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# Horizon refinement by synthesis of seismic and well log data

*Morgan Brown*[1]

***keywords:*** *helix, spectral factorization, preconditioning, anisotropy*

**ABSTRACT**

Subsurface rock strata are often conceptualized in three dimensions as depth surfaces, or **horizons**. By picking the corresponding reflection event from 3-D seismic data, an approximate "seismic horizon" can be obtained, but in general the inconsistencies in the approximation are nontrivial. Picks from well log data give the depth to the horizon to high accuracy, but only at a few points. To achieve the final goal, which is a representation of the actual horizon more accurate than the seismic horizon, I present a least squares optimization scheme to optimally "warp" the seismic data to match the well log measurements. I then test the method on a small group of data from offshore West Africa, consisting of two picked seismic horizons data from about 15 well logs. The method shows promise in more far-reaching problems such as anisotropic parameter estimation and global velocity update.

## INTRODUCTION

### Motivation

The ultimate goal of nearly all geophysical methods is to obtain an accurate model of one or more measurable subsurface properties. Similarly, the goal of this paper is to create a model of a given geological stratum, or horizon, in depth as a function of the spatial coordinates, $x$ and $y$. This model should be well resolved both vertically and horizontally, densely map a wide area, and of course be obtained cheaply from a computational standpoint. Let us see if any of the current methods at our disposal fit the requirements of this problem.

[1]**email:** morgan@sep.stanford.edu

| **Imaging Method** | **Resolution** | | **Areal Coverage** | **Relative Cost** |
|---|---|---|---|---|
| | Vertical | Horizontal | | |
| *Ideal* | *Fine* | *Fine* | *Wide* | *Low* |
| Seismic | Coarse | Medium | Wide | Low |
| Crosswell | Medium | Medium | Medium | High |
| Well Logs | Fine | - | Narrow | High |

Singularly, none of these methods meets all the criteria of the "ideal" method. If we are to make the most accurate map possible, we must master the art of compromise and design an intelligent hybrid method. Seismic data, though poorly resolved vertically, does a decent job of delineating general horizontal trends over large areas. Well logs provide accurate vertical measurements, but little to no information on horizontal trends and are prohibitively expensive[1]. The hybrid method must account for these strengths and weaknesses.

From the oil industry's perspective, this problem has traditionally been of great importance, since the first source of information on any oilfield is usually seismic data. Seismic data is used to create the first velocity model, upon which many future operations in depth are strongly dependant (van Riel Paul and Mesday (1988), van der Made P. et al. (1990)). Therefore, any procedure which estimates relative seismic misfit should by definition estimate the corresponding error in the velocity model.

The problem of making an accurate depth map of subsurface geology is not unique to exploration geophysics. Hydrogeologists and petroleum engineers interested in reservoir monitoring obtain optimal models by minimizing the error between some unbiased linear estimate of the output model and some known data, a technique known as *kriging*. (Kitanidis, 1997)

Previous SEP approaches to solve the same problem have been expressed similarly as "missing data" problems: i.e., given the known values of the output model at some points, perform a least squares inversion, subject to other model constraints, to obtain the value of the output model over all space (Claerbout (1997b), Berlioux (1995)). My methodology does not deviate from this tradition, though I make use of some newly developed techniques and tools to obtain a solution.

Assume that in depth, the actual horizon is mathematically representable by a surface, which in general can be discontinuous, multivalued, or both. Compute an approximation to this horizon by picking from seismic data in time the reflection event corresponding to the actual horizon, and then convert it to depth. I iteratively solve a least squares optimization problem to calculate a model which is constrained to match the well log measurements at the well locations, but assumes the general shape of the picked seismic horizon elsewhere - an approach that honors each type of data in its regions of maximum statistical reliability.

[1]The cost of acquiring well log data is not high, but I include the high cost of drilling a well when I give well logs and crosswell techniques a "high" relative cost

The idea of optimized horizon refinement lends itself well to almost any recursive "layer-stripping" updating algorithm. As potential future areas of reseach, I discuss application of these ideas to the problems of anisotropic parameter estimation and global velocity update.

## DATA

We are given two irregular picked seismic horizons of 578 samples each, shown in the right panel of Figures 1 and 2 after nearest-neighbor binning into a 40x40 grid. To obtain these horizons, an interpreter picked "iso-velocity surfaces" in time from a 3-D cube of stacking velocities, then converted the surfaces to depth via the same velocities. Note that the bounds of the data do not cover the entire grid - a disturbing problem which we wish did not exist, but will rectify shortly.

