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Regularizing time tomography with steering filters

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ABSTRACT

Standard depth tomography methods often do not converge, converge slowly, or converge to a model that is geologically unrealistic. By using dip based steering filters rather than standard isotropic regularization operators, convergence speed and final model quality are improved. By formulating the tomography operator in two-way vertical traveltime (τ,ξ) rather than in depth (z,x) coordinates, we can uncouple velocity estimation from reflector position estimation. In this coordinate system we avoid many of the problems associated with the depth tomography problem, and converge quickly to a reasonable solution.

INTRODUCTION

Depth velocity estimation is one of the most difficult problems in reflection seismology. The tomography problem is by nature non-linear and under-determined. A standard technique is to perform a non-linear loop over a linearized inversion problem. Unfortunately, such a technique is prone to converge to a local minimum of the objective function (Phillips and Fehler, 1991). In addition, each of the linearized inversion problems are under-determined, requiring some type of regularization. Often an isotropic operator is used to regularize the tomography problem (Biondi, 1990), but these operators tend to fill-in the null space with isotropic features that are often geologically unreasonable (Etgen, 1997).

Fortunately, we often have other sources of information, such as well logs, stacked sections, or geologist's interpretation, that we can use to construct anisotropic operators (steering filters) (Clapp et al., 1997, 1998) that fill the null space with more geologically reasonable velocities. Convergence speed can be improved by changing from a regularized to a preconditioned problem (Claerbout and Nichols, 1993). By forming the regularization operator in a helical coordinate system we can efficiently obtain an inverse operator by polynomial division (Claerbout, 1998). This new operator can be used as a preconditioner, creating an equivalent optimization problem (Fomel et al., 1997) that converges significantly faster.

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Another major difficulty in depth tomography is the strong connection between depth and velocity. We can avoid some of the problems caused by this connection, by transforming the whole problem into vertical-traveltime coordinates (τ,ξ). In the time domain reflector position is less sensitive to velocity changes. This modified coordinate system still allows for complex velocity structures, but significantly reduces the map migration term in tomography (Biondi et al., 1997).

We construct a synthetic anticline velocity model and apply a standard ray based tomography technique to estimate velocity. We show that the inversion result is improved by the use of steering filters to precondition our tomography operator over a more standard isotropic regularization technique. We then apply the same basic tomography method in (τ,ξ) space again significantly improving our velocity estimate.

REGULARIZATION USING STEERING FILTERS

We can write our fitting goals for a linear problem as

$$\mathbf{d} \approx \mathbf{T}\mathbf{m}. \tag{1}$$

Where $\mathbf{d}$ is the data, $\mathbf{T}$ is the linearized tomographic operator, and $\mathbf{m}$ is our model. Often the problem is under-determined, so we need to add some type of regularization

$$\mathbf{0} \approx \mathbf{A}\mathbf{m}. \tag{2}$$

Ideally, $\mathbf{A}$ should be the inverse of the model covariance matrix (Tarantola, 1987). Unfortunately, we are estimating the model so we don't have the covariance matrix. The lack of knowledge about the model often leads to the Laplacian or some other isotropic operator being used for $\mathbf{A}$. As a result, we fill the null-space of the model with isotropic features, that, while explaining the data, may be unreasonable when judging the results with geologic criteria.

Fortunately, we often do have other sources of information, such as a geologist's model for the region or reflector dip from well logs, that can be used to better constrain our inversion. For example, we can the find the general dip direction by interpreting an early migration result and use this information to construct a space variant filtering operator that annihilates dips with the given direction. (Figure 1), that can be used as $\mathbf{A}$ in equation (2). The inverse of the dip-annihilation operator is really a first-order approximation for the model covariance matrix. We know that in general we have some isotropic smoothness in our velocity function, therefore adding some isotropic smoothness to our regularization operator is appropriate. This creates a space variant filter direction and produces an anisotropic blob oriented in the dip direction (Figure 2). By forming our operators in helix-space Claerbout (1997), we can find a stable inverse for our steering filters ($\mathbf{A}$) and change our regularization problem into a preconditioning problem. By substituting:

