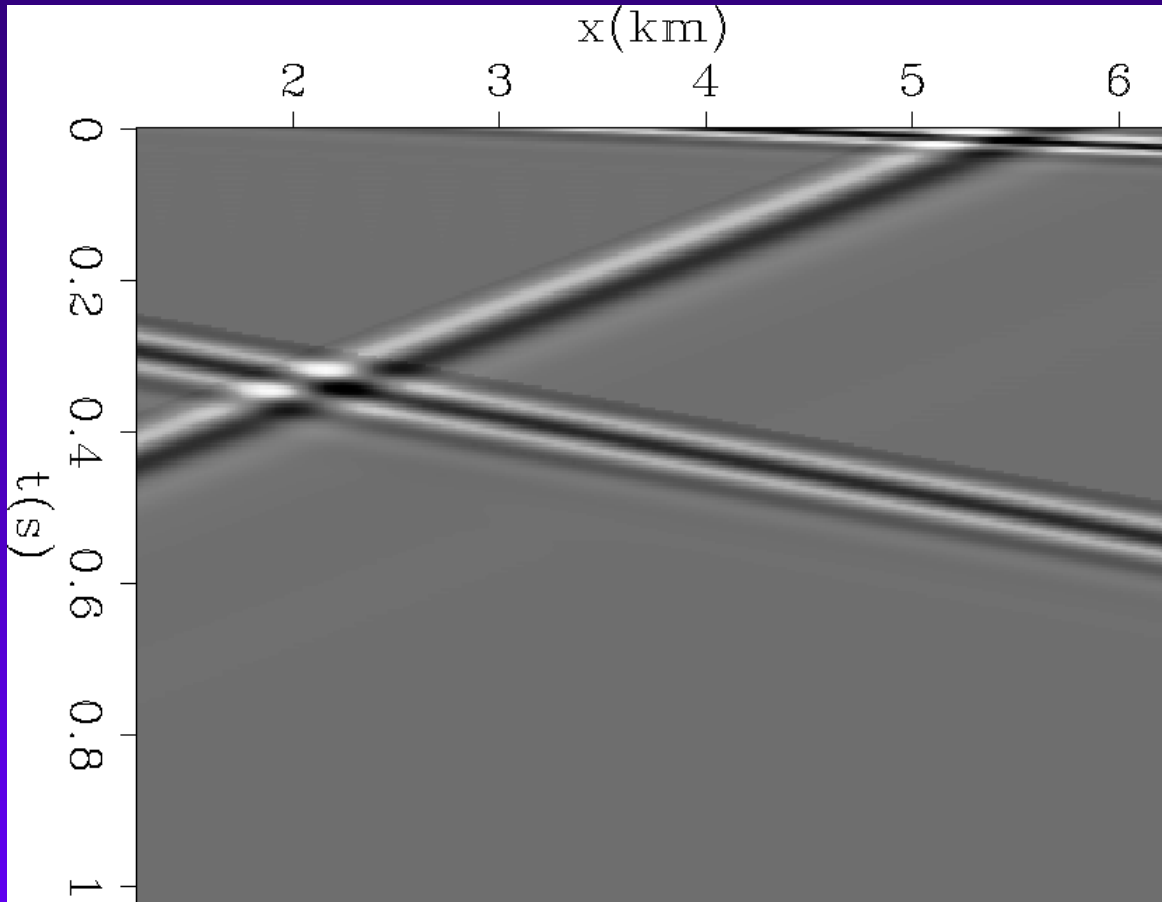
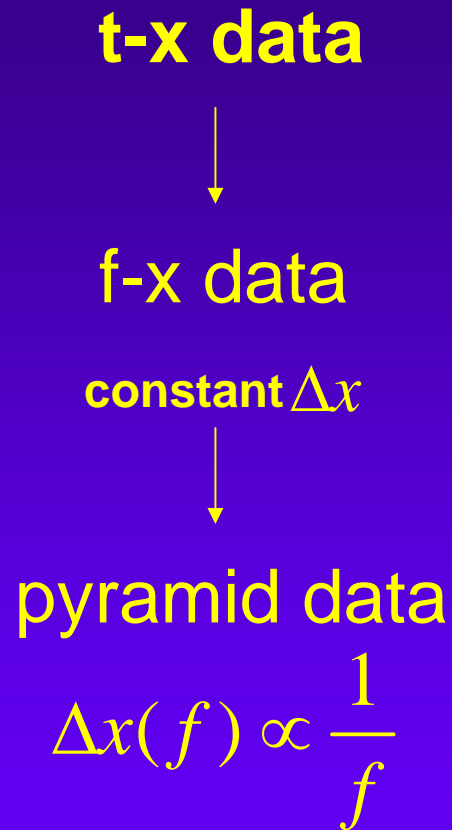
$$\Delta x(f) \propto \frac{1}{f}$$