20 wells reach the shallow horizon, while only 17 reach the deeper one. The left panel of Figures 1 and 2 display the depth to the actual subsurface horizons in which we're interested, as measured by the well logs. Due to a further subwindowing of the data on my part, only about 15 wells are included in the calculations.

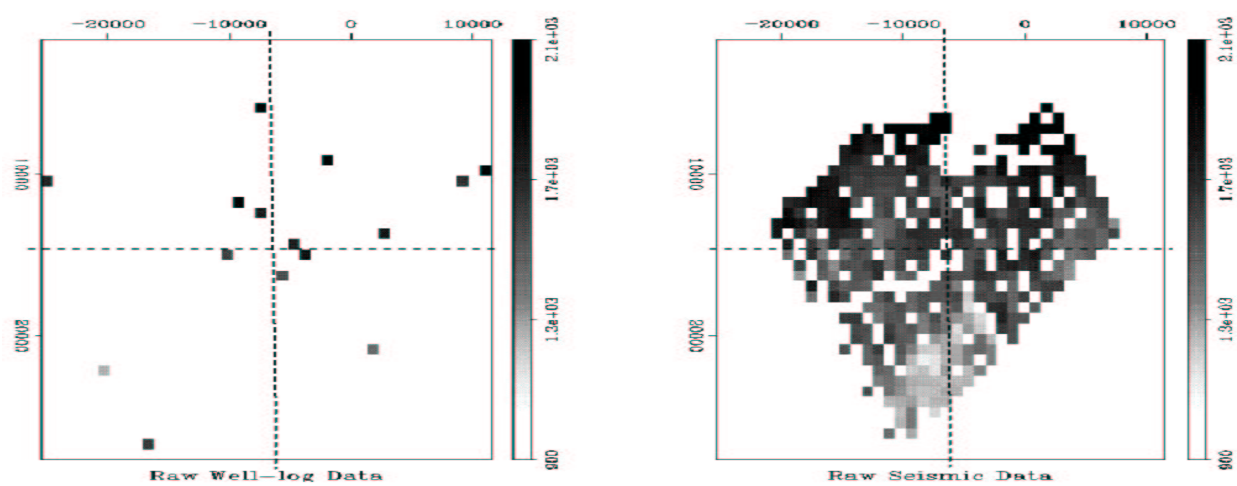


Figure 1: Well Log and Seismic data for shallow horizon, binned into a 40x40 grid. morgan1-well-seis-init-2000 [NR]

## ANALYSIS

For convenience, define the following quantities

1. $\mathbf{H}_i(x, y)$: Depth to actual horizon $i$ as a function of $x$ and $y$.
2. $\mathbf{H}'_i(x, y)$: Picked seismic horizon $i$, defined for all $x$ and $y$.

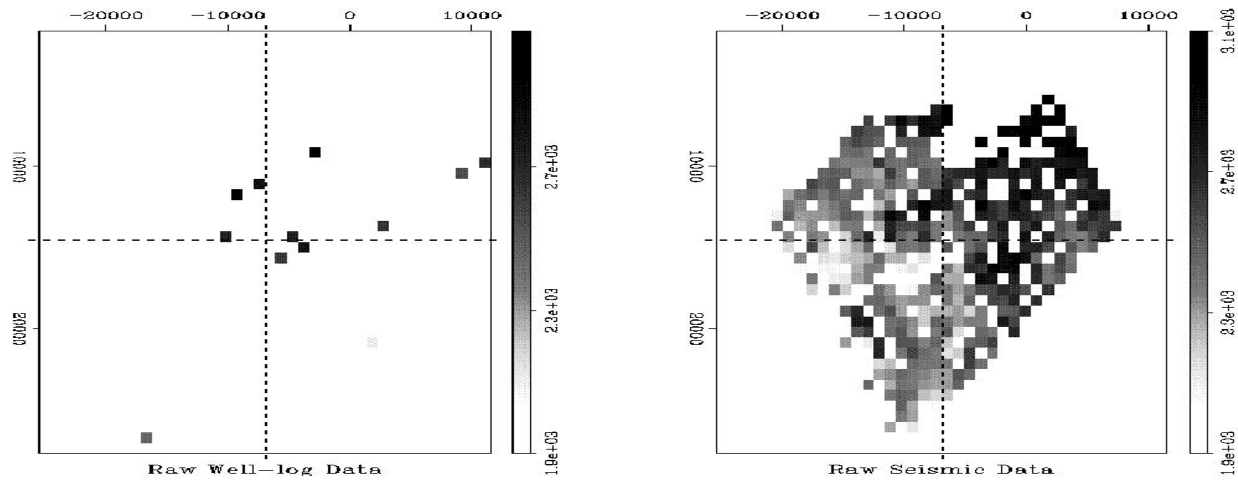


Figure 2: Well Log and Seismic data for deep horizon, binned into a 40x40 grid. $\boxed{\text{morgan1-well-seis-init-2300}}$ [NR]

3. $\tilde{\mathbf{H}}_i(x,y)$: Updated depth model of $i^{th}$ horizon.

