Seismic Data Regularization

SEP-107

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Seismic Data Descriptions

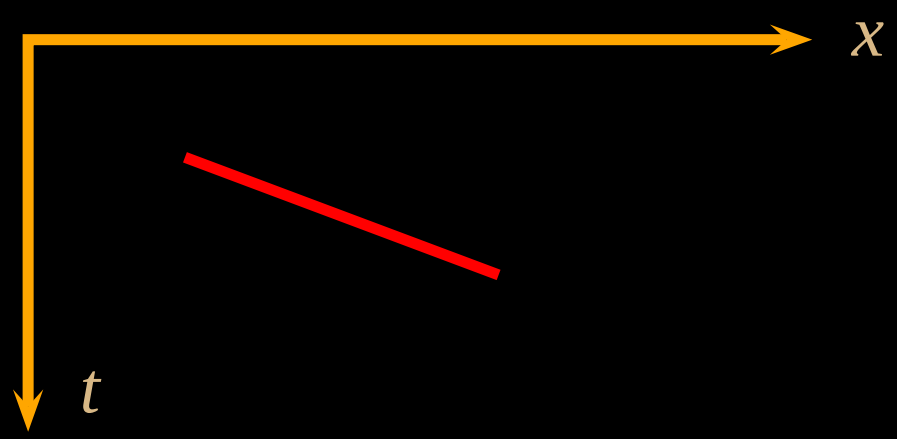
- Local plane waves
  - plane-wave destruction filters
- Differential offset and shot continuation
  - offset and shot continuation filters

Local Plane-wave Description

$$\frac{\partial P}{\partial x} + s \frac{\partial P}{\partial t} = 0 \qquad (1)$$

Explicit solution (for constant slope s):

$$P(t,x) = f(t - sx) \qquad (2)$$



Plane-wave Destruction in F-X

$$\frac{d\hat{P}}{dx} + i\omega s\,\hat{P} = 0 \qquad (3)$$

Explicit solution (for constant slope $s(x)$):

$$\hat{P}(x+\Delta x) = \hat{P}(x)\,e^{i\omega s(x)\,\Delta x} \qquad (4)$$

Linear phase-shift operator.

Equivalent to F-X prediction-error filtering.

Plane-wave Destruction in T -X

$$e^{i\omega s} = Z^s \approx \frac{B(Z)}{B(1/Z)} \tag{5}$$

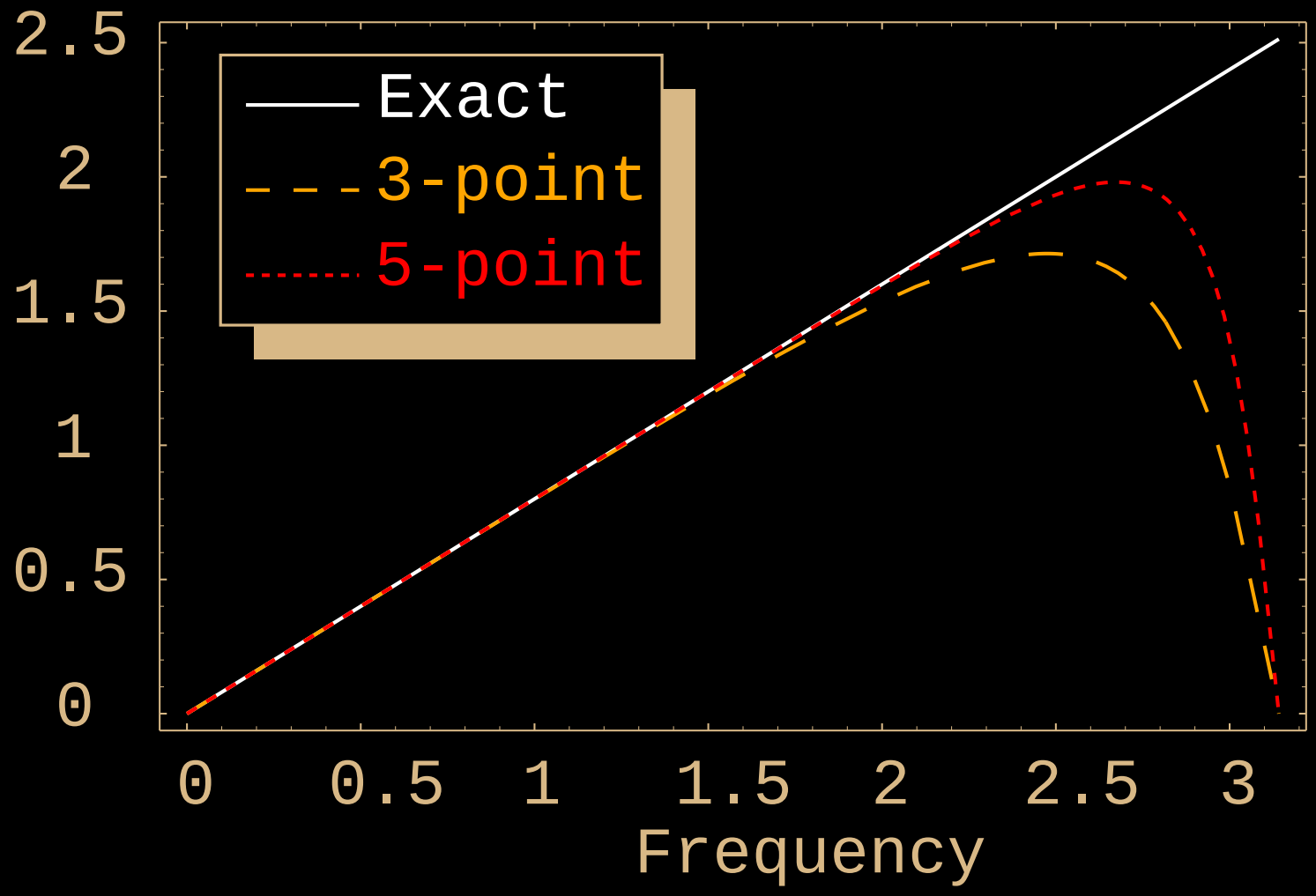
All-pass filter, implicit finite differences.

$$\hat{P}_{x+1}(Z) = \hat{P}_x(Z)\frac{B(Z)}{B(1/Z)} \tag{6}$$

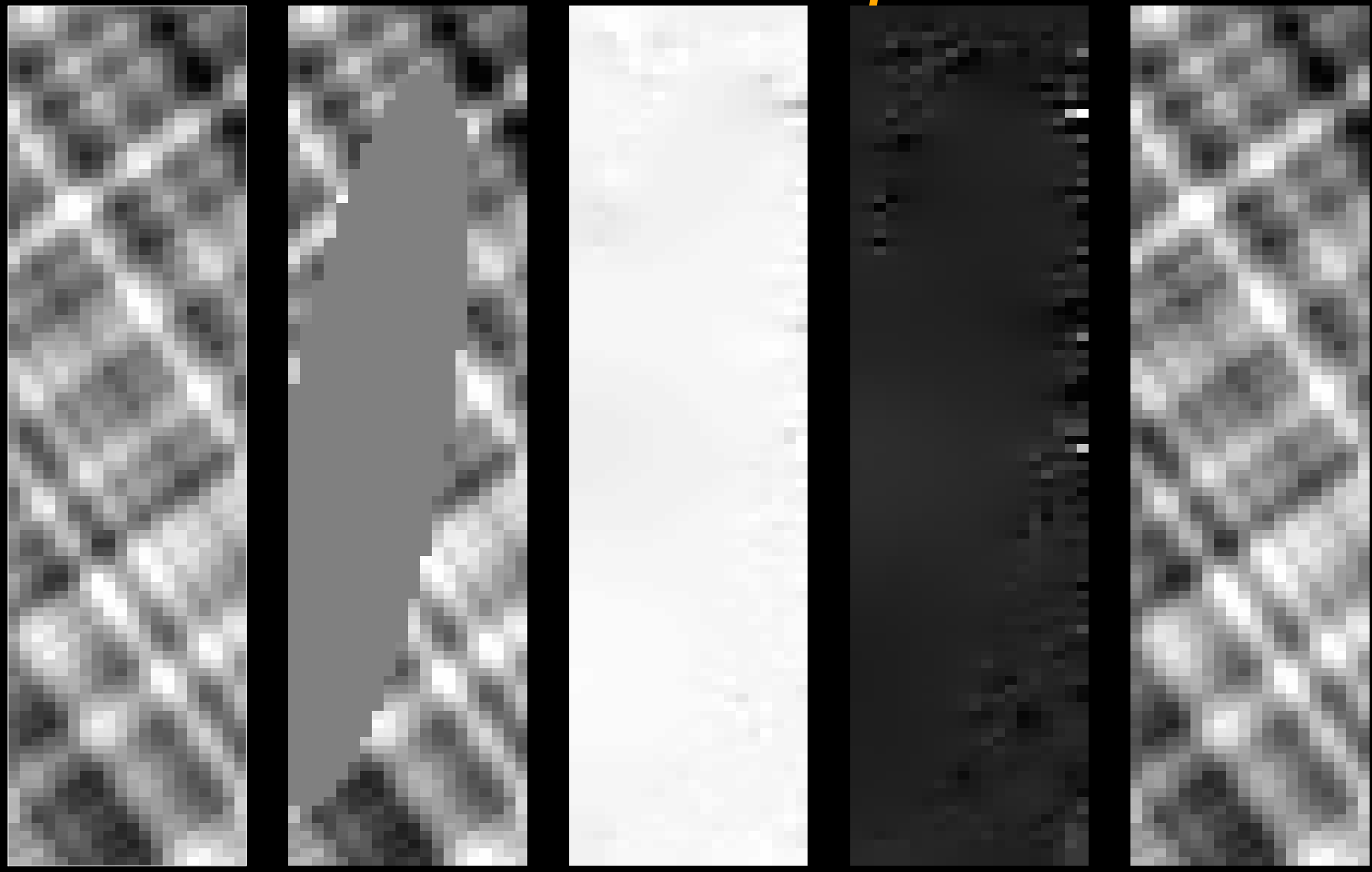
Assured preservation of energy.