$$\mathbf{m} = \mathbf{A}^{-1}\mathbf{p} \tag{3}$$

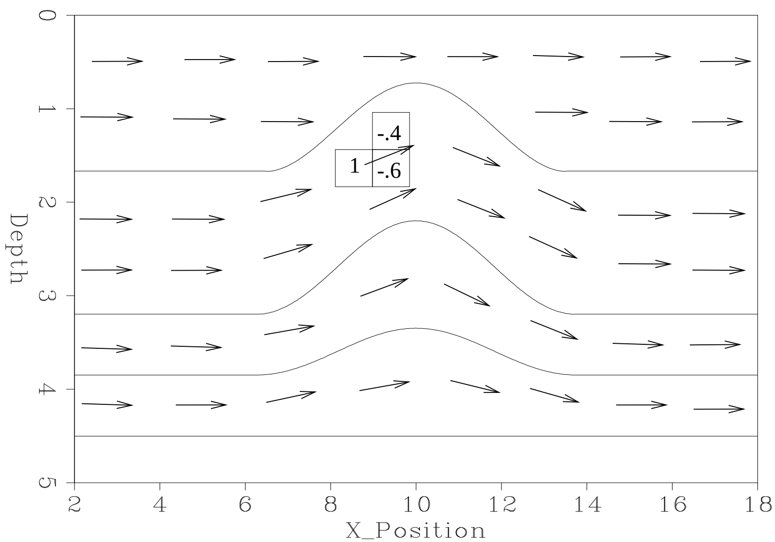

Figure 1: Steering filter directions as a function of geologic dip. bob2-filters [NR]

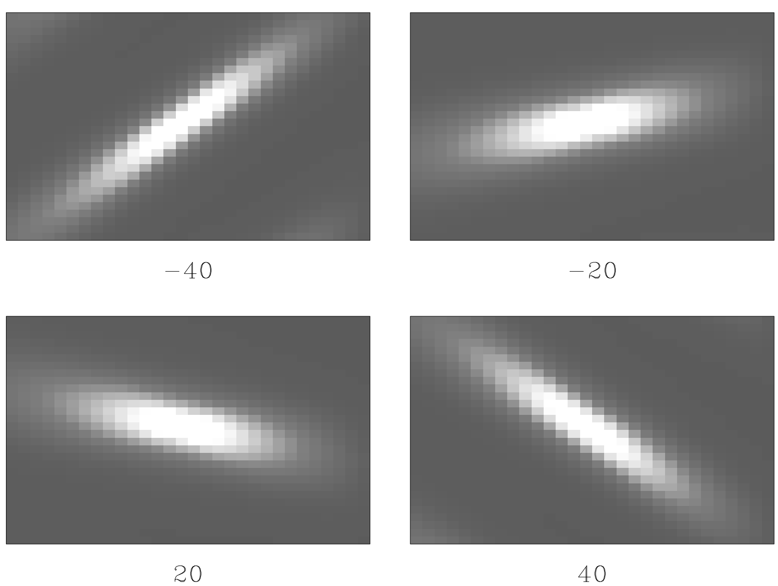

Figure 2: Preconditioning operator impulse response when oriented at -40, -20, 20, and 40 degrees from horizontal. bob2-sweep [ER]

we get

$$
\begin{aligned}
\mathbf{d} &\approx \mathbf{T}\mathbf{A}^{-1}\mathbf{p} \\
\mathbf{0} &\approx \epsilon\mathbf{I}\mathbf{p}.
\end{aligned} \tag{4}
$$

The inverse operator ($\mathbf{A}^{-1}$) spreads information long distances at every iteration, quickly filling the null-space with reasonable values.