The path to our final solution has two major steps.

1. In-Fill and extrapolation of irregular picked seismic horizon
2. Warping $\mathbf{H}_i'(x,y)$ to match the well logs

### 1: In-Fill and extrapolation of irregular picked seismic horizon

The irregular picked seismic horizons of Figures 1 and 2 are not defined at many $x$ and $y$ locations. So to create $\mathbf{H}_i'(x,y)$, we must fill all the interior "holes" in the irregular picked seismic horizons and also extrapolate them to the edges of the grid.

A human attempting to fill and extrapolate the data by hand would first discern, then manually extend, its dominant trends into the empty regions. Unfortunately, the human approach is a "non-linear" one; tough to reproduce and even tougher to encode into a computer algorithm. However, by computing a 2-D Prediction Error Filter (PEF) from the irregular picked seismic horizon, we encapsulate the spatial spectrum of the known data, and thus systematically extrapolate by imposing this spectrum on the output model. (Crawley, 1995) discusses a closely related example using sparse side-scan sonar bathymetry data.

First we must contend with a detail: the data used to estimate a PEF must obey the stationarity assumption. In other words, the spectrum of the data must be spatially invariant in order to encapsulate the inverse spectrum with a single PEF. Though the spatial spectra of the irregular picked seismic horizons in this example are roughly constant, the stationarity assumption is commonly violated for real-world problems. I make use of data "patching" to subdivide the data into smaller regions where the assumption *is* assumed to hold, and then estimate a PEF from the data contained in each patch (Claerbout, 1992). The dashed lines on Figures 1 and 2 delimit the four equal-sized patches I use in this example.

The problem of finding $\mathbf{H}'_1(x,y)$ and $\mathbf{H}'_2(x,y)$ is underdetermined, since we have only 578 known seismic data values, but 1600 model points. However, the classical least squares solution to the problem is valid only for *over*determined (Strang, 1986) systems. To convert this underdetermined problem to an overdetermined one, we must constrain the output model with additional regression equations, a process known as *regularization*. Normally the regularization operator imposes a "minimum-energy," or other similarly "safe" constraint on the free model variables, but adds little or no meaningful statistical information to the problem. However, by using a PEF as the regularization operator, we impose a fundamental statistical property of the known data on the model. In symbols, the problem can be stated through the following least squares "fitting goals."

$$\mathbf{J_{seis}}(\mathbf{H'_i} - \mathbf{S}) \approx 0 \tag{1}$$

$$\epsilon \mathbf{A} \mathbf{H'_i} \approx 0 \tag{2}$$

In Equations (1) and (2), the output is $\mathbf{H}'_i(x,y)$. The "$\approx$" means that we minimize the squared $L_2$ norm of the residual. Equation (1) forces $\mathbf{H}'_i$ to match the irregular picked seismic horizon $\mathbf{S}$, where it is known. $\mathbf{J_{seis}}$ is known-data "selector" operator, which effectively ignores the difference $\mathbf{H'_i} - \mathbf{S}$ wherever $\mathbf{S}$ does not exist. $\boldsymbol{\epsilon}$ is a so-called "damping factor," which weights the effective strength of the regularization equations (2) relative to the "data-matching" equations (1). The operator $\mathbf{A}$ is convolution with the patch-variant PEF. The problem is solved iteratively, using a conjugate direction-type (CD) algorithm.

The result is shown in Figure 3. Now that we have the surfaces $\mathbf{H}'_1(x,y)$ and $\mathbf{H}'_2(x,y)$.

### 2: Warping $\mathbf{H}'_i(x,y)$ to match the well logs

Qualitatively, we can express the desired attributes of the output model $\tilde{\mathbf{H}}_i(x,y)$:

1. At the well locations, the difference between $\tilde{\mathbf{H}}_i(x,y)$ and the well log data should vanish.

2. At distances far from any well, the output model should match (in a least squares sense) $\mathbf{H}'_1(x,y)$ and $\mathbf{H}'_2(x,y)$ as closely as possible – in magnitude and also in relative shape.