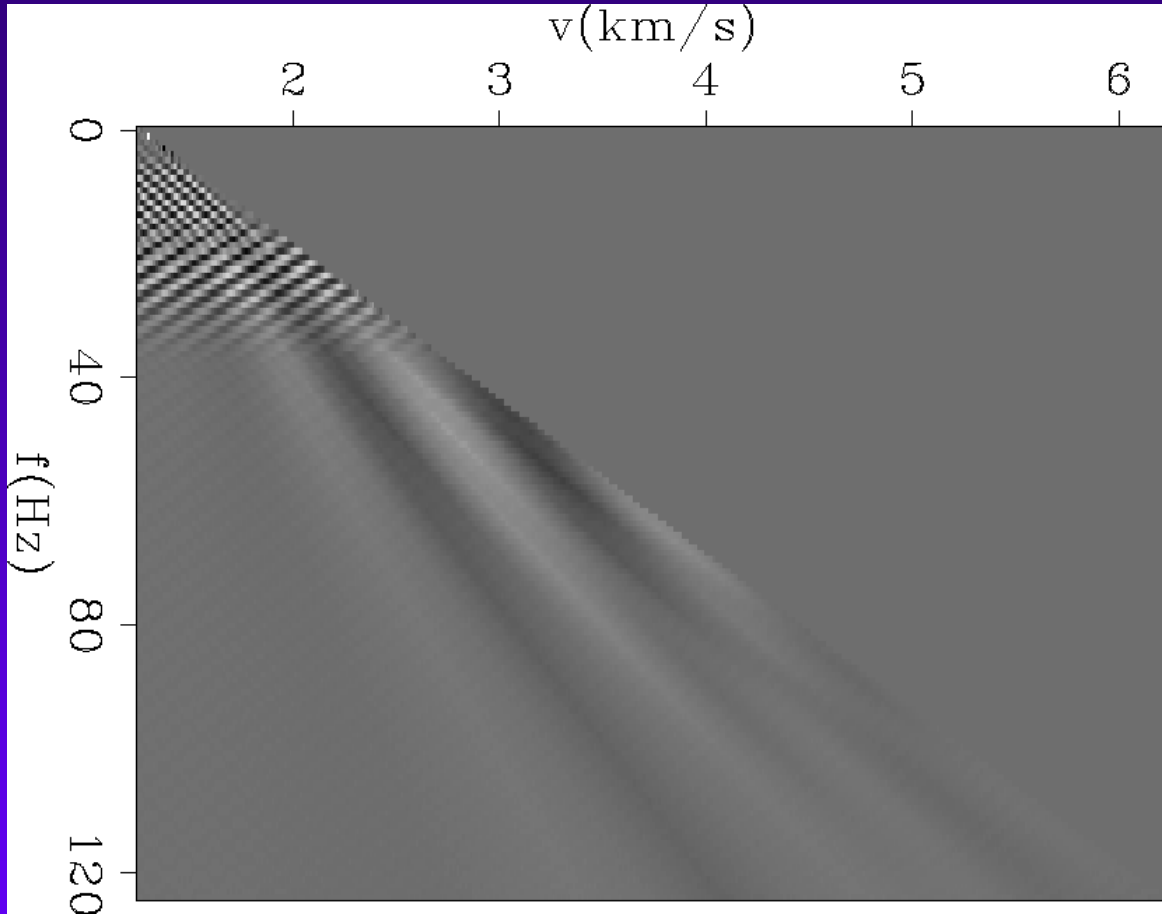
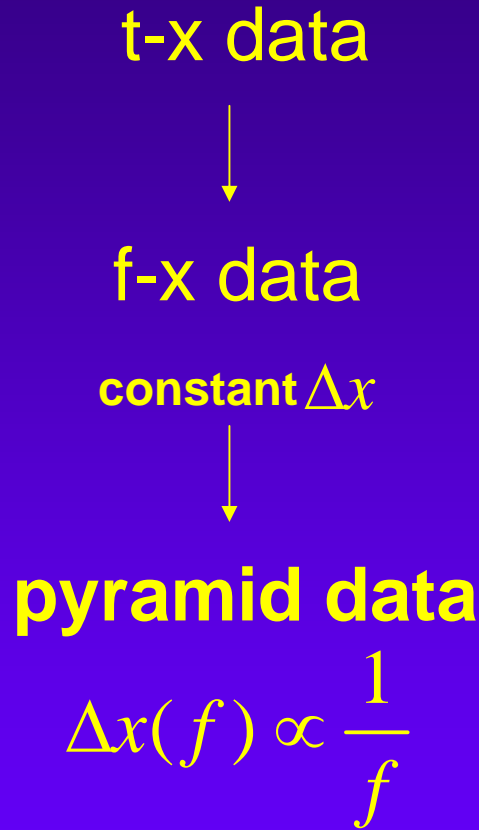
real part