In general, tomography problems are not as straightforward as the one presented above. First and foremost, the tomography problem is non-linear. Perturbations in the slowness model change the raypaths, making the tomography problem non-linear. We can get around this by imposing an outer, non-linear raytracing loop over a linearized back projection operation that assumes stationary raypaths. In addition, we must deal with the inherent velocity-depth coupling problem: any changes in traveltimes can be caused by either reflector movement or slowness model changes. Therefore, we must take into account reflector movements when evaluating the linearized tomographic operator (van Trier, 1990):

$$
\mathbf{\Delta t_m^i} \approx \mathbf{L_s^i}\mathbf{\Delta s} + \mathbf{G^i}\mathbf{\Delta R^i} = (\mathbf{L_s^i} + \mathbf{G^i}\mathbf{H^i})\mathbf{\Delta s}, \tag{5}
$$

where

$\mathbf{\Delta t_m^i}$ is the difference between the modeled and the correct travel times,

$\mathbf{L_s^i}$ is the back-projection operator along our modeled ray paths,

$\mathbf{G^i}$ maps changes in reflector position to changes in traveltimes,

$\mathbf{\Delta R^i}$ is the change in reflector position,

$\mathbf{H^i}$ maps slowness changes to reflector movement,

$\mathbf{\Delta s}$ is -ur change in slowness.

Finally, we add in our preconditioning operator to obtain our final set of tomography goals,

$$
\begin{aligned}
\mathbf{\Delta t_m^i} &\approx (\mathbf{L_s^i} + \mathbf{G^i}\mathbf{H^i})\mathbf{A^{-1}}\mathbf{p} \\
\mathbf{0} &\approx \epsilon\mathbf{I}\mathbf{p}.
\end{aligned} \tag{6}
$$

Synthetic Example

To test the steering methodology versus a more tradition isotropic smoothing approach we created a synthetic velocity model with four reflectors (Figure 3). To simulate picked traveltimes we shot rays through the correct model using a paraxial ray tracing code (Rekdal and Biondi, 1994) and found ray pairs where Snell's Law was obeyed at the reflectors. We use as our starting guess a $v(z)$ velocity model

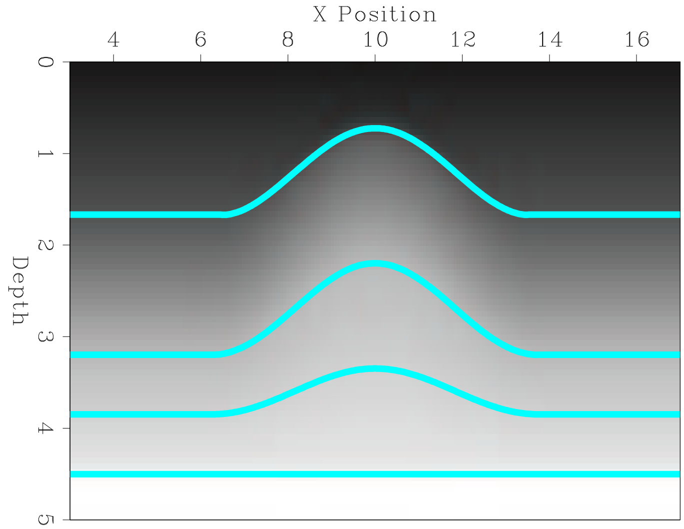

Figure 3: Initial model with reflectors superimposed. bob2-overlay-vel-cor [CR]