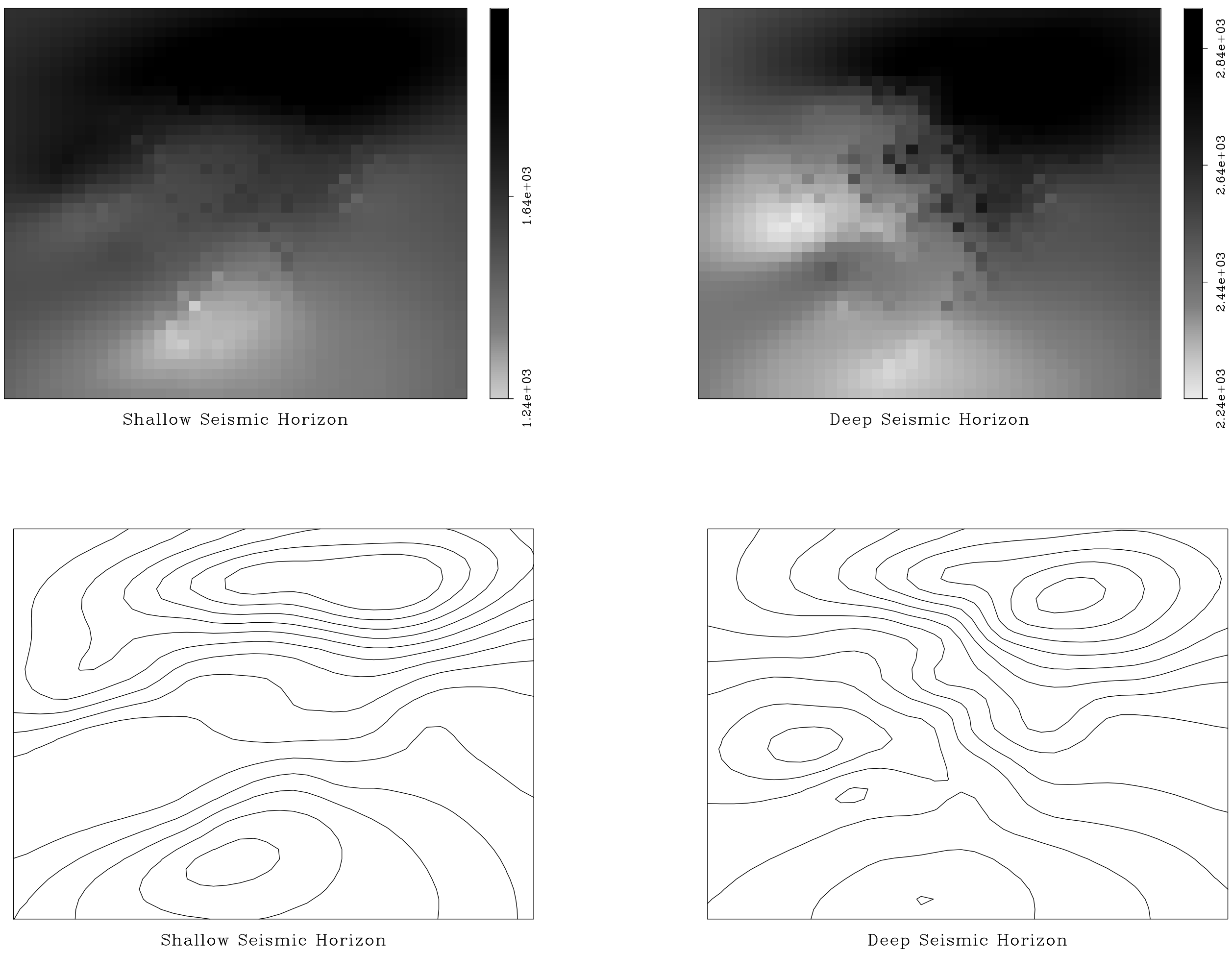


Figure 3: Top: $\mathbf{H}'_1(x,y)$ and $\mathbf{H}'_2(x,y)$. Bottom: contour plots of surfaces shown above. Note that the predominant trend in both figures is roughly "east-north-east", consistent with the trends predicted by the patch-variant PEF. morgan1-pef-seisfill-both [ER]

This is sound logic; the well log data is (for the sake of simplicity) assumed to be perfectly accurate, so at the well locations, the output model is rigidly constrained to match these measurements. On the other hand, when we are far from any well location, the only source of information is $\mathbf{H}'_i(x,y)$.

This method is strictly a vertical correction. It is naïve to assume that the misfit between the seismic horizon and the well logs is purely vertical, but well log measurements place no direct horizontal constraints on the output model. However, in the "Future Work" section I discuss how simple anisotropy can manifest itself in seismic data through vertical misfit, and how, through a simple inversion scheme, we may be able to quantify the degree of anisotropy.