Analogous to finite-difference migration.

Phase Approximation



Local Plane-Wave Interpolation



Slope Estimation

Plane-wave destruction:

$$\mathbf{C}(\mathbf{s})\,\mathbf{d} \approx 0 \tag{7}$$

Linearization:

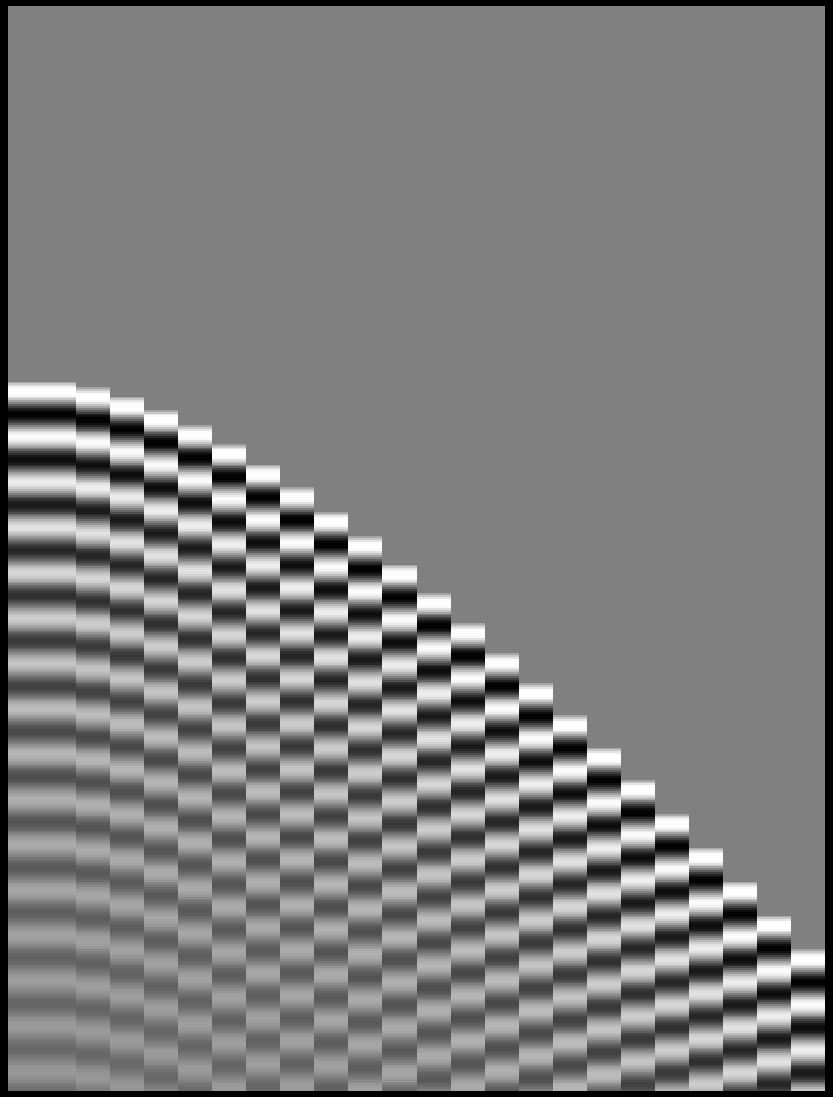
$$\mathbf{C}'(\mathbf{s}_0)\,\Delta\mathbf{s}\,\mathbf{d} + \mathbf{C}(\mathbf{s}_0)\,\mathbf{d} \approx 0 \tag{8}$$

Regularization:

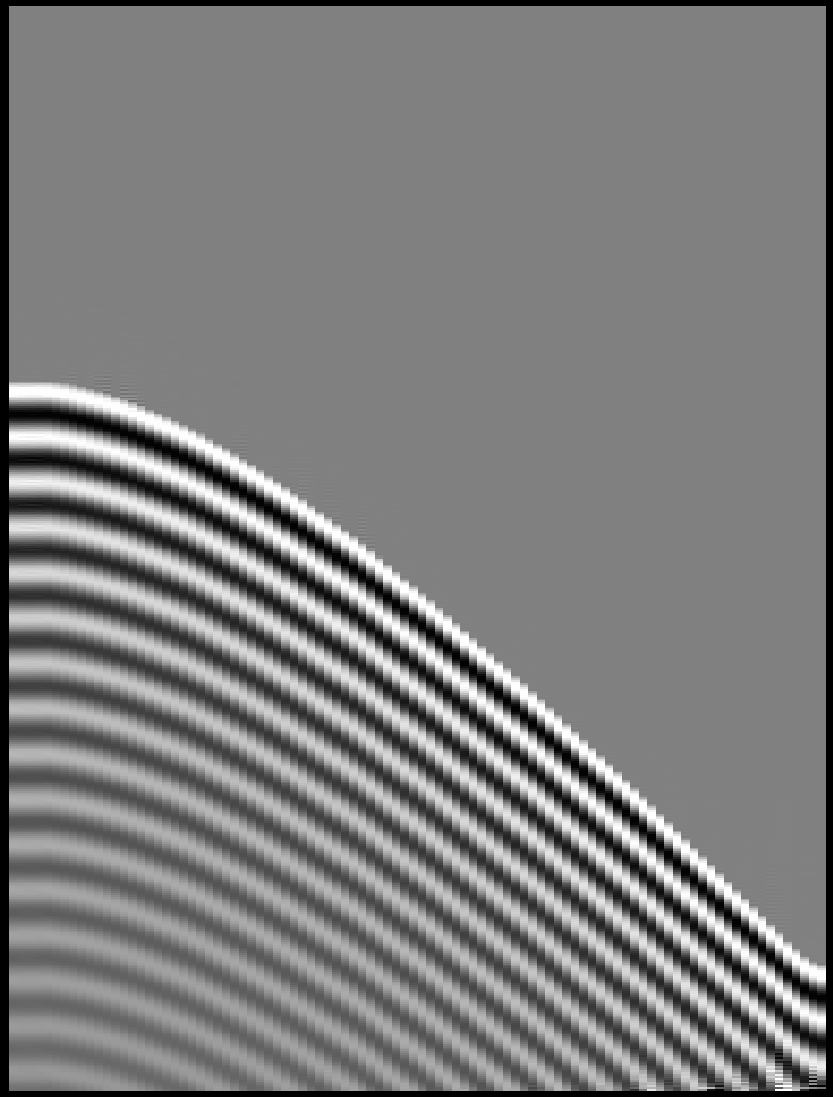
$$\epsilon\mathbf{D}\,\Delta\mathbf{s} \approx 0 \tag{9}$$

Linearized non-linear iteration.