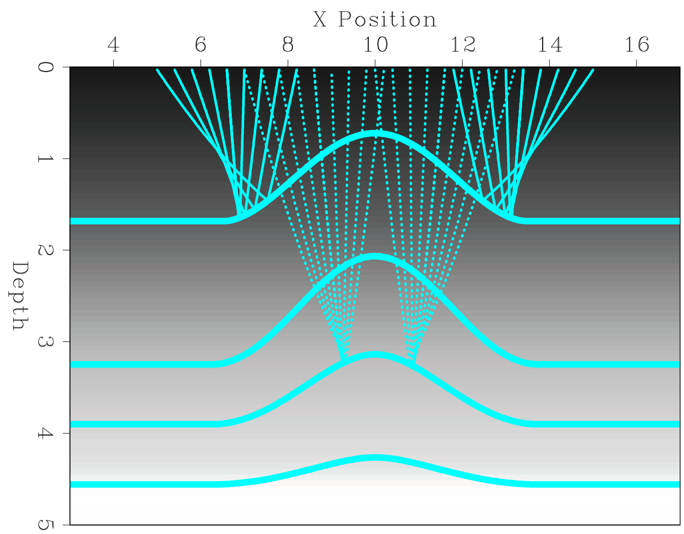

Figure 4: Starting model and reflector position with model rays superimposed. bob2-rays-vel0 [CR]

calculated by averaging over x at each depth. We map migrated the correct depth surfaces according to our initial guess at the velocity model to obtain our first guess at their position (Figure 4). We then shot and matched rays using the initial model and map-migrated reflector position. The difference between the modeled reflection times and the correct reflection times (Figure 5) was then used as input to our tomography problem fitting goals (6). We performed two non-linear iterations of tomography using both the inverse Laplacian and steering filters, oriented along the map migrated reflector position (Figure 6), as our preconditioner. As Figure 7 shows, the Laplacian model estimate has already produced an unwanted isotropic anomaly to explain the w-shaped pattern in the time differentials of reflector 3 and 4 (Figure 5). On the other hand, the steering filter preconditioned result has introduced velocity perturbations that follow our reflector geometry. To test whether or not the steering filter approach would eventually fall into the same trap as the Laplacian preconditioned scheme we performed two more non-linear iterations. Figure 8 shows the results of all four non-linear iterations using the steering filters. Note how the dome like shape

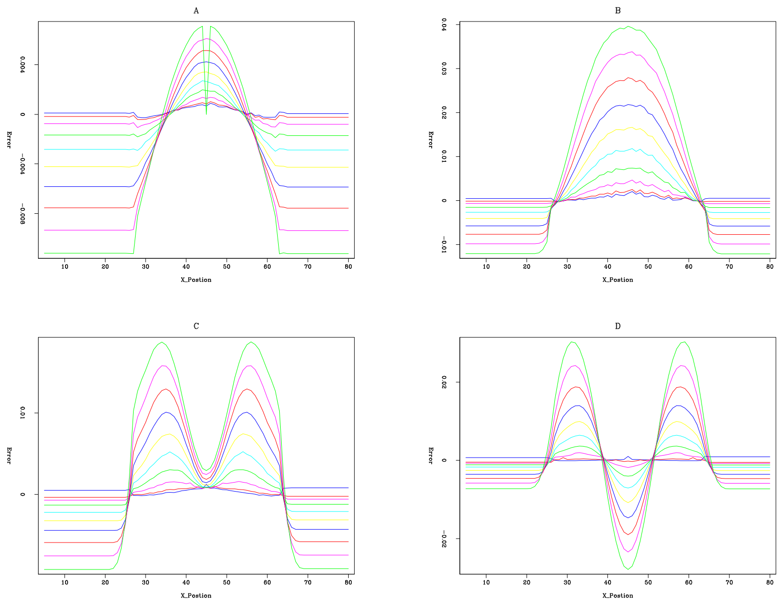

Figure 5: Arrival time differences for the correct model and initial model for the four reflectors (A, top reflector; B, second reflector; C, third reflector; D, bottom reflector.) The various curves for each reflector represent different offsets. The inconsistent difference seen in the top curve of A is due to a bad ray and is weighted to zero in the inversion. bob2-time-diff [CR]