We can directly measure the vertical discrepancy between $\mathbf{H}(x,y)$ and $\mathbf{H}'_i(x,y)$ at the well locations, $(x_w, y_w)$:

$$
\begin{aligned}
\text{Well Log} &= \text{Seismic} + \text{Vertical Misfit} \\
\\
\mathbf{H}_i(x_w,y_w) &= \mathbf{H}'_i(x_w,y_w) + \Delta z \\
&= \left(1 + \frac{\Delta z}{\mathbf{H}'_i(x_w,y_w)}\right) \cdot \mathbf{H}'_i(x_w,y_w) \\
&= \boldsymbol{\alpha}_{\boldsymbol{known}}(x_w,y_w) \cdot \mathbf{H}'_i(x_w,y_w) \qquad (3)
\end{aligned}
$$

Now solve a missing data problem to obtain a smooth 2-D function $\boldsymbol{\alpha}(x,y)$, whose value is already known at the well locations ($\boldsymbol{\alpha}_{\boldsymbol{known}}(x,y)$).

$$
\begin{aligned}
\mathbf{J}_{well}(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{\boldsymbol{known}}) &\approx 0 \qquad (4) \\
\boldsymbol{\epsilon}\,\nabla^2\boldsymbol{\alpha} &\approx 0 \qquad (5)
\end{aligned}
$$

$\mathbf{J}_{well}$ is the "well location selector" operator, similar to $\mathbf{J}_{seis}$ in Equation (1). I choose the 2-D Laplacian, $\nabla^{\mathbf{2}}$, as the regularization operator, because the vertical misfit is in general not dependent on the geology of the horizon which we're trying to update. Therefore we opt for the isotropic smoothing qualities of $\nabla^{\mathbf{2}}$, rather than those of a PEF. However, as a future project I suggest implementing this methodology in a recursive layer-stripping scheme. In this case, making the regularization operator dependent on the geometrical and physical properties of the overlying horizons makes sense, though the details have not yet been formalized.

This problem is grossly underdetermined ($\approx$ 15 known values vs. 1600 points in model space), so even for a good choice of $\boldsymbol{\epsilon}$, convergence is slow – too slow, in fact, to make this method practical for application to real-world problems. To solve this problem, I use **preconditioning,** defined as any change of variables which speeds convergence of an iterative scheme.

In general, the preconditioning operator $\mathbf{S}$ is defined by

$$\mathbf{m} = \mathbf{S}\mathbf{p} \qquad (6)$$

where **m** is the model variable and **p** is the so-called "preconditioned variable." The choice of **S** is arbitrary. However, (Fomel et al., 1997) proposes a simple, yet highly effective choice for **S**: the inverse of the regularization operator. Our regularization operator ($\nabla^2$) is 2-D convolution, so its inverse is obtained by 2-D *de*convolution - in general an ill-defined process. But the Helix Transform (Claerbout, 1997a) comes to the rescue, allowing us to use the simple 1-D algorithm to perform 2-D deconvolution.

First wrap $\boldsymbol{\nabla^2}$ onto a helix, then unroll it to create a long 1-D filter, as shown in Figure 4. Through the process of **spectral factorization**, we compute a minimum phase filter **a** whose autocorrelation is $\boldsymbol{\nabla^2}$: $\mathbf{a}'\mathbf{a} = \boldsymbol{\nabla^2}$. Since **a** is minimum phase, successive convolution with **a** and **a**$'$ is multiplication with lower and upper diagonal matrices, respectively. Therefore, the inverse operation, deconvolution, is matrix inversion: $(\boldsymbol{\nabla^2})^{-\mathbf{1}} = (\mathbf{a}'\mathbf{a})^{-\mathbf{1}} = \mathbf{a}^{-\mathbf{1}}(\mathbf{a}')^{-\mathbf{1}}$, which can be done very quickly in this case with simple backsubstitution, an $\mathcal{O}(n)$ operation! For more details on the spectral factorization methods used at SEP, see (Claerbout, 1997b), or more recently, (Sava et al., 1998). Now that we can compute the preconditioner $(\boldsymbol{\nabla^2})^{-\mathbf{1}}$ with confidence,

