Poroelastic Fluid Effects on Shear for Rocks

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OUTLINE

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- Backus Averaging for Finely Layered Media
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With σ_{ij} being the ij component of stress, and

e_{kl} being the kl component of strain:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f & & & \\ b & a & f & & & \\ f & f & c & & & \\ & & & 2l & & \\ & & & & 2l & \\ & & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix},$$

where $a = b + 2m$ (e.g., Musgrave, 1970; Auld, 1973).

Indices i, j, k, l range from 1 to 3 in Cartesian coordinates.

Following Backus (1962), we suppose that the region of interest is composed of fine layers having isotropic elastic constants, $\lambda(z)$ and $\mu(z)$, being functions of depth z. Then, the average over an arbitrary stack of such layers can be computed using a layer averaging method. This involves a Legendre transform that I will not present here. We use the layer averaging operator symbolized, for example, by brackets

$$\langle \mu \rangle \equiv \frac{1}{D} \int_0^D \mu(z) dz.$$

where D is the depth of the stack of layers.

The elastic anisotropy coefficients are then related to the layer parameters by the following expressions:

$$c = \left\langle \frac{1}{\lambda + 2\mu} \right\rangle^{-1}, \qquad \text{and} \qquad l = \left\langle \frac{1}{\mu} \right\rangle^{-1},$$

$$f = c \left\langle \frac{\lambda}{\lambda + 2\mu} \right\rangle,$$

$$a = \frac{f^2}{c} + 4m - 4 \left\langle \frac{\mu^2}{\lambda + 2\mu} \right\rangle,$$

$$m = \langle \mu \rangle, \qquad \text{and} \qquad b = a - 2m.$$

Thomsen's Parameters for Weak Anisotropy

The Thomsen (1986) parameters ϵ, δ, and γ are related to these stiffness coefficients by

$$\epsilon \equiv \frac{a-c}{2c},$$

$$\delta \equiv \frac{(f+l)^2-(c-l)^2}{2c(c-l)},$$

$$\gamma \equiv \frac{m-l}{2l}.$$

We can immediately write down four singular vectors (or eigenvectors):

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

and their corresponding singular values (eigenvalues), are respectively: $2l$, $2l$, $2m$, and $a - b = 2m$.

All four correspond to shear modes of the system.

Uniaxial shear strain can be applied to this system in the form:

$$\begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Then, the associated energy per unit volume for such a vector (of unit magnitude) can be shown to be:

Uniaxial Shear Strain: Energy per Unit Volume

$$G_{eff} \equiv [a - m + c - 2f]/3.$$

For this and several other reasons I will not have time to discuss, G_{eff} acts like an effective shear modulus and it is the only one of the five shear moduli that ever contains information about pore fluids.

Useful Facts for Layered Media

$$c_{44} = l = \langle 1/\mu \rangle^{-1}, \qquad c_{66} = m = \langle \mu \rangle$$

$$G_{Voigt} = \tfrac{1}{5}\left(G_{eff} + 2l + 2m\right)$$

$$c_{44} = l \leq G_{eff} \leq m = c_{66}$$

$$\gamma = \tfrac{1}{2}\left(\langle \mu \rangle \langle 1/\mu \rangle - 1\right)$$

This parameter $\gamma = (c_{66}/c_{44} - 1)/2$ is one of the Thomsen parameters.

Definition and Use of $ratio_{\alpha B}$

It is not hard to show that $l \leq G_{eff} \leq m$, the precise value depending on αB. Therefore, define for utility

$$ratio_{\alpha B} \equiv \frac{m - G_{eff}}{m - l},$$

a quantity that ranges from 0 to 1. Then, the quantity

$$1 - \frac{ratio_{\alpha B}}{ratio_0}$$

also ranges at most from 0 to 1 and measures the effect of pore fluids on shear (for $B > 0$, $\alpha > 0$).

The general behavior of seismic waves in anisotropic media is well known, and the equations are derived in many places including Berryman (1979) and Thomsen (1986). The results are

$$\rho\omega_{\pm}^2 = \frac{1}{2}\Bigg[(a+l)k_1^2 + (c+l)k_3^2$$

$$\pm\sqrt{[(a-l)k_1^2 - (c-l)k_3^2]^2 + 4(f+l)^2k_1^2k_3^2}\Bigg]$$

for compressional (+) and vertically polarized shear (−) waves and

Dispersion Relations (continued)

$$\rho\omega_s^2 = mk_1^2 + lk_3^2,$$

for horizontally polarized shear waves, where ρ is the overall density, ω is the angular frequency, k_1 and k_3 are the horizontal and vertical wavenumbers (respectively), and the velocities are given simply by $v = \omega/k$ with $k = \sqrt{k_1^2 + k_3^2}$.

The SH wave depends only on elastic parameters l and m, which are not dependent in any way on layer λ and, therefore, play no role in the poroelastic analysis. Thus, we can safely ignore SH except when we want to check for shear wave splitting (birefringence) – in which case the SH results will be useful for the comparisons.

Dispersion Relations Simplified

$$\rho\omega_+^2 \equiv ak_1^2 + ck_3^2 - \Delta,$$

and

$$\rho\omega_-^2 \equiv lk^2 + \Delta,$$

with Δ determined approximately by

$$\Delta \simeq \frac{[(a-l)(c-l)-(f+l)^2]}{(a-l)/k_3^2+(c-l)/k_1^2}.$$

Recall that

$$(a - l)(c - l) - (f + l)^2 = 2c(c - l)(\epsilon - \delta).$$

We can also rewrite the first elasticity factor in the denominator as $a - l = (c - l)[1 + 2c\epsilon/(c - l)]$. Combining these results in the limit of $k_1^2 \to 0$ (for relatively small horizontal offset), we find that

$$\rho\omega_+^2 \simeq ck^2 + 2c\delta k_1^2,$$

and

$$\rho\omega_-^2 \simeq lk^2 + 2c(\epsilon - \delta)k_1^2,$$

since, in this limit, we have $\Delta \simeq 2c(\epsilon - \delta)k_1^2$.

Improved approximations to any desired order can be obtained with only a little more effort by keeping more terms in the expansion.

CONCLUSIONS

- Of the five shear moduli of a VTI system, only G_{eff} as defined here can ever contain information about pore fluids.

- Pore fluids have their biggest effects at $\theta = 45°$, on both quasi-P and quasi-SV waves in layered VTI media.

- The stiffening effects of pore fluids can be substantial (i.e., 20% of the difference between anisotropic shear moduli l and m).