Interpolation Beyond Aliasing

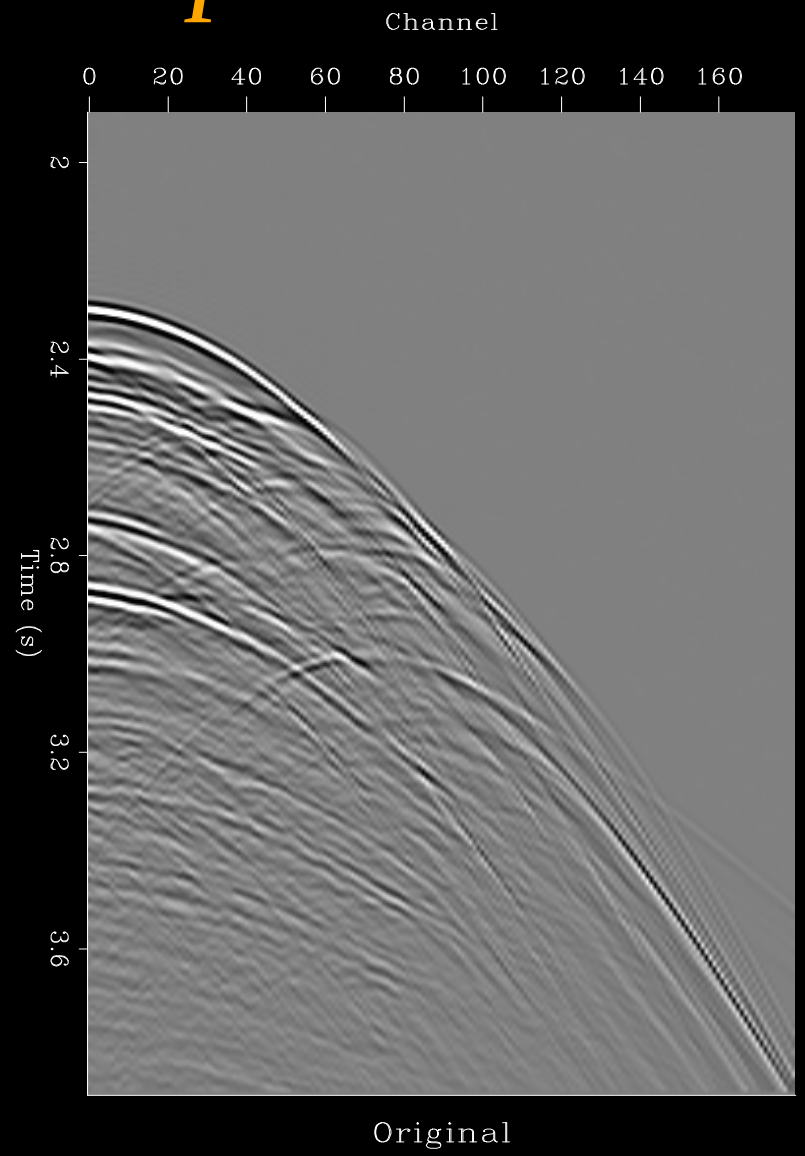


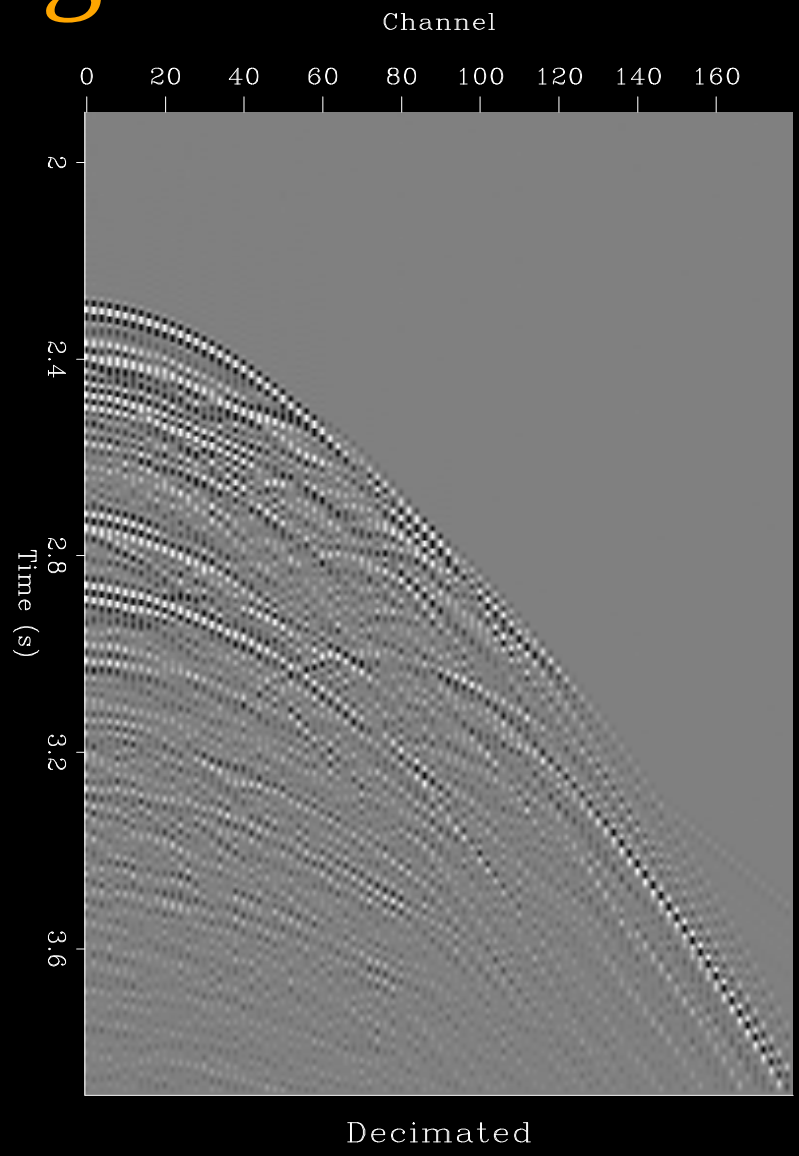
Input



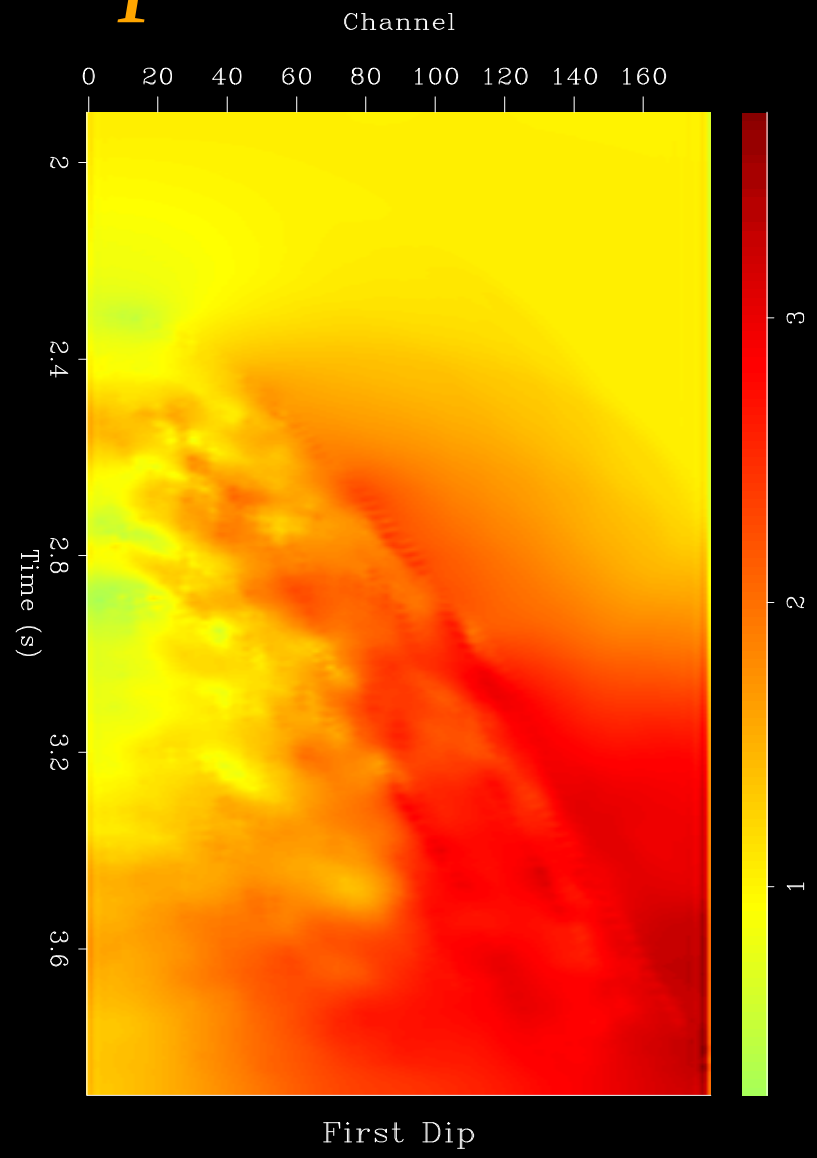
Output

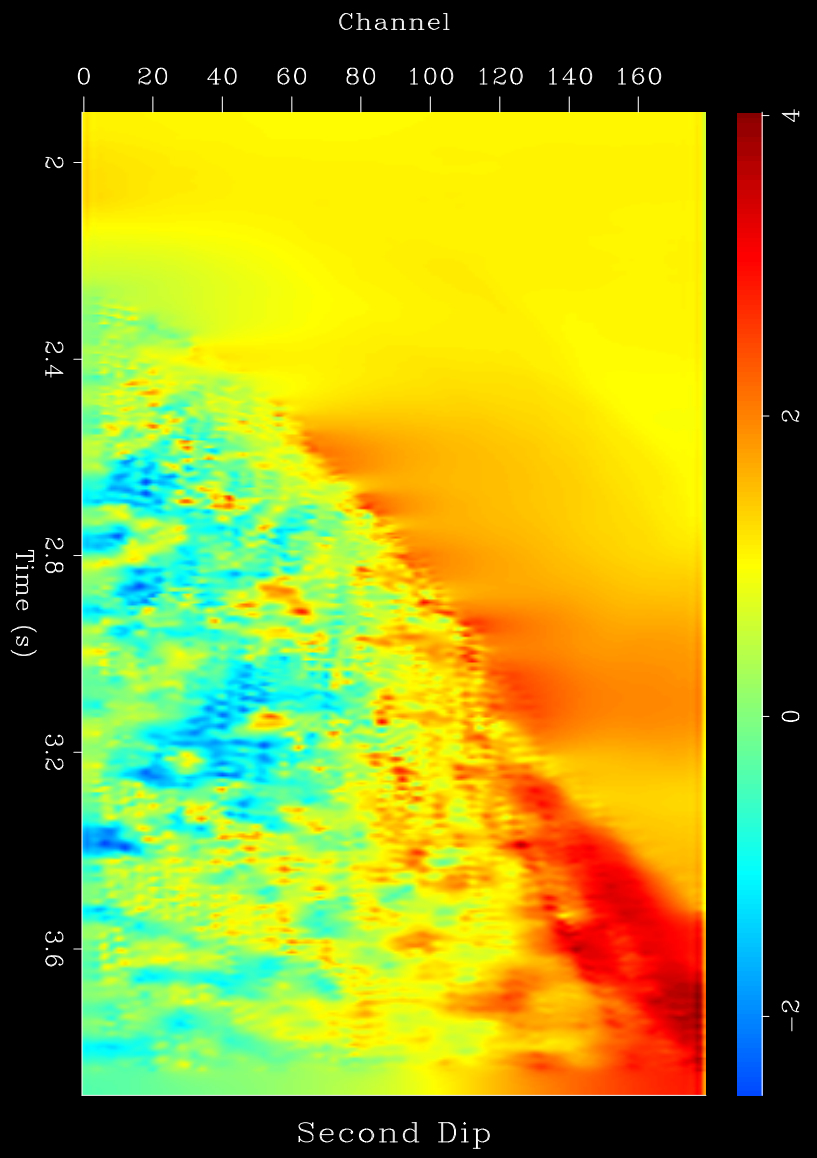
Interpolation Challenge



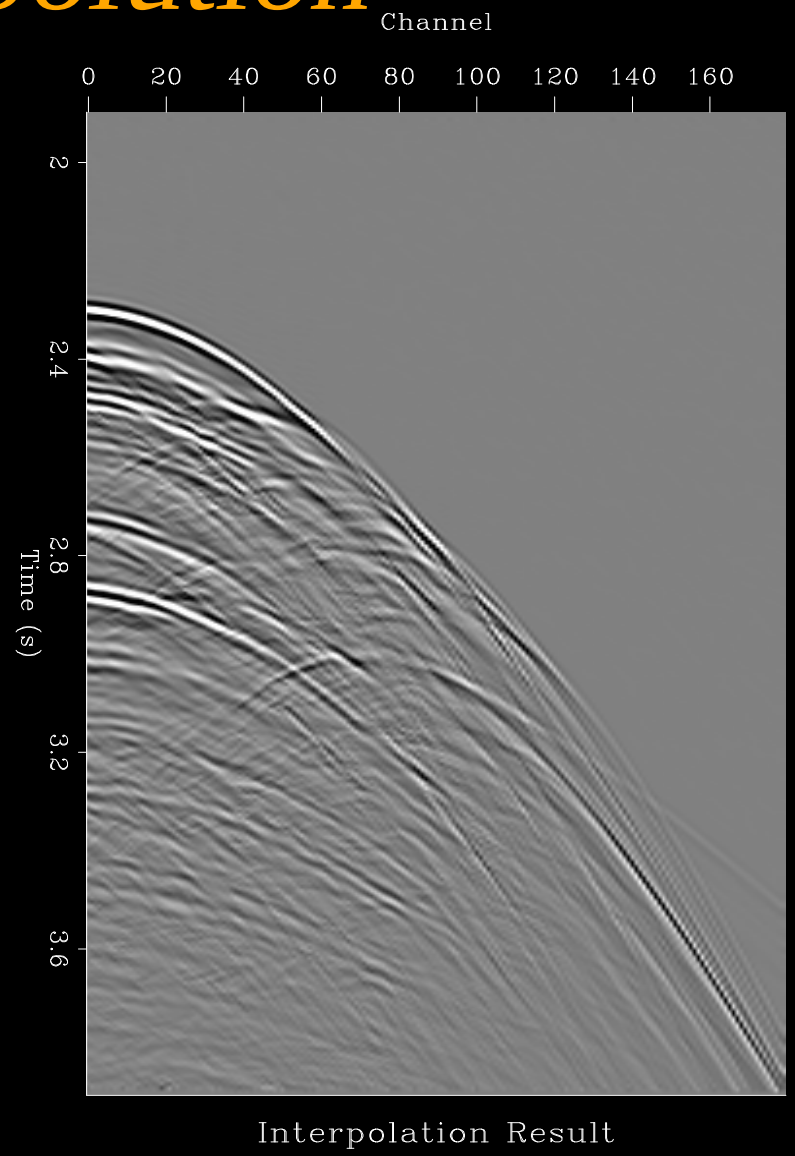


Slope Estimate

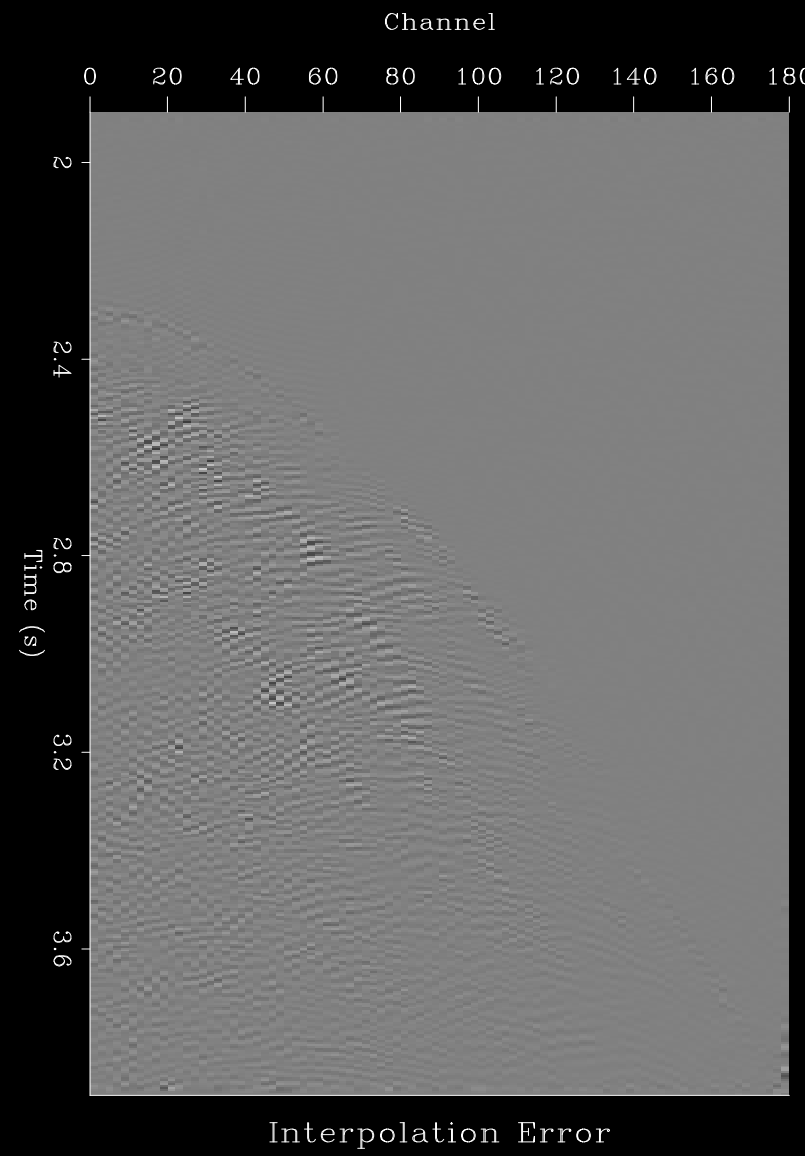




Solution



Interpolation Result



Interpolation Error

Data Interpolation Algorithm

- Estimate all present slopes.
- Neglect aliased slopes.
- Interpolate missing data using the plane-wave destruction filter.
- 3-D: pair of filters in T-X and T-Y planes.

Plane-Wave Destruction versus T-X Prediction-Error Filtering

Similarities:

- predict local plane waves in T-X
- applied in a similar way

Differences:

- PWD: deterministic; PEF: statistical
- PWD: estimate slope; PEF: filter coefficients
  - PWD: non-linear; PEF: linear
  - PWD: interpretable; PEF: non-intuitive

