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SEISMIC WAVE PROPAGATION IN ROCKS WITH FLUIDS AND FRACTURES

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OUTLINE

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STIFFNESS VERSUS COMPLIANCE

The isotropic stiffness matrix (inverse of the compliance matrix) for an elastic material is often written as $C =$

$$\begin{pmatrix} \lambda + 2G & \lambda & \lambda & & & \\ \lambda & \lambda + 2G & \lambda & & & \\ \lambda & \lambda & \lambda + 2G & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{pmatrix}$$

where λ and G are the two Lamé parameters,

G is shear modulus, and $K = \lambda + 2G/3$ is bulk modulus.

ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1+\nu)$ is the shear modulus.

ANISOTROPY DUE TO FRACTURES (2)

In the following discussion, I will assume that all the fractures/cracks are in the form of penny-shaped cracks. These are defined as oblate spheroids, having round and flat holes, circular in cross-section with area $A = \pi b^2$, and volume $V_c = 4\pi a b^2/3$,
or $V_c = 4\pi\alpha b^3/3$, where $\alpha \equiv a/b$
is the aspect ratio. We assume $\alpha = a/b << 1$,
so the crack porosity $\phi = V_c/V << 1$.

ANISOTROPY DUE TO FRACTURES (3)

Sayers and Kachanov (1991) show that corrections to the isotropic matrix S_{ij}, caused by low crack densities ($\rho << 1$), can be written as

$$\Delta\left(\frac{1}{G}\right) = 4\eta_2\rho/3,$$

$$\Delta\left(-\frac{\nu}{E}\right) = 2\eta_1\rho/3,$$

$$\Delta\left(\frac{1}{E}\right) = 2(\eta_1 + \eta_2)\rho/3,$$

where η_1 and η_2 are parameters to be found from EMT.

ANISOTROPY DUE TO FRACTURES (4)

Thus, in the isotropic case, we have

$$\Delta S_{ij} = (2\rho/3)\times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1 & & & \\ \eta_1 & \eta_1 & (\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (5)

For horizontal cracks, we get an anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (6)

For vertical cracks whose axis of symmetry is randomly oriented in the xy-plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1/2 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1/2 & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (7)

Examples of the values of the η's found from various effective medium theories are:

$\underline{EMT}$	$\underline{\eta_1}$	$\underline{\eta_2}$
NI	-0.000216	0.0287
DS	-0.000216	0.0290
CPA	-0.000258	0.0290
SC	$-.0000207$	0.0290

THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures,

two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = -\eta_2\rho\frac{E}{4(1+\nu)} = -\eta_2\rho\frac{G}{2}$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq -\eta_2\rho\frac{G}{1-\nu}.$$

The remaining Thomsen parameter δ, which is the one that determines the degree of anellipticity in angular dependence of the wave speeds is given exactly by $\delta = \epsilon$, which means there is no deviation from ellipticity.

THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = \eta_2 \rho G$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq \eta_2 \rho \frac{2G}{1-\nu}.$$

Note that these results both differ exactly by a factor of -2 from the previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

RAYLEIGH WAVE SPEED

For a transversely isotropic medium with vertical axis of symmetry (which is true of both the cases described so far), the Rayleigh surface wave has a speed determined by the following equation:

$$\frac{1}{16}q^3 - \frac{1}{2}q^2 + \left(\frac{3}{2} - \frac{C_{66}}{C_{11}}\right)q + \left(\frac{C_{66}}{C_{11}} - 1\right) = 0,$$

where $q \equiv v_R^2/v_s^2$ and $v_s^2 = C_{66}/\rho_0$,
with ρ_0 being the inertial mass density of the medium.
Recall that $C_{66} = C_{44}(1 + 2\gamma)$ and that $C_{11} = C_{33}(1 + 2\epsilon)$ in terms of Thomsen parameters.

GASSMANN'S EQUATIONS (1)

$$K_u = K_d + \frac{\alpha^2}{(\alpha-\phi)/K_m + \phi/K_f},$$

where K_u is the undrained bulk modulus, K_d is the drained bulk modulus, K_m is the mineral (or solid) modulus, K_f is the pore fluid bulk modulus, ϕ is the porosity, and $\alpha = 1 - K_d/K_m$.