real part



real part



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Inverse pyramid transform

For each frequency f , perform (loop over output)

$$\mathbf{d} = \mathbf{L} \mathbf{m}$$

\mathbf{m} • pyramid data

\mathbf{d} • f-x data

\mathbf{L} • Linear interpolation operator

Forward pyramid transform

For each frequency f , solve

$$\mathbf{L}\mathbf{m} - \mathbf{d} = \mathbf{r} \approx 0$$

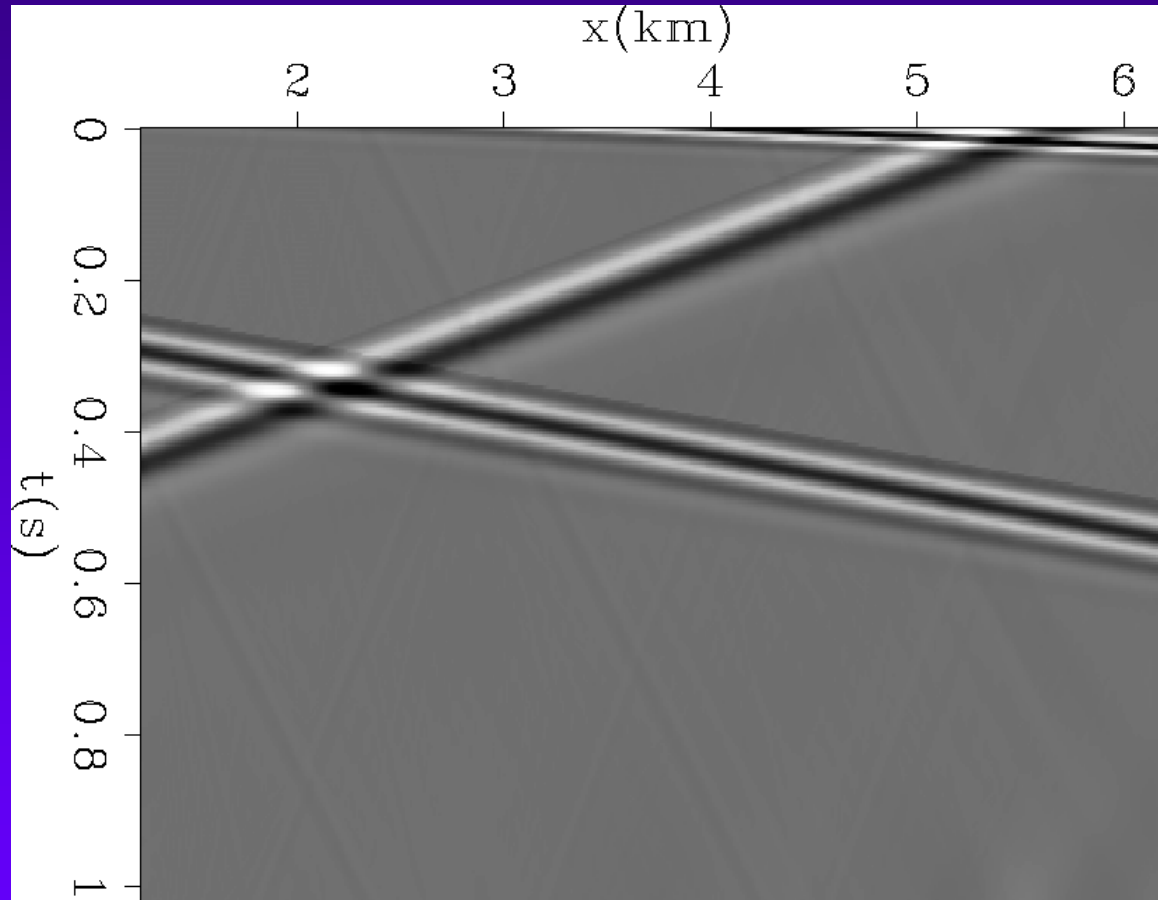
\mathbf{m} • pyramid data

\mathbf{d} • f-x data

\mathbf{L} • Linear interpolation operator