Differential Offset Continuation

$$h\left(\frac{\partial^2 P}{\partial y^2}-\frac{\partial^2 P}{\partial h^2}\right)=t_n\,\frac{\partial^2 P}{\partial t_n\,\partial h} \qquad (10)$$

- Describes reflection seismic data.
- Incorporates NMO and DMO effects
- h: half-offset
- y: midpoint
- t_n: time after NMO

Differential Shot Continuation

$$h\left(2\frac{\partial^2 \tilde{P}}{\partial s \partial h}-\frac{\partial^2 \tilde{P}}{\partial h^2}\right)=i\,\Omega\left(\frac{\partial \tilde{P}}{\partial h}-\frac{\partial \tilde{P}}{\partial s}\right) \qquad (11)$$

- h: half-offset
- s: shot position
- Ω: frequency after NMO and log-stretch

Shot Continuation Filters

Phase-shift operator:

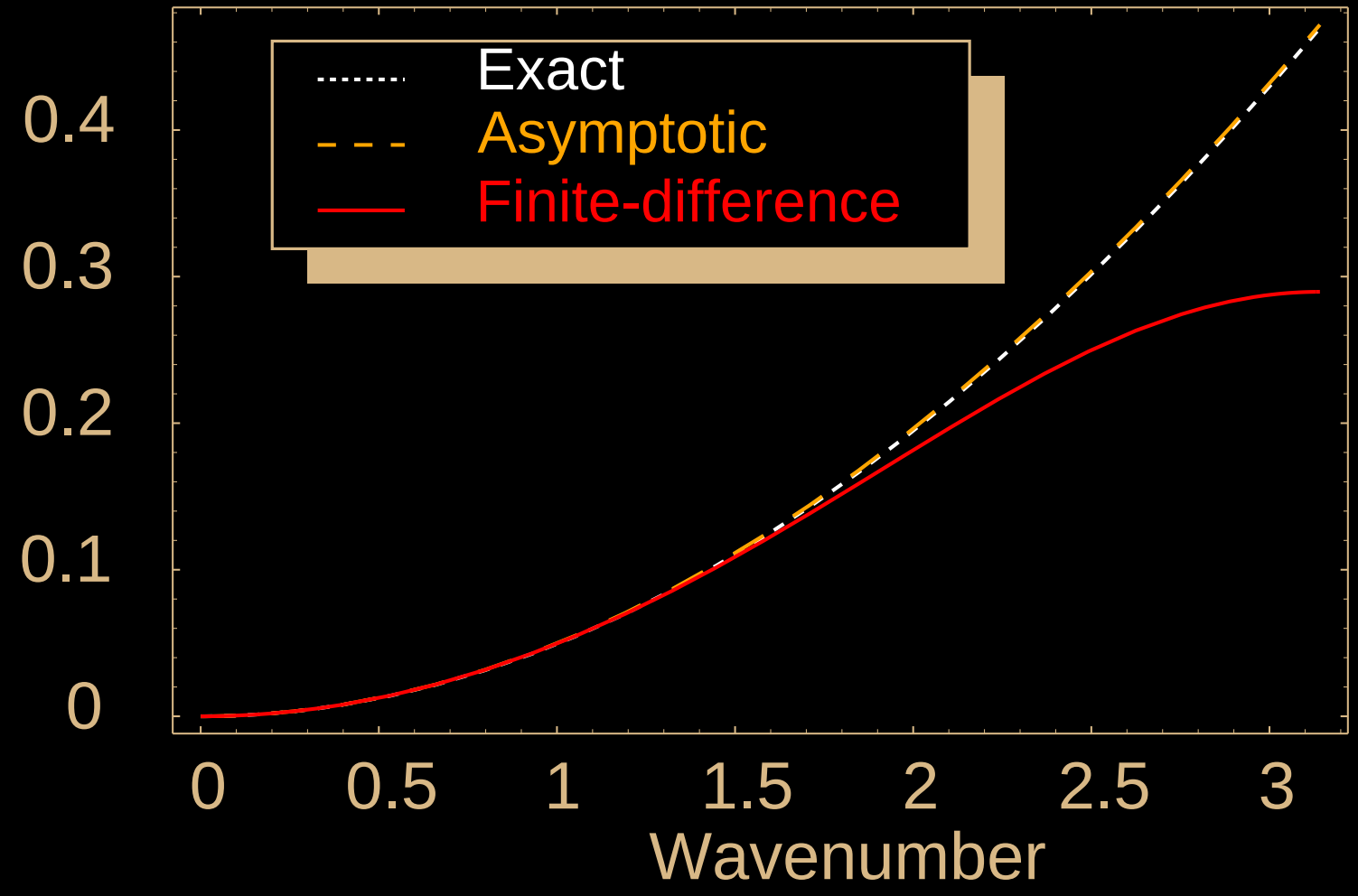
$$\hat{\hat{P}}(s+\Delta s) = \hat{\hat{P}}(s) \exp\left[i\, k_h\, \Delta s\, \frac{k_h\, h - \Omega}{2\, k_h\, h - \Omega} \right] \qquad (12)$$