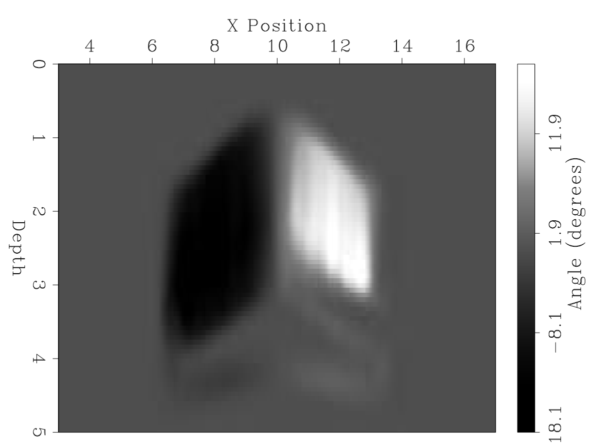

Figure 6: Angle (from horizontal) of the steering filters. bob2-angles [CR]

is developing with successive iterations and the low velocity doublet (at 3 km depth and approximately at x=10 km) is diminishing.

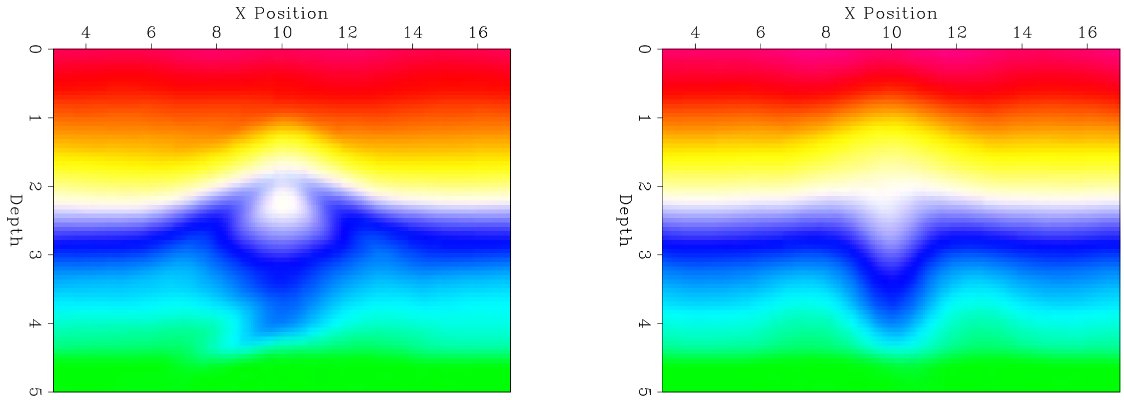

Figure 7: Left, inversion result after two non-linear iterations using steering filters as a preconditioner. The right, after two non-linear iterations using the inverse Laplacian as a preconditioner. Note how the dome shape is better resolved using the steering filters. bob2-lap-compare [CR]

TOMOGRAPHY IN TIME VS. TOMOGRAPHY IN DEPTH

Because of the velocity-depth coupling, non-linear tomographic velocity estimation in the depth coordinates often does not converge effectively. To avoid some of the drawbacks of traditional tomography Biondi et al. (1997) proposed a tomographic velocity estimation in the two-ways traveltime coordinates (τ,ξ). The method is based on a transformation of the eikonal equation from the depth coordinates (z,x) into the time coordinates (τ,ξ), according the following transformation ov variables:

$$\begin{aligned} \tau(z,x) &= \int_0^z \frac{2}{V(z',x)} dz' \\ \xi(z,x) &= x. \end{aligned} \qquad (7)$$

Starting from the eikonal equation in the time coordinates, we can remap the tomography goals of equation (6) into the time coordinates. The operators $\mathbf{L_{s^i}}$, $\mathbf{H}$, $\mathbf{G}$ are different in the time domain from the equivalent depth-domain operators; and also $\mathbf{A^{-1}}$ has a slightly different orientation, because it now operates on a velocity function defined in (τ,ξ).

