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SEISMIC ANISOTROPY FOR POLAR MEDIA: EXTENDED THOMSEN FORMULATION FOR LONGER OFFSETS

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OUTLINE

- Review of Anisotropy Due to Fractures
  - Sayers-Kachanov method
  - Thomsen parameters
  - Extensions to larger offsets
- P-wave and SV-wave Connections
  - Approximate and exact peak finding
  - Results for SV-waves, P-waves
- Discussion and Conclusions
- References

STIFFNESS VERSUS COMPLIANCE

The isotropic stiffness matrix (inverse of the compliance matrix) for an elastic material is often written as $C =$

$$\begin{pmatrix} \lambda + 2G & \lambda & \lambda & & & \\ \lambda & \lambda + 2G & \lambda & & & \\ \lambda & \lambda & \lambda + 2G & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{pmatrix}$$

where λ and G are the two Lamé parameters,

G is shear modulus, and $K = \lambda + 2G/3$ is bulk modulus;

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66} \end{pmatrix} \text{ for VTI media.}$$

ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1+\nu)$ is the shear modulus.

ANISOTROPY DUE TO FRACTURES (2)

In the following discussion, I will assume that all the fractures/cracks are in the form of penny-shaped cracks. These are defined as oblate spheroids, having round and flat holes, circular in cross-section with area $A = \pi b^2$, and volume $V_c = 4\pi a b^2/3$,
or $V_c = 4\pi\alpha b^3/3$, where $\alpha \equiv a/b$
is the aspect ratio. We assume $\alpha = a/b << 1$,
so the crack porosity $\phi = V_c/V << 1$.

ANISOTROPY DUE TO FRACTURES (3)

Sayers and Kachanov (1991) show that corrections to the isotropic matrix S_{ij}, caused by low crack densities ($\rho << 1$), can be written as

$$\Delta\left(\frac{1}{G}\right) = 4\eta_2\rho/3,$$

$$\Delta\left(-\frac{\nu}{E}\right) = 2\eta_1\rho/3,$$

$$\Delta\left(\frac{1}{E}\right) = 2(\eta_1 + \eta_2)\rho/3,$$

where η_1 and η_2 are parameters to be found from EMT.

ANISOTROPY DUE TO FRACTURES (4)

Thus, in the isotropic case, we have

$$\Delta S_{ij} = (2\rho/3)\times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1 & & & \\ \eta_1 & \eta_1 & (\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (5)

For horizontal cracks, we get an anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (6)

For vertical cracks whose axis of symmetry is randomly oriented in the xy-plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1/2 & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1/2 & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (7)

Examples of the values of the η's found from various effective medium theories are:

EMT	η_1	η_2
NI	-0.000216	0.0287
DS	-0.000216	0.0290
CPA	-0.000258	0.0290
SC	$-.0000207$	0.0290

THOMSEN WEAK ANISOTROPY FORMULAS

$$v_p(\theta) = v_p(0)\left(1 + \delta\, sin^2\theta\, cos^2\theta + \epsilon\, sin^4\theta\right),$$

$$v_{sv}(\theta) = v_s(0)\left(1 + (c_{33}/c_{44})(\epsilon - \delta)\, sin^2\theta\, cos^2\theta\right),$$

$$v_{sh}(\theta) = v_s(0)\left(1 + \gamma\, sin^2\theta\right).$$

where

$$\gamma = \tfrac{c_{66}-c_{44}}{2c_{44}},\ \epsilon = \tfrac{c_{11}-c_{33}}{2c_{33}},\ \text{and}$$

$$\delta = \left(\tfrac{c_{13}+c_{33}}{2c_{33}}\right)\left(\tfrac{c_{13}+2c_{44}-c_{33}}{c_{33}-c_{44}}\right)$$

THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures,
two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = -\eta_2\rho\frac{E}{4(1+\nu)} = -\eta_2\rho\frac{G}{2}$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq -\eta_2\rho\frac{G}{1-\nu}.$$

The remaining Thomsen parameter δ, which is the one that determines the degree of anellipticity in angular dependence of the wave speeds is given exactly by $\delta = \epsilon$,
which means there is no deviation from ellipticity.

THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}} = \eta_2 \rho G$$

$$\epsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}} \simeq \eta_2 \rho \frac{2G}{1-\nu}.$$

Note that these results both differ exactly by a factor of -2 from the previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

THOMSEN WEAK ANISOTROPY FORMULAS AGAIN

$$v_p(\theta) = v_p(0)\left(1 + \delta\, sin^2\,\theta\, cos^2\,\theta + \epsilon\, sin^4\,\theta\right),$$

$$v_{sv}(\theta) = v_s(0)\left(1 + (c_{33}/c_{44})(\epsilon - \delta)\, sin^2\,\theta\, cos^2\,\theta\right),$$

$$v_{sh}(\theta) = v_s(0)\left(1 + \gamma\, sin^2\,\theta\right).$$

When $\delta = \epsilon$, as we have shown happens for these crack models at low crack density, the SV-wave has constant velocity for all angles in this approximation. Even if $\epsilon \neq \delta$, the Thomsen formulation has another peculiarity which is that the peak value of the SV-wave is ALWAYS at $\theta = 45^o$ – BUT usually NOT true.

EXACT FORMULAS FOR VTI WAVE SPEEDS

$$v_p^2(\theta) = \frac{1}{2\rho}[c_{44} + c_{11}\, sin^2\,\theta + c_{33}\, cos^2\,\theta] + R(\theta)$$

$$v_{sv}^2(\theta) = \frac{1}{2\rho}[c_{44} + c_{11}\, sin^2\,\theta + c_{33}\, cos^2\,\theta] - R(\theta)$$

where

$$R^2(\theta) = [(c_{11} - c_{44})\, sin^2\,\theta - (c_{33} - c_{44})\, cos^2\,\theta]^2 + [c_{13} + c_{44}]^2\, sin^2\,\theta\, cos^2\,\theta.$$

These two formulas can be greatly simplified by noting that

$$R(\theta) = [c_{11}\, sin^2\,\theta + c_{33}\, cos^2\,\theta - c_{44}]\sqrt{1 - \zeta(\theta)},$$

where ζ can be shown to be given by:

$$\zeta(\theta) = \zeta_m\, sin^2\,\theta_m\, cos^2\,\theta/[1 - cos\,2\theta_m\, cos\,2\theta]^2$$

The maximum value of $\zeta(\theta_m) = \zeta_m$ and occurs when

$$tan\,\theta_m = (c_{33} - c_{44})/(c_{11} - c_{44}).$$

LOCATION OF SV-WAVE MAXIMUM (1)

Let's define the angle at which the SV-wave has its maximum as θ_{ex}. Two approximations to this angle are Thomsen's weak anisotropy approximation (twa) $\theta_{twa} \equiv 45^o$, and the θ_m value already discussed.

It can be shown that

$$tan^2\,\theta_{ex} \simeq tan\,45^o\,tan\,\theta_m.$$

So the geometric mean of these two tangents is an excellent approximation to the tangent of the true angle of the maximum.

LOCATION OF SV-WAVE MAXIMUM (2)

Furthermore, it is obviously possible to compute the location exactly for the maximum from these equations, but there does NOT appear to be any simple formula for this result. It requires instead an iterative solution to find the true value of θ_{ex}.

I now compare these various results in some numerical examples.

CONCLUSIONS

- The Sayers and Kachanov (1991) approach for cracks can be combined with Thomsen equations for seismic wave anisotropy. This permits useful interpretation of measured Thomsen parameters in terms of fracture properties.
- The true SV-wave behavior is sufficiently different from that assumed by Thomsen's original weak-anisotropy model that some care should be taken to put its peak in the right place – as this does also have implications for the P-wave behavior.

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