Implicit finite differences:

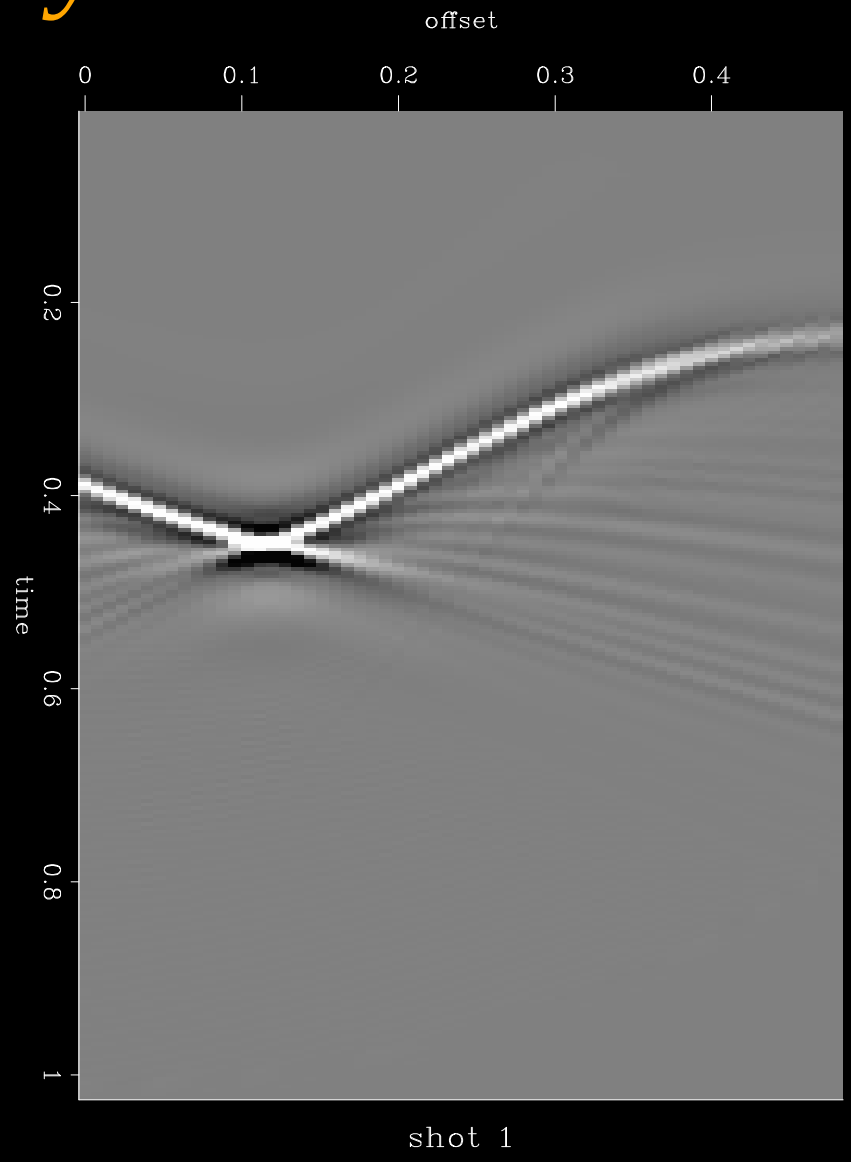
$$\hat{P}_{s+1}(Z_h) = \hat{P}_s(Z_h) \frac{S(Z_h)}{S(1/Z_h)}\,, \qquad (13)$$

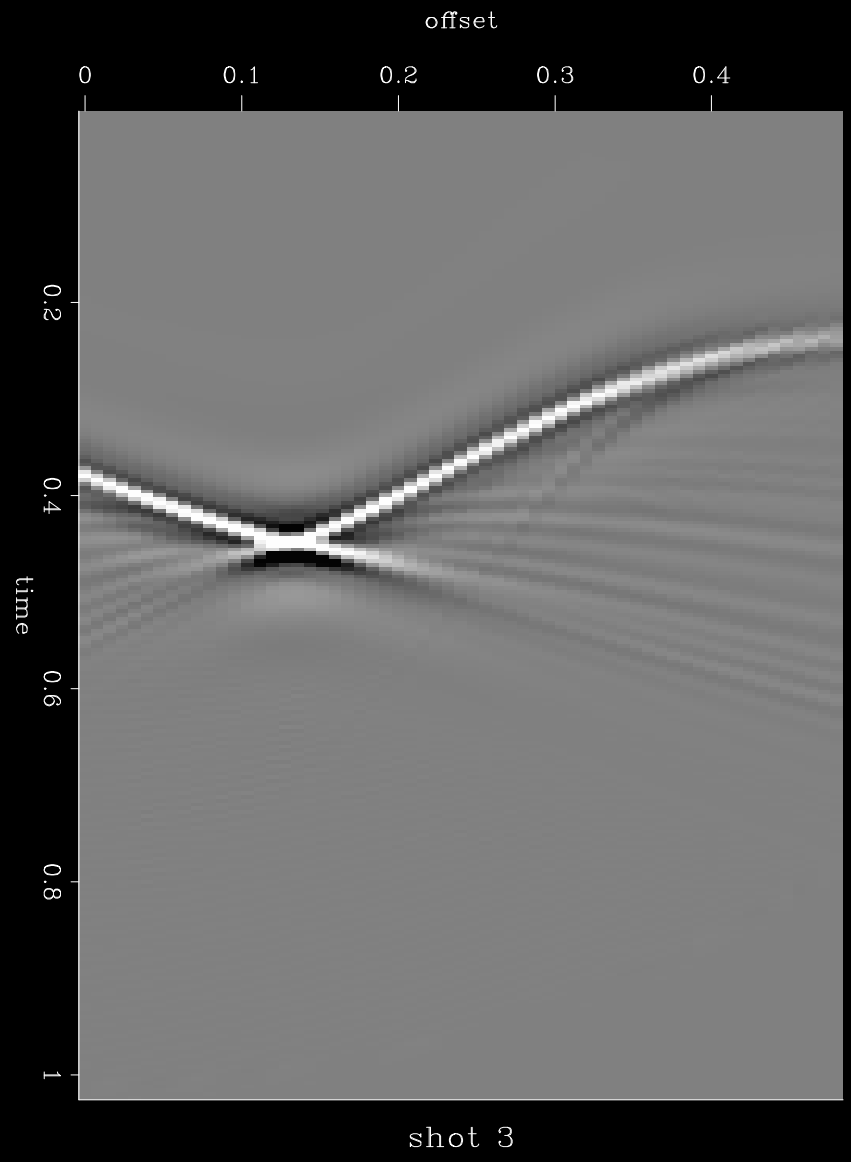
Assured preservation of energy.