Rearranging into compliance form, we have

$$\frac{1}{K_u} - \frac{1}{K_d} = -\frac{\alpha}{K_d} \times$$

$$\left[1 + \frac{K_d\phi}{K_f\alpha}\left(1 - \frac{K_f}{K_m}\right)\right]^{-1}.$$

Also, porosity $\phi = \frac{4\pi a}{3b}\frac{Nb^3}{V} = \frac{4\pi a}{3b}\rho$.

GASSMANN'S EQUATIONS (2)

Compliance correction matrix for fluid inclusions:

$$\Delta S_{ij} = -\gamma^{-1} \begin{pmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & & & \\ \beta_1\beta_2 & \beta_2^2 & \beta_2\beta_3 & & & \\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

The fluid effects (K_f) appear only in the overall factor γ. The coefficients β_i, $i = 1, 2, 3$, satisfy a sumrule of the form $\beta_1 + \beta_2 + \beta_3 = 1/K_d - 1/K_m \equiv \alpha/K_d$. K_d is the drained bulk modulus. K_m is the mineral (solid) modulus.

GASSMANN CORRECTIONS TO SAYERS-KACHANOV FORMULAS

With $\rho = Nr^3/V$ being the crack density parameter, and η_2 being the Sayers-Kachanov parameter that perturbs the shear compliance, we have

$$\Delta S_{ij} \simeq \frac{2\eta_2\rho}{3}\times$$

$$\begin{pmatrix} (1-K_f/K_m) & 0 & 0 & & & \\ 0 & (1-K_f/K_m) & 0 & & & \\ 0 & 0 & (1-K_f/K_m) & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

MAVKO-JIZBA REDERIVED

It is now easy to show that

$$\frac{1}{K_u} - \frac{1}{K_d} \equiv \Delta\left(\frac{1}{K_u}\right) = 2\eta_2\rho(1 - K_f/K_m)$$

and

$$\frac{1}{G_u} - \frac{1}{G_d} \equiv \Delta\left(\frac{1}{G_u}\right) = \frac{2}{5}\left(\frac{1}{G^r_{eff}}\right) + \frac{3}{5} \times 0$$

where

$$\Delta\left(\frac{1}{G^r_{eff}}\right) = 4\eta_2\rho(1 - K_f/K_m)/3$$

So

$$\Delta\left(\frac{1}{G_u}\right) = \frac{4}{15}\Delta\left(\frac{1}{K_u}\right).$$

CONCLUSIONS

• The Sayers and Kachanov (1991) approach has some powerful advantages for both forward and inverse modeling in fractured systems, especially when used in conjunction with measured Thomsen parameters.

• For a given material (*e.g.*, quartz) and a given crack shape (*e.g.*, penny-shaped), η_1 and η_2 can be computed once and for all.

• Measured Thomsen parameters can then be inverted for crack density.

• Incorporating fluid dependence rigorously into this problem is easy using the Sayers-Kachanov parameters.

REFERENCES (1)

- J. G. Berryman, 1995, Mixture theories for rock properties: in *American Geophysical Union Handbook of Physical Constants*, edited by T. J. Ahrens, AGU, New York, pp. 205–228.
- J. G. Berryman, 2005, Bounds and self-consistent estimates for elastic constants of random polycrystals with hexagonal, trigonal, and tetragonal symmetries: *J. Mech. Phys. Solids* **53**(10), 2141–2173.
- J. G. Berryman, 2006, Seismic waves in rocks with fluids and fractures: **SEP-124**.

REFERENCES (2)

- V. Grechka, 2005, Penny-shaped fractures revisited: *Stud. Geophys. Geod.*, **49**, 365–381.
- V. Grechka and M. Kachanov, 2006, Effective elasticity of fractured rocks: *The Leading Edge*, **25**, 152–155.

REFERENCES (3)

• C. M. Sayers and M. Kachanov, 1991, A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics: *Int. J. Solids Struct.*, **27**, 671–680.

• R. W. Zimmerman, 1991, *Compressibility of Sandstones*, Elsevier, Amsterdam.

REFERENCES (4)

- R. Jones, 1958, In situ measurement of the dynamic properties of soil by vibration methods: *Geotechnique* **8**, 1–21.
- H. P. Bucker, J. A. Whitney, and D. L. Keir, 1964, Use of Stoneley waves to determne the shear velocity in ocean sediments: *J. Acoust. Soc. Am.* **36**, 1595–1596.

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