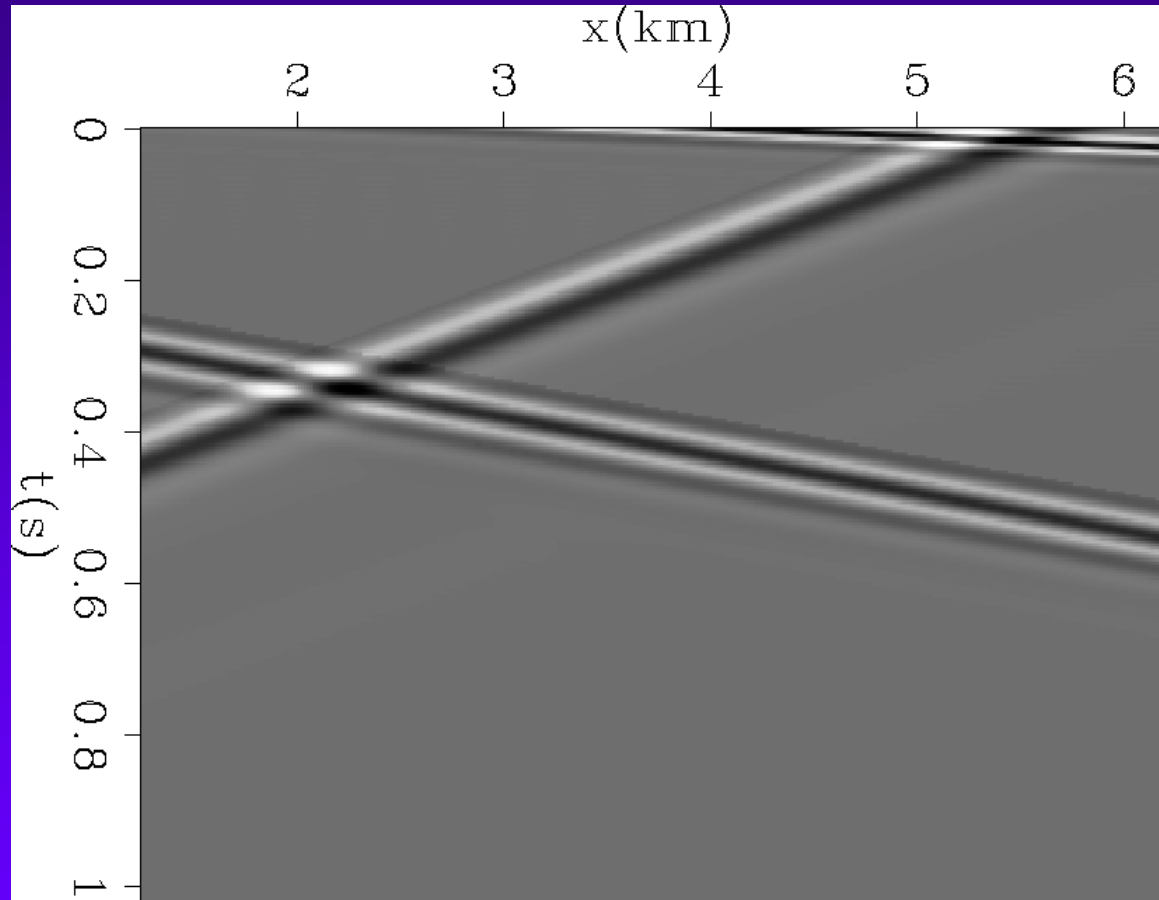
Example

reconstructed data



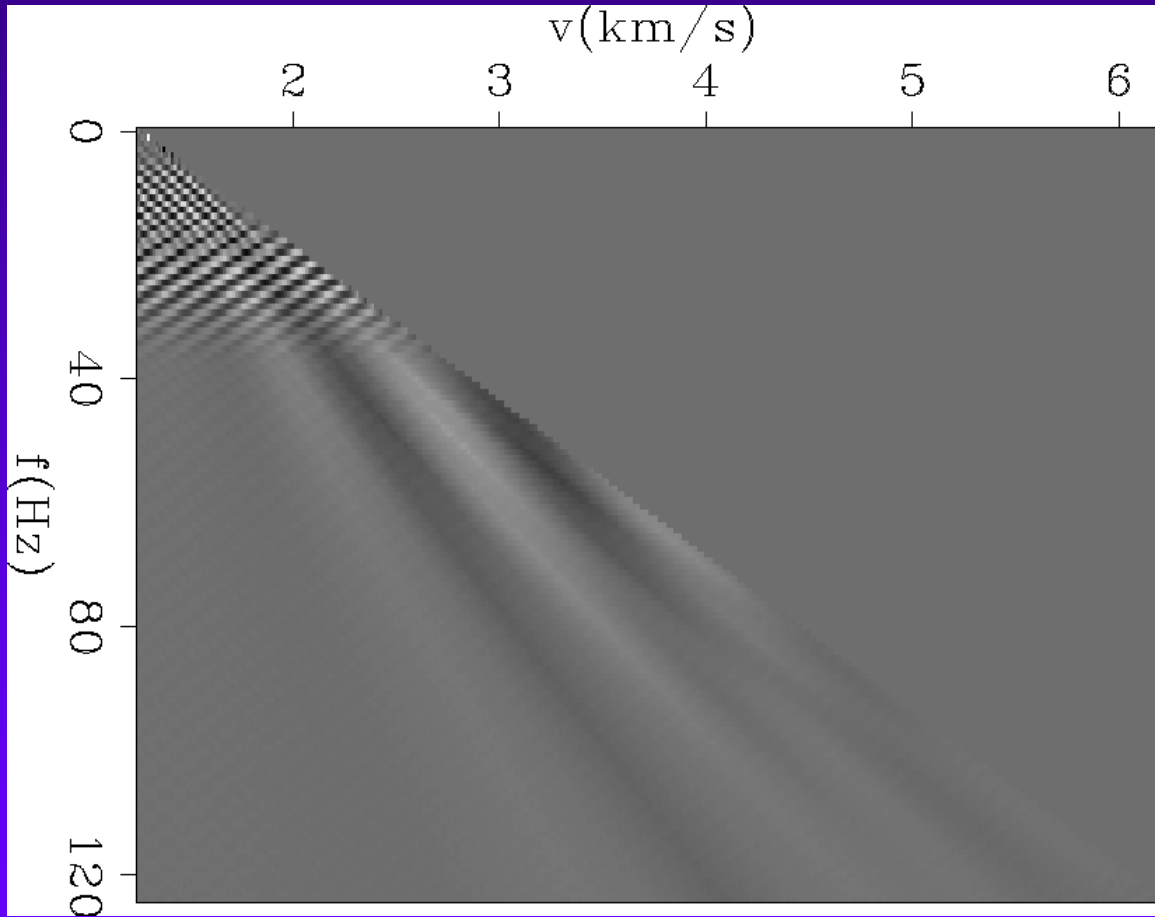
Example

original data



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1 Derive a PEF \mathbf{A} from original data