Phase Approximation

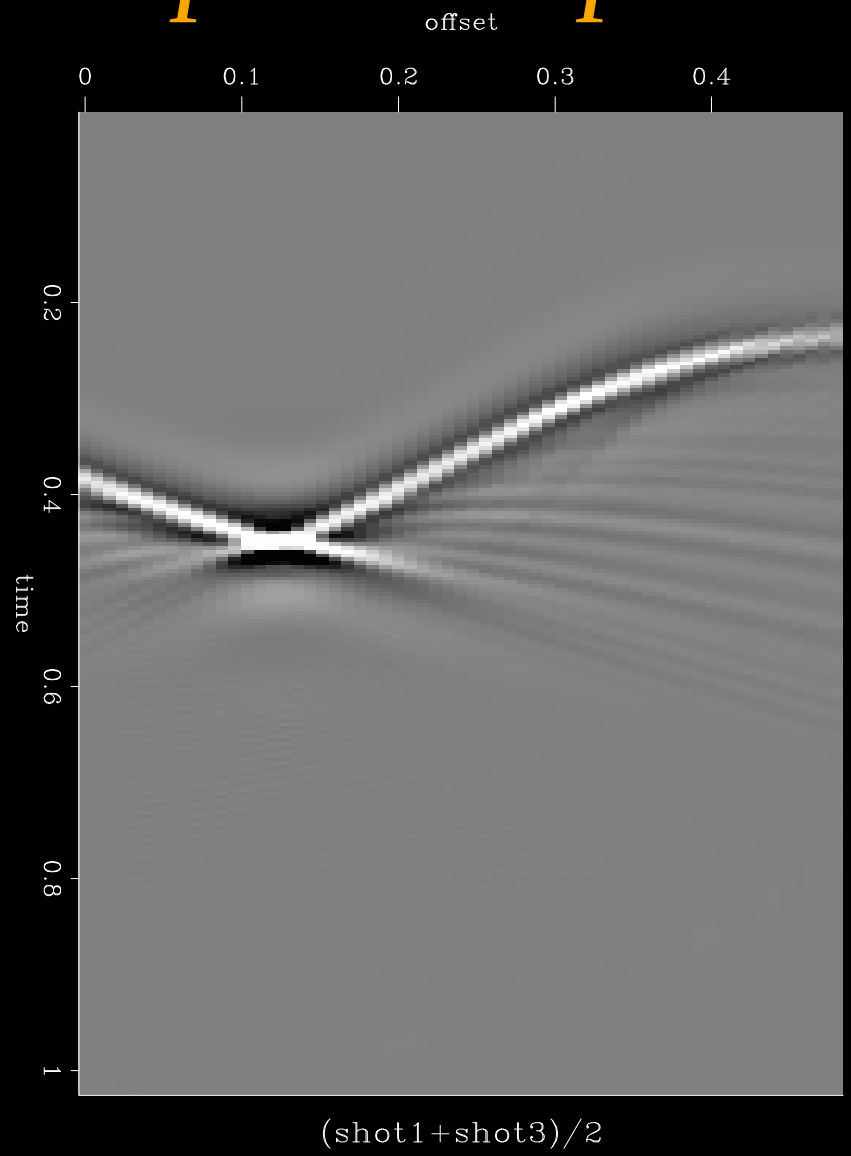


Synthetic Sinusoidal Reflector

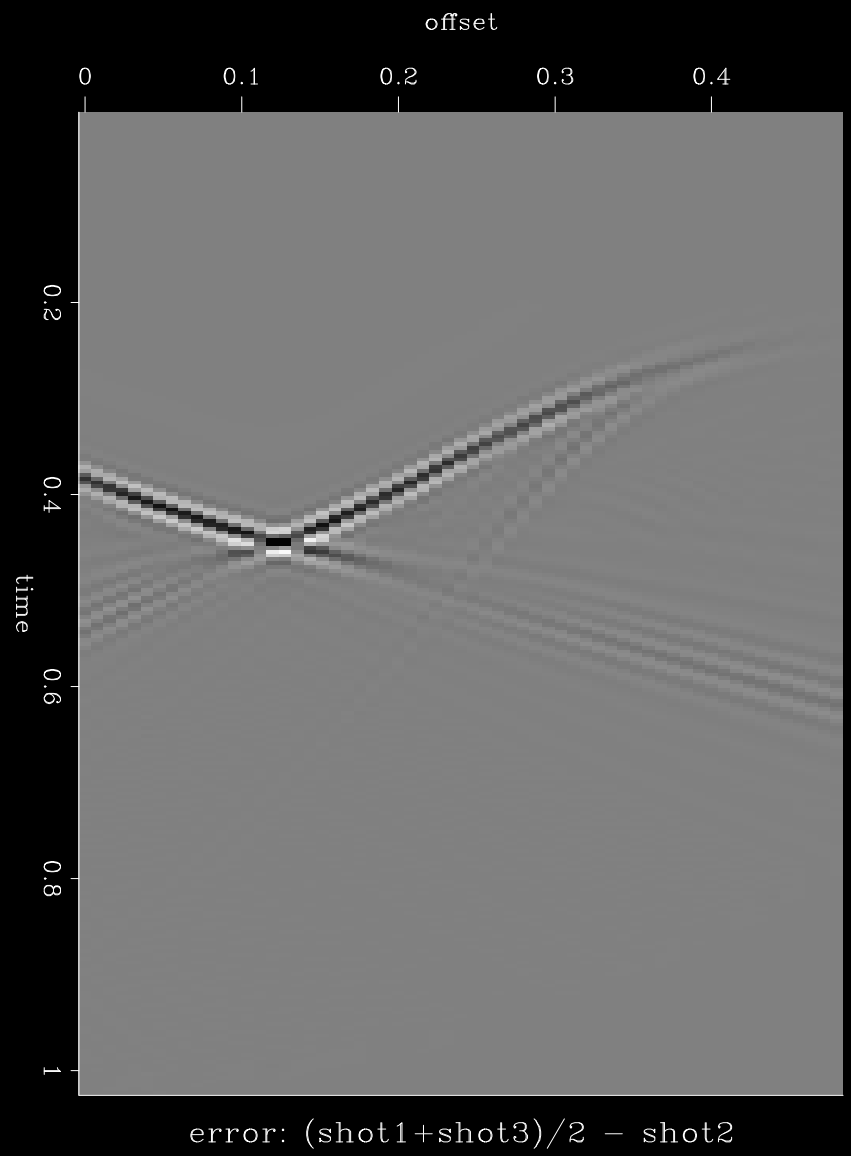




Simple Interpolation

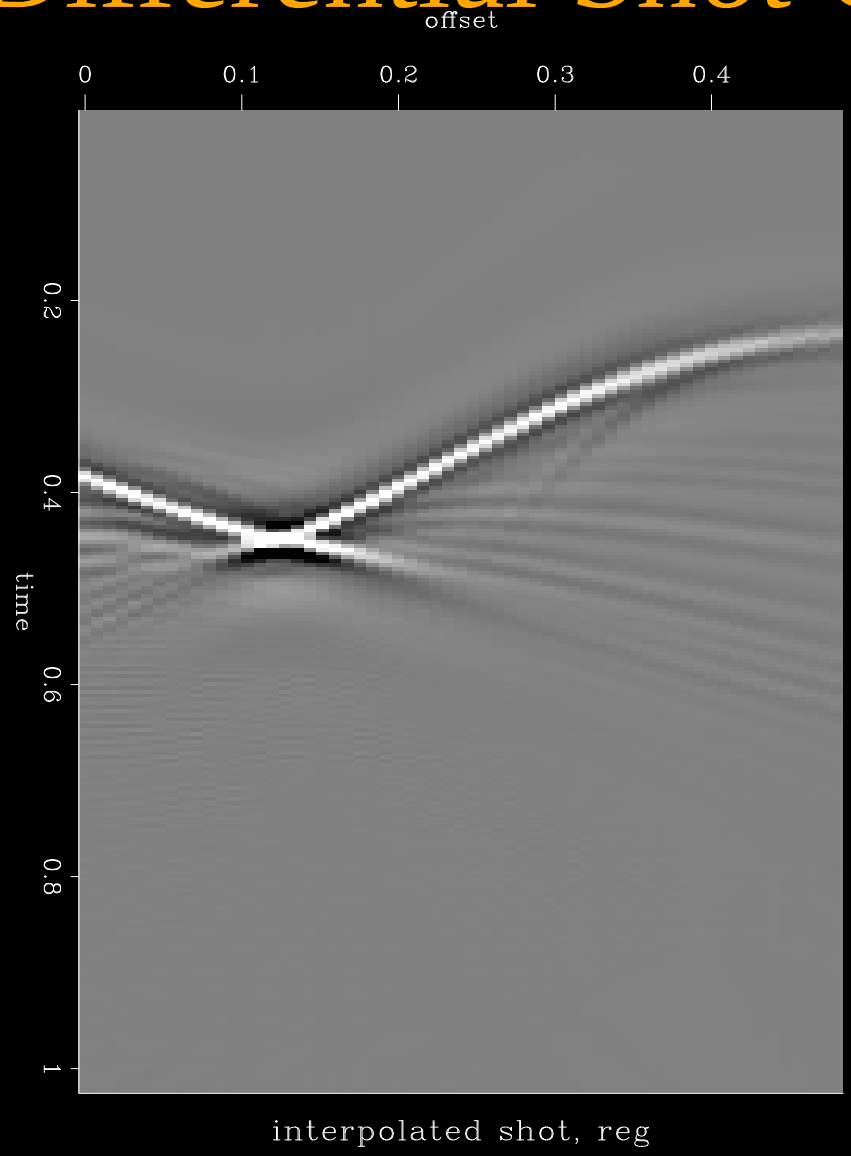


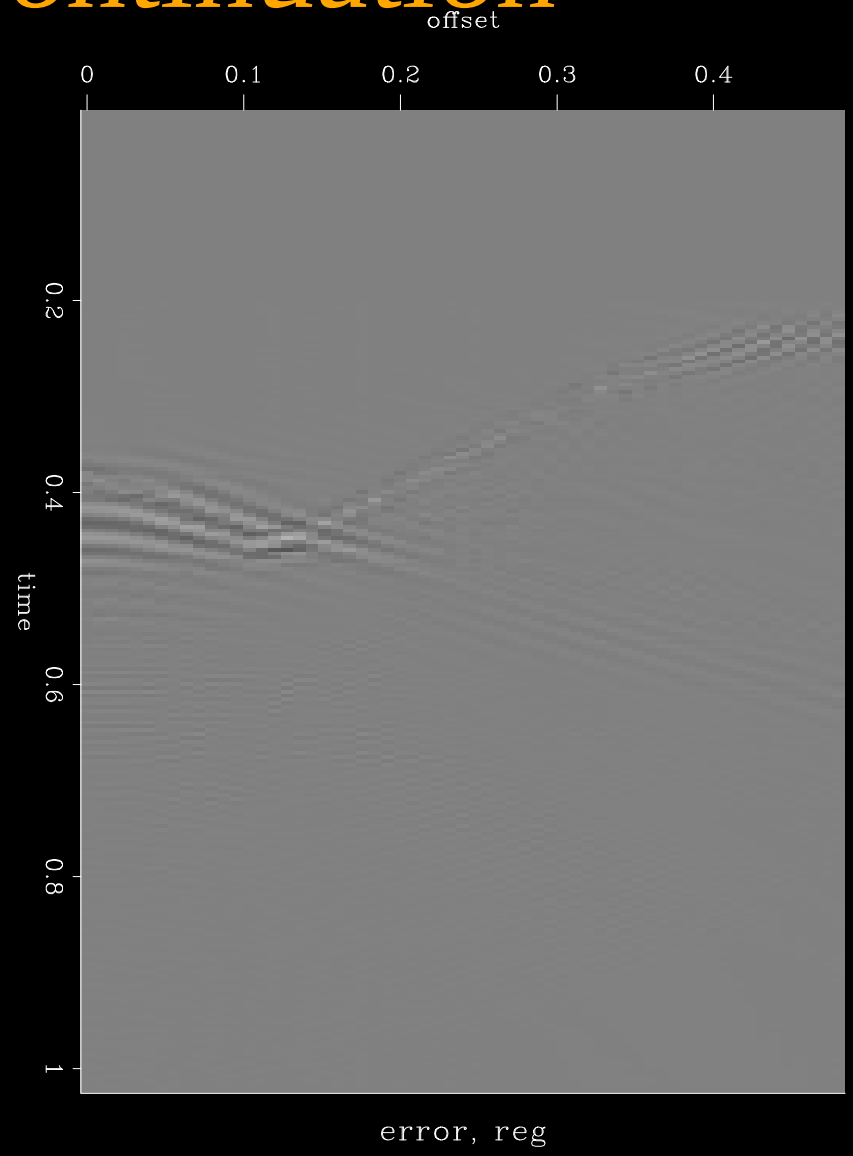
(shot1+shot3)/2



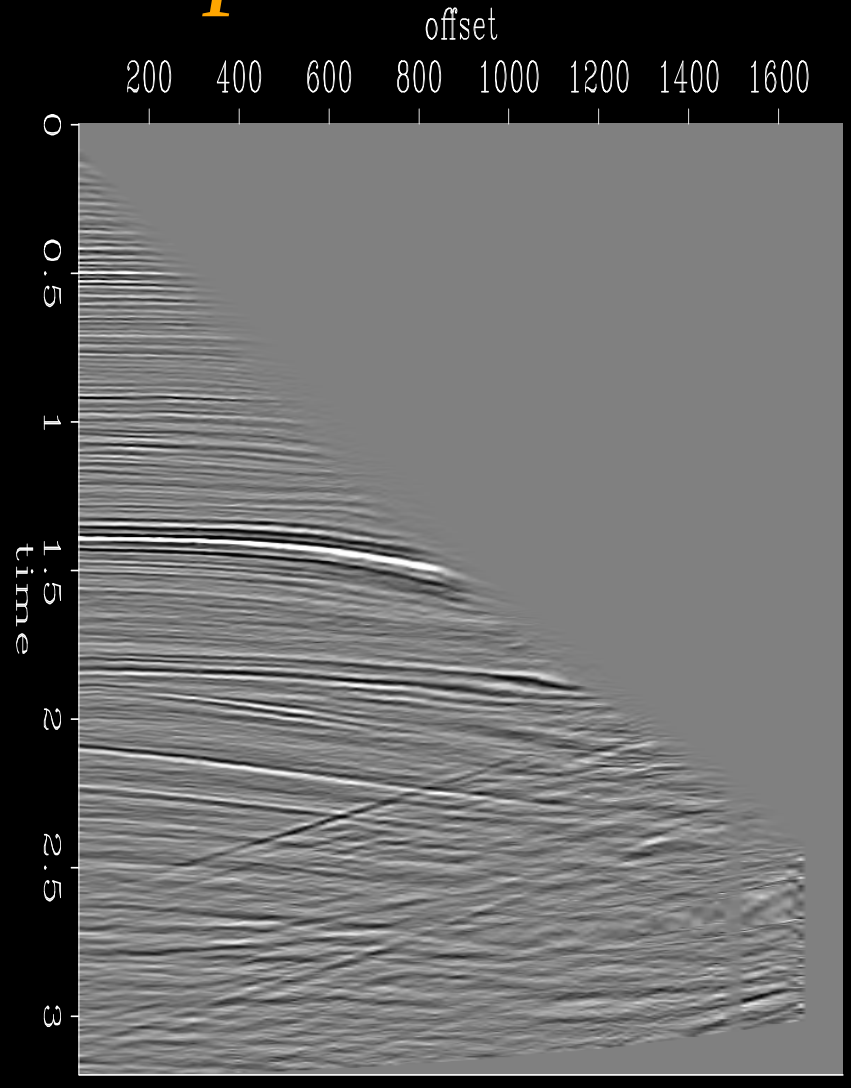
error: (shot1+shot3)/2 − shot2

Differential Shot Continuation

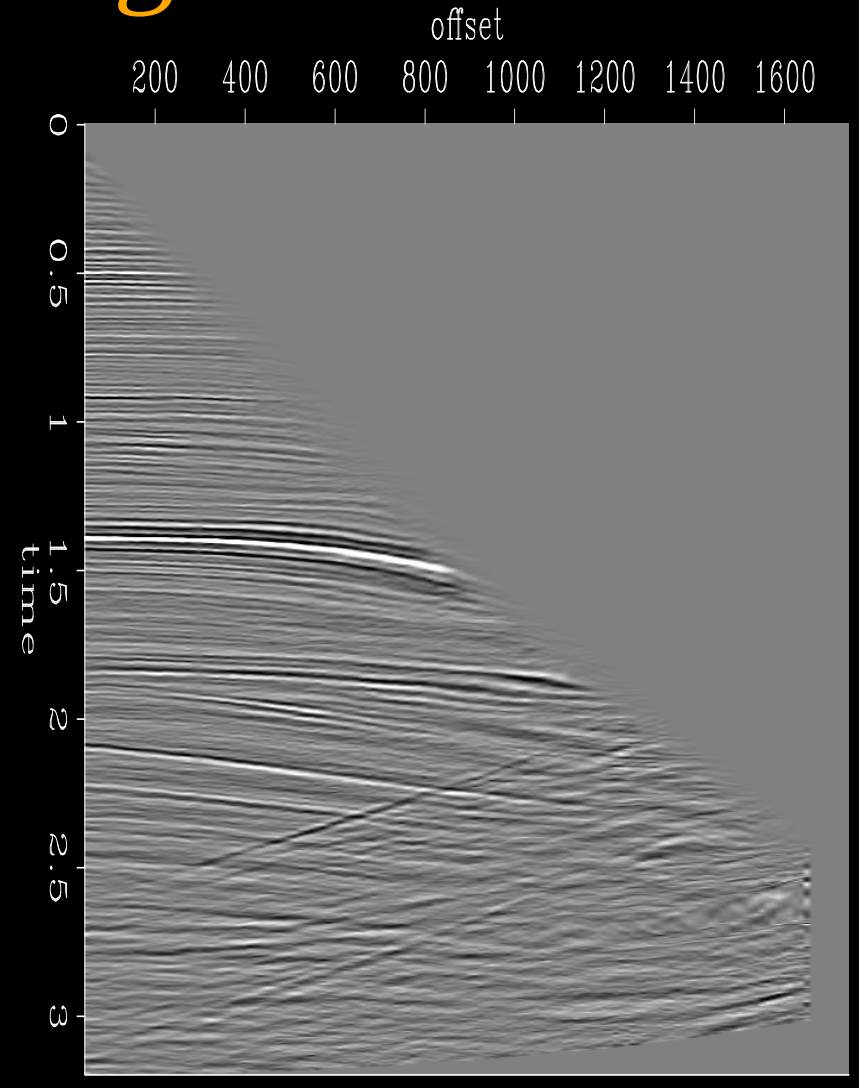




Interpolation Challenge

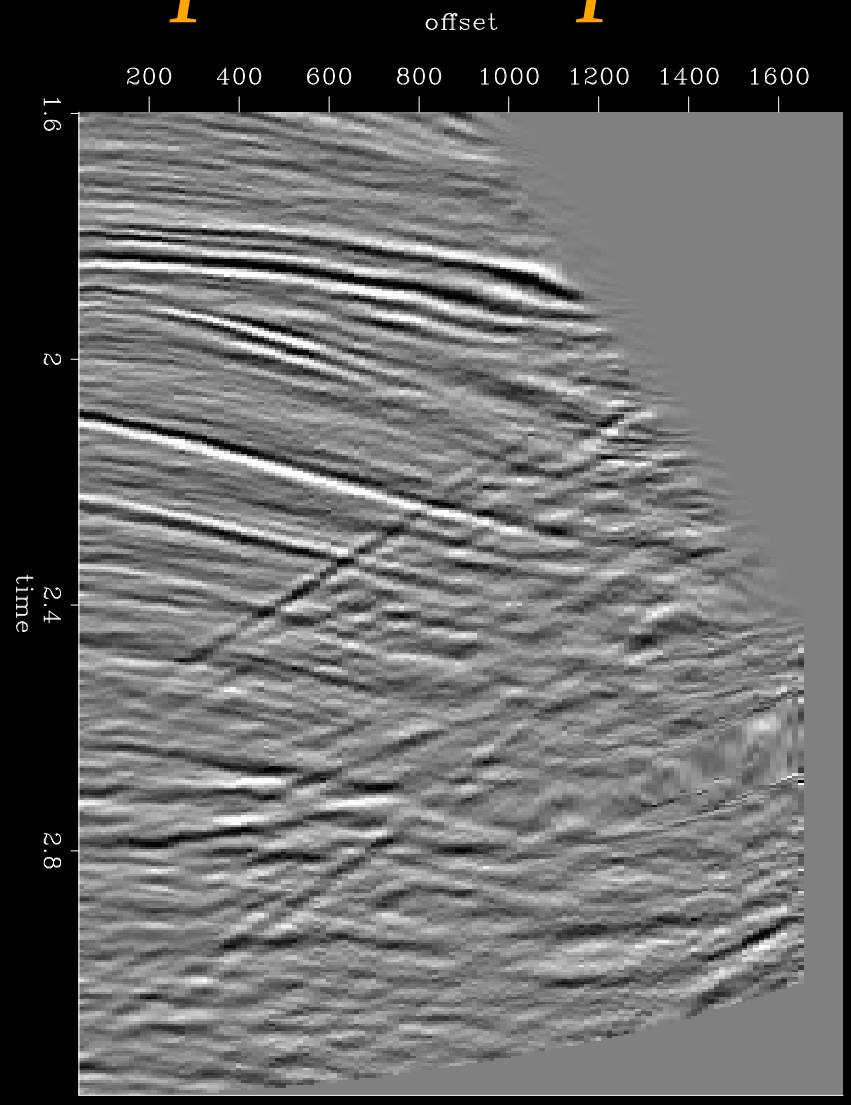


Shot 1

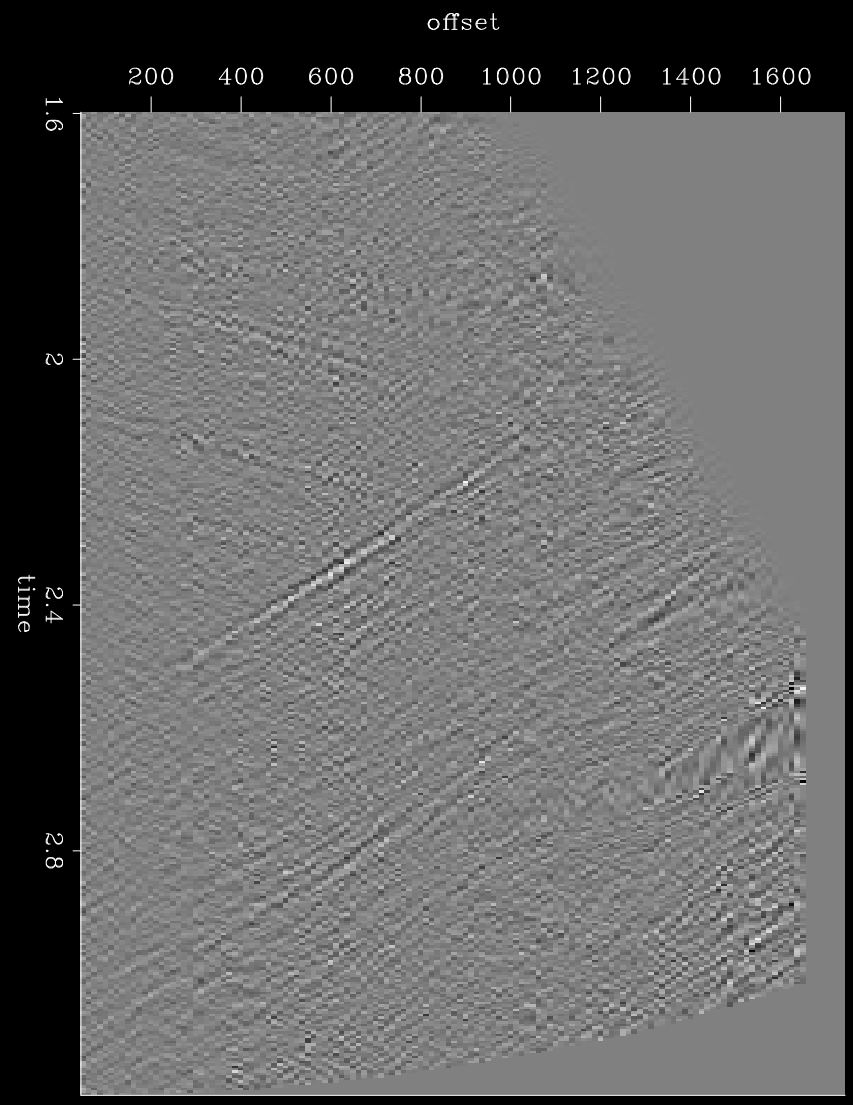