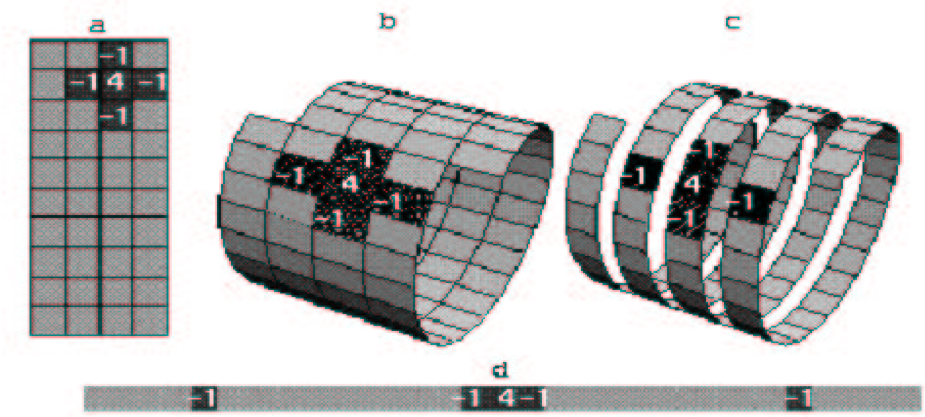


Figure 4: Schematic of the helical representation of $\nabla^2$. Adapted from a Mathematica drawing by Sergey Fomel. morgan1-helix [NR]

repose the original problem. First apply the following change of variables:

$$\alpha = (\boldsymbol{\nabla^2})^{-\mathbf{1}}\mathbf{p} \tag{7}$$

Substitute Equation (7) into Equations (4) and (5) to obtain

$$\begin{aligned} \mathbf{J_{well}}((\boldsymbol{\nabla^2})^{-\mathbf{1}}\mathbf{p} - \boldsymbol{\alpha_{known}}) &\approx 0 \\ \epsilon\mathbf{p} &\approx 0 \end{aligned} \tag{8}$$

We solve for **p** iteratively, then convert back to $\boldsymbol{\alpha}$ via equation 7. Figure 5 shows the resultant $\boldsymbol{\alpha}(x, y)$ for both the shallow and deep horizons. Does preconditioning work? **Yes!** In this example, the rate of convergence is increased by two orders of

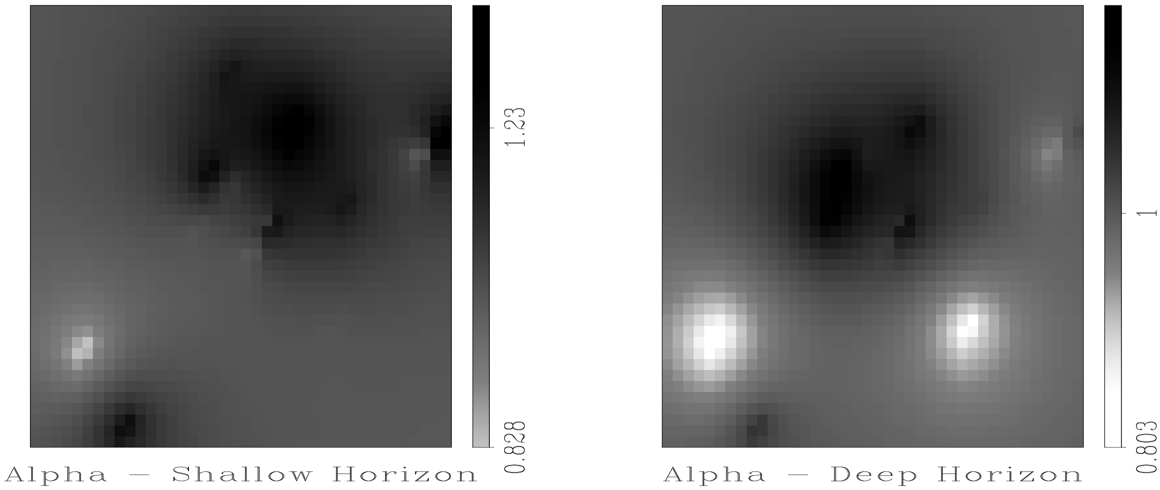


Figure 5: Seismic warp functions $\alpha(x,y)$, for the shallow and deep horizons. morgan1-alpha-both [ER]

magnitude by preconditioning, e.g., roughly 1000 versus 10 CD iterations. Now we compute the final output model, $\tilde{\mathbf{H}}_{\mathbf{i}}(\mathbf{x},\mathbf{y})$

$$\tilde{\mathbf{H}}_{\mathbf{i}}(\mathbf{x},\mathbf{y}) = \mathbf{H}'_{\mathbf{i}}(\mathbf{x},\mathbf{y}) \cdot \boldsymbol{\alpha(x,y)} \tag{9}$$

The results for both shallow and deep horizons are shown in figures 6 and 7, respectively.