$\rightarrow (1, a_1, a_2, a_3)$

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

$$\mathbf{L}(\mathbf{d}_{interp} - \mathbf{d}_{known}) = \mathbf{r}_l$$

- \mathbf{d}_{interp} • interpolated data
- \mathbf{d}_{known} • known data
- \mathbf{L} • Mask operator being 1 where data are known

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

$$\varepsilon \mathbf{W} \mathbf{A} \mathbf{d}_{interp} = \mathbf{r}_2$$

- \mathbf{A} • PEF Convolution operator
- ε • Weighting coefficient
- \mathbf{W} • Mask operator being 1 where convolution equation can be applied
- \mathbf{d}_{interp} • interpolated data

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

$$\min \left(\|\mathbf{r}_1\|^2 + \|\mathbf{r}_2\|^2 \right)$$

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

$$\min \left(\|\mathbf{r}_1\|^2 + \|\mathbf{r}_2\|^2 \right)$$

Match the data at known locations

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

$$\min \left(\|\mathbf{r}_1\|^2 + \|\mathbf{r}_2\|^2 \right)$$

Match the data represented by the PEF

2 Solve missing data from \mathbf{A} and known data \mathbf{d}_{known}

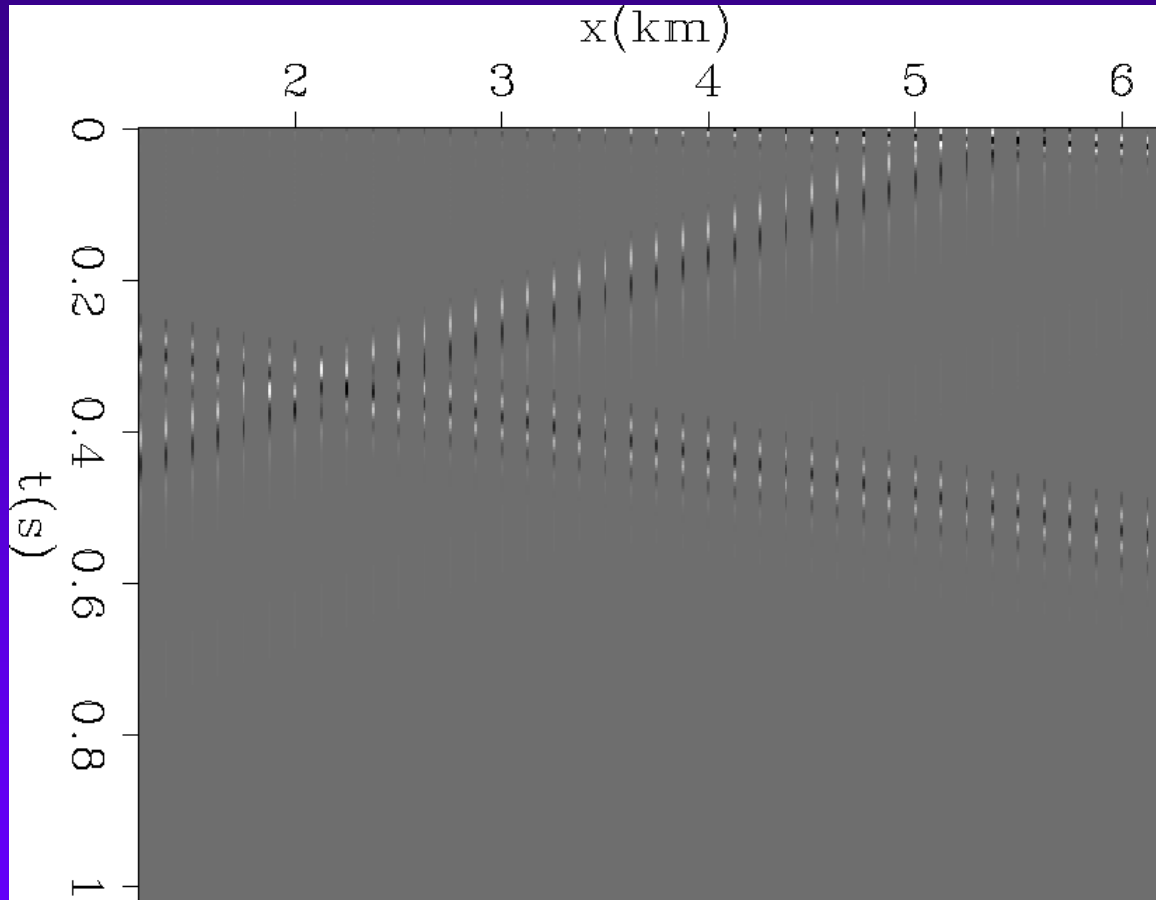
$$\mathbf{L}(\mathbf{d}_{interp} - \mathbf{d}_{known}) = \mathbf{r}_1$$