Shot 3

Simple Interpolation

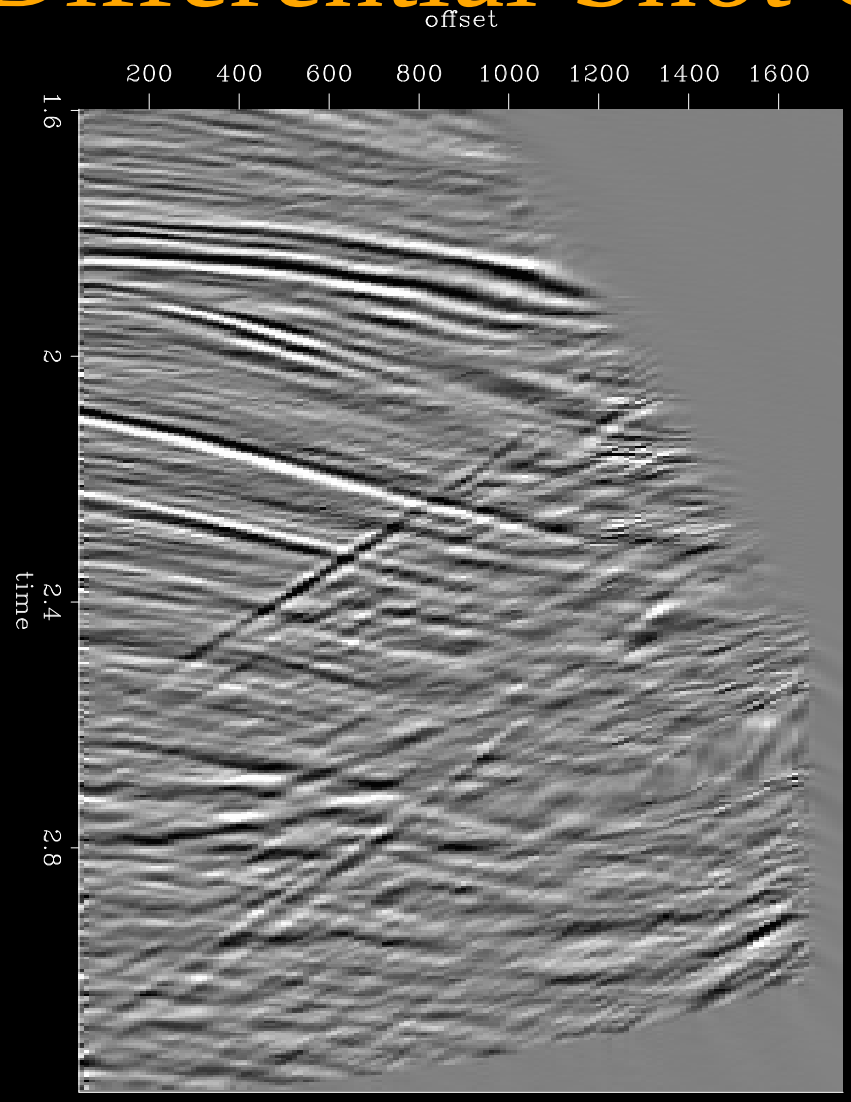


(shot1+shot3)/2

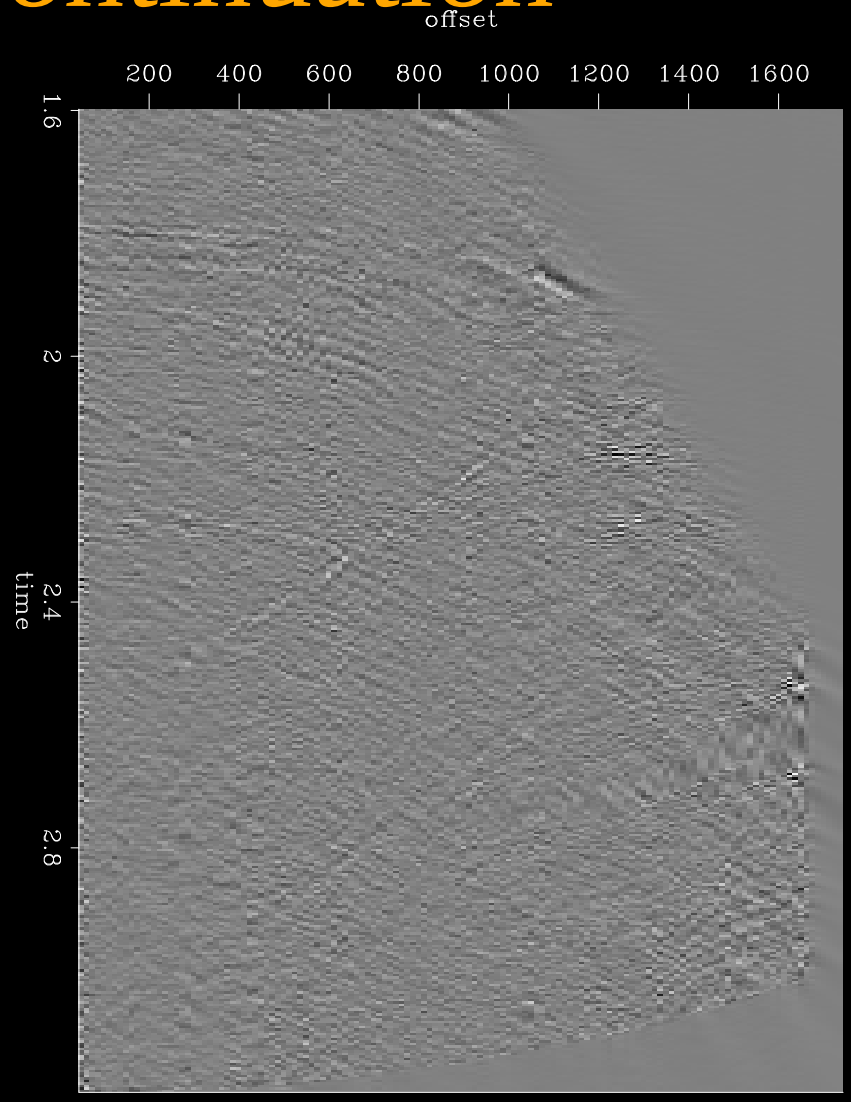


error: (shot1+shot3)/2 − shot2

Differential Shot Continuation



interpolated shot



error

Data Interpolation Algorithm

- NMO, logarithmic stretch, FFT in time.
- Interpolate missing data using the offset or shot continuation filters.
- 3-D: pair of filters in X and Y hyperplanes.
- Inverse FFT, inverse stretch, inverse NMO.

Plane-wave Destruction

- Based on local plane-wave model.
- Efficiently implemented with compact finite-difference filters.
- Preserves propagating energy.
- Constructs easily interpretable slope estimates.
- Applicable to a wide class of problems in geophysical data analysis.

Offset and Shot Continuation

- Based on differential offset continuation model for seismic reflection data.
- Efficiently implemented with compact finite-difference filters.
- Preserves propagating energy.
- Applicable to a wide class of problems in geophysical data analysis.