Some of the advantages of formulating the tomography problem in (τ,ξ) domain can be seen in Figure 9. In this new domain reflector movement is minimal, significantly decreasing the velocity-depth connectivity problem that we saw in (z,x) space. The initial reflector position is closer to the correct reflector position. As a result, the orientation of the steering filters more accurately follows true reflector dip. To

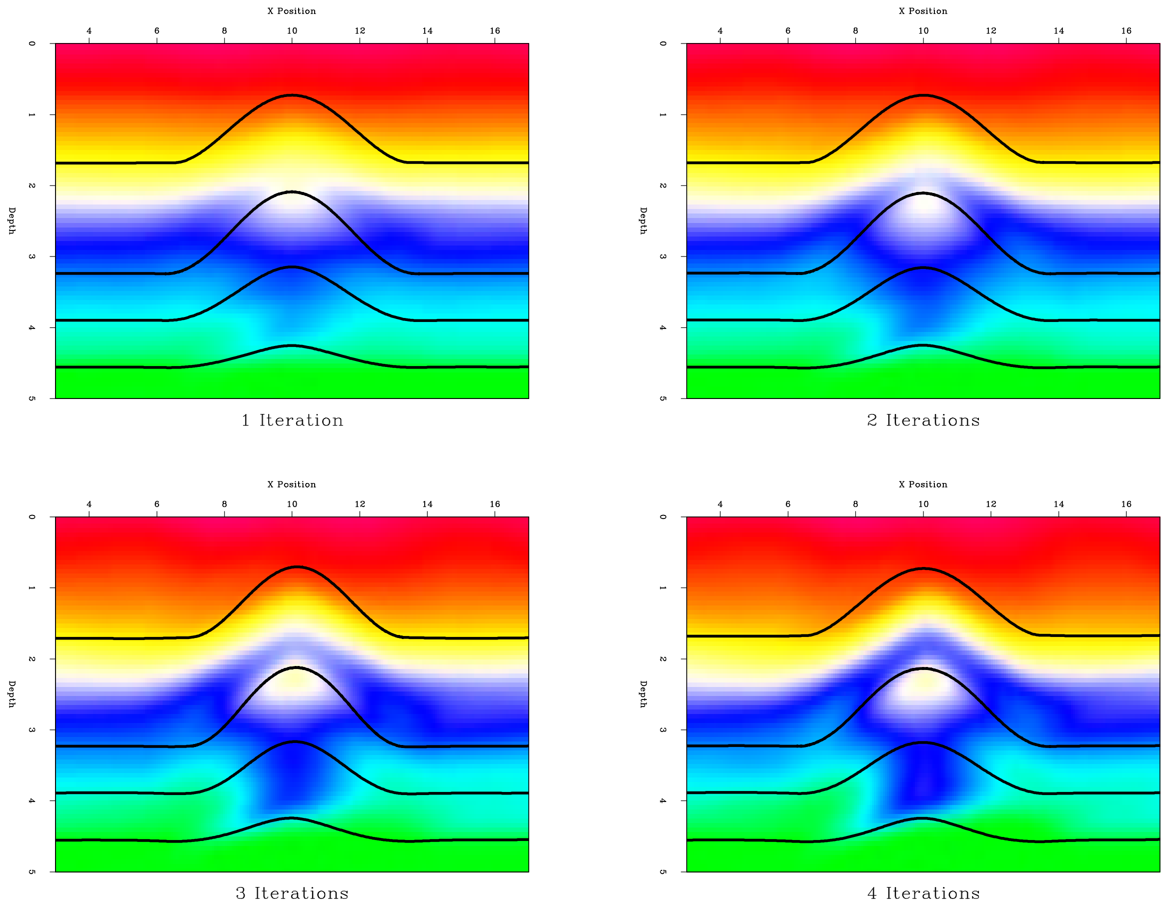

Figure 8: Velocity model after one, two, three, and four non-linear iterations. bob2-panel [CR]