## CONCLUSIONS/FUTURE WORK

Currently, this paper makes no new claims, nor does it apply new methods, but to borrow from Thomas Kuhn, a major part of scientific research consists of "mop-up" work. To gain credibility and acceptance, new tools must be applied to existing problems, and should solve them more effectively than before. Here I use the Helix Transform to compute an effective preconditioner for the problem, thus speeding convergence significantly. The updated horizons honor the shape of the picked seismic horizons and also match the well log data at the wells - a result similar to those obtained by Claerbout (1997b) and Berlioux (1995).

### Data limitations – and solutions

I feel that the data used here has been stretched to the limits of its inherent usefulness. In my estimation, the following attributes (not an exhaustive list) describe "ideal" data for this problem:

1. Dense, wide-area seismic coverage.

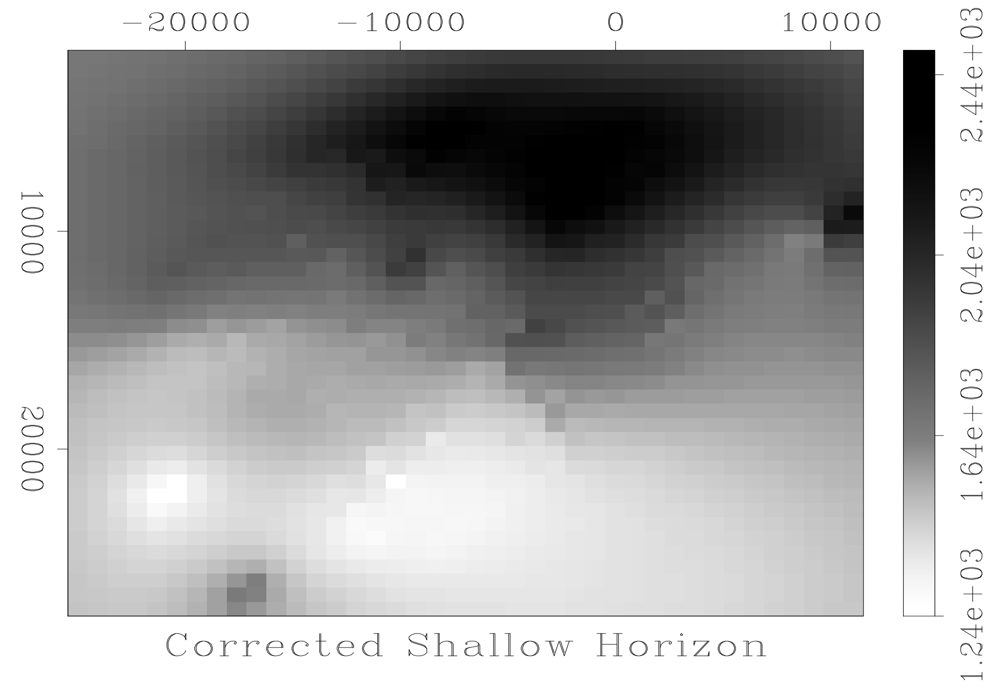


Figure 6: Final result: updated shallow horizon. morgan1-wellmap-2000 [ER]