$$\varepsilon \mathbf{W} \mathbf{A} \mathbf{d}_{interp} = \mathbf{r}_2$$

$$\min \left(\|\mathbf{r}_1\|^2 + \|\mathbf{r}_2\|^2 \right)$$

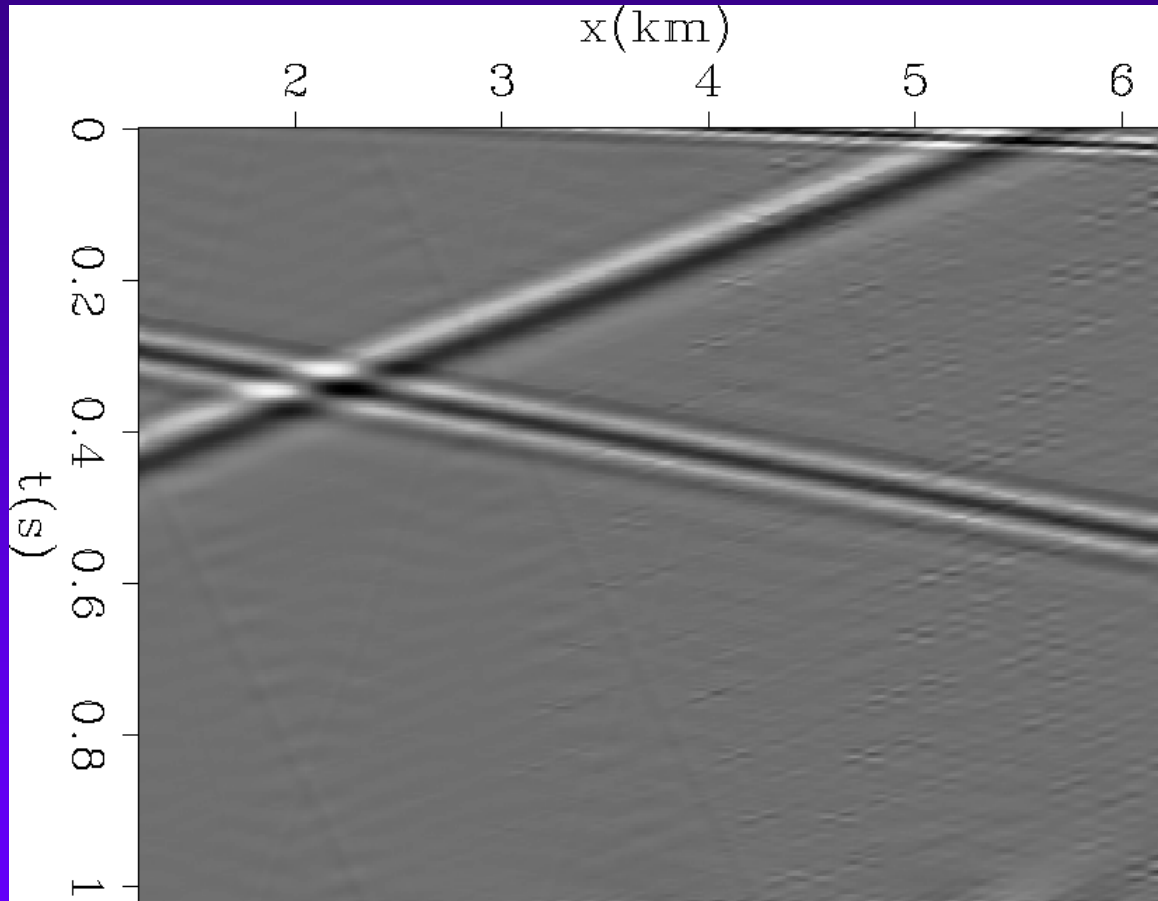
Example

sub-sampled t-x data



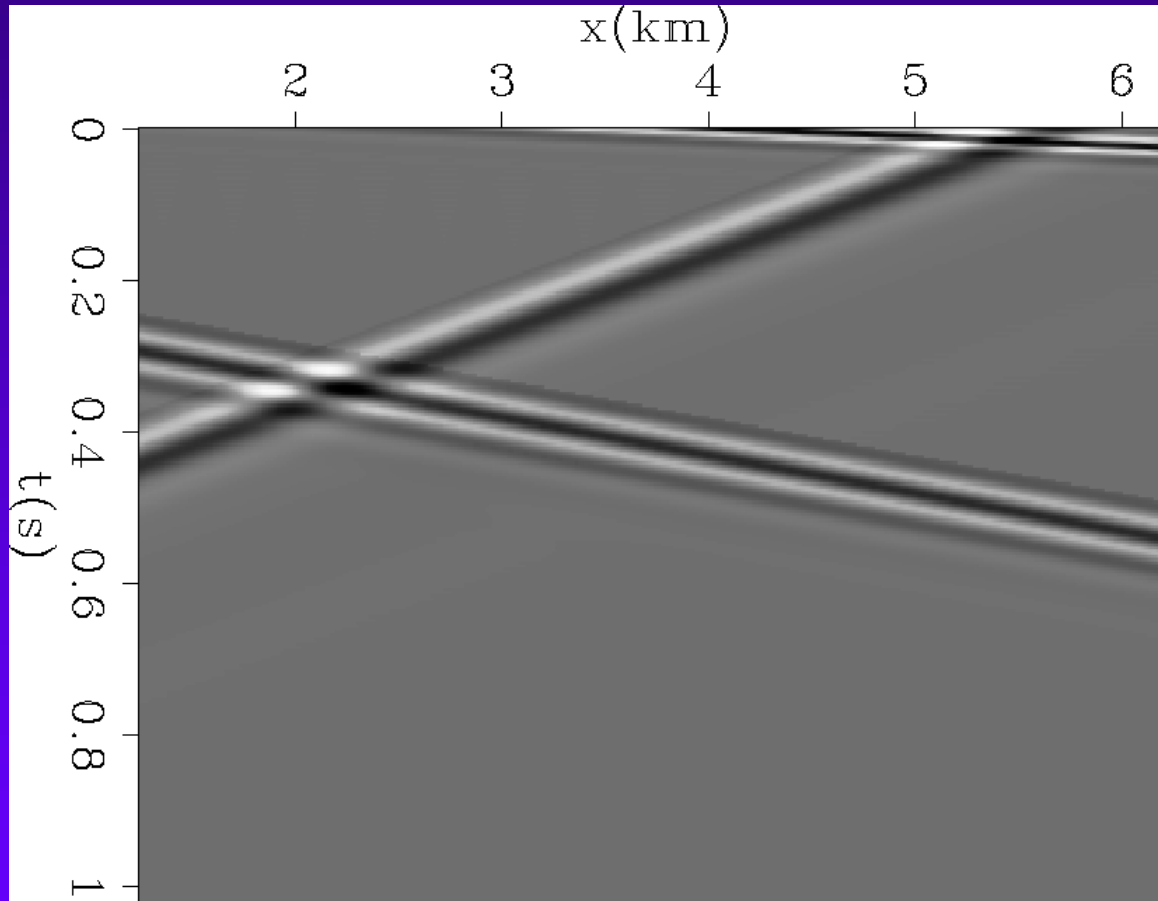
Example

interpolated data



Example

original t-x data



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1. Interpolate missing data without using the PEF that estimated from original data
2. Compare this data interpolation with Bill's methods

Acknowledgments

- Jon and Shuki for data interpolation ideas
- Antoine and Bill for numerous debugging suggestions
- Other SEP members for various suggestions