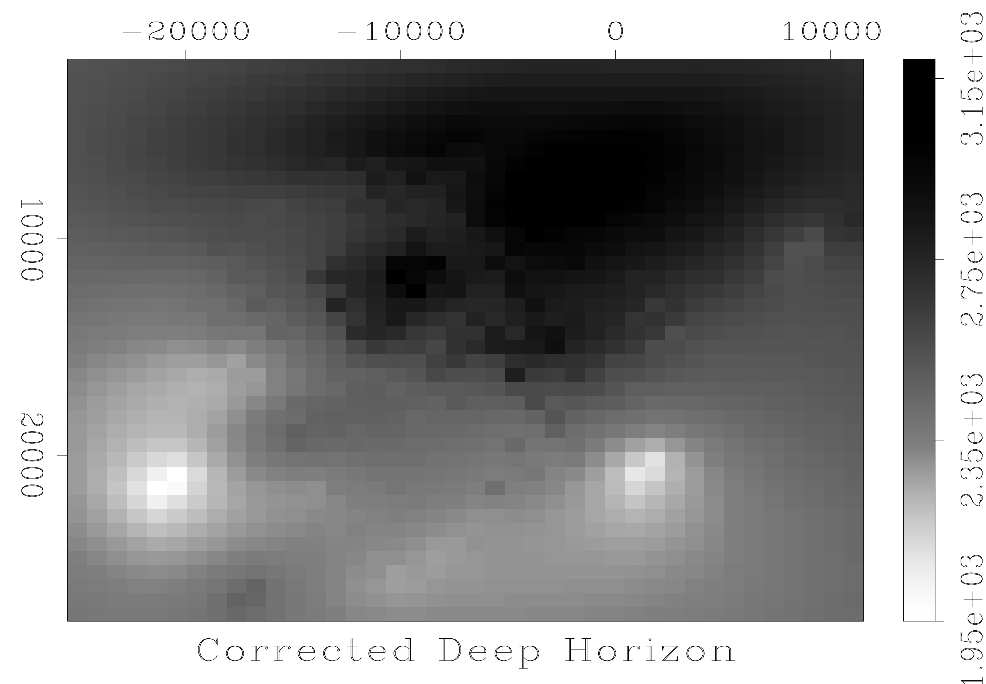


Figure 7: Final result: updated deep horizon. morgan1-wellmap-2300 [ER]

2. 20 or more wells inside seismic coverage.
3. Five or more picked seismic horizons.
4. Access to stacking velocities.

I combine "Conclusions" and "Future Work" into one section, because SEP is in the process of acquiring a large group of North Sea data from Mobil that will better allow us to develop and test new ideas, thanks to a collaboration with the **Stanford Center for Reservoir Forecasting** (SCRF). The data consists of five densely sampled seismic horizons and over twenty wells with well log data.

Verification of the validity of the final model $\tilde{\mathbf{H}}_i(x, y)$ is a difficult and ill-defined task. The method of *cross-validation* seems most promising. The optimization problem of Equation (8) is solved, but one of more of the well logs is disregarded. At the disregarded well locations, compare the depth to the actual horizon from the well log and that predicted by $\tilde{\mathbf{H}}_i(x, y)$. Since the first and second "ideal data attributes" should be satisfied to make this scheme viable, I do not utilize it in this paper, but I think the Mobil data is well-suited for the task. One can imagine a more complicated cross-validation scheme, in which many values of $\tilde{\mathbf{H}}_i(x, y)$ are determined from Equation (8) by removing different well logs. The result is a map of cross-validation errors, which is then minimized in another least-squares scheme.

### Anisotropy/velocity update

Anisotropy introduces error into seismically-estimated velocities. In a homogeneous, elliptically anisotropic medium[1] with a single flat reflector, standard velocity analysis measures $v_H$, the horizontal wave velocity, rather than the vertical velocity $v_V$. An elliptically anisotropic medium is defined by the parameter

$$\boldsymbol{\alpha_{aniso}} = \frac{v_V}{v_H} \tag{10}$$

Equation (3) appears to be a complicated way to express the vertical misfit between $\mathbf{H}'_i(x, y)$ and $\tilde{\mathbf{H}}_i(x, y)$, but it can be shown that $\boldsymbol{\alpha_{known}}$ in Equation (3) is equivalent to $\boldsymbol{\alpha_{aniso}}$ in Equation (10) for the simple medium discussed above. Given the flat reflector and constant $\boldsymbol{\alpha_{aniso}}$ assumptions, Equation (8) computes $\boldsymbol{\alpha_{aniso}}$ for all $x$ and $y$, solely from the vertical seismic/well log misfit.

Though I have not yet been able to prove it, I believe that in a more complex medium, the $\boldsymbol{\alpha}(x, y)$ from Equation (8) is an "RMS" $\boldsymbol{\alpha}$. From Equation (10) we obtain at the well locations the horizontal wave velocity $v_H$ in the overlying medium. At other locations, the $\boldsymbol{\alpha}(x, y)$ from Equation (8) gives a reasonable estimate of $v_H$.

[1] For most real materials, the observed anisotropy is somewhat anelliptic. However, the simplicity of the elliptical assumption makes it easy to conceptualize and to manipulate mathematically. Future views of this problem should address more complicated anisotropic assumptions, i.e., TI.

For multiple $\tilde{\mathbf{H}}_i(x, y)$, one could imagine vertically interpolating the $\boldsymbol{\alpha}(x, y)$ surfaces corresponding to each one, thus yielding a 3-D $\boldsymbol{\alpha}$ cube which could then be subjected to a Dix-like inversion to obtain a 3-D "interval" $\boldsymbol{\alpha}$ cube. The RMS $\boldsymbol{\alpha}$ cube could be, after conversion back to time, used as a direct multiplicative correction on the stacking velocities.

## ACKNOWLEDGEMENTS

Thanks to discussions with Bob Clapp and James Rickett, I gained many valuable insights on theoretical and practical issues pertaining to this paper. Jon Claerbout and Sergey Fomel developed and presented many of the recent advances in SEP's methods of practical inversion. Biondo Biondi and Matthias Schwab independently suggested that I approach this problem in terms of anisotropy. Chevron Overseas Petroleum donated the data shown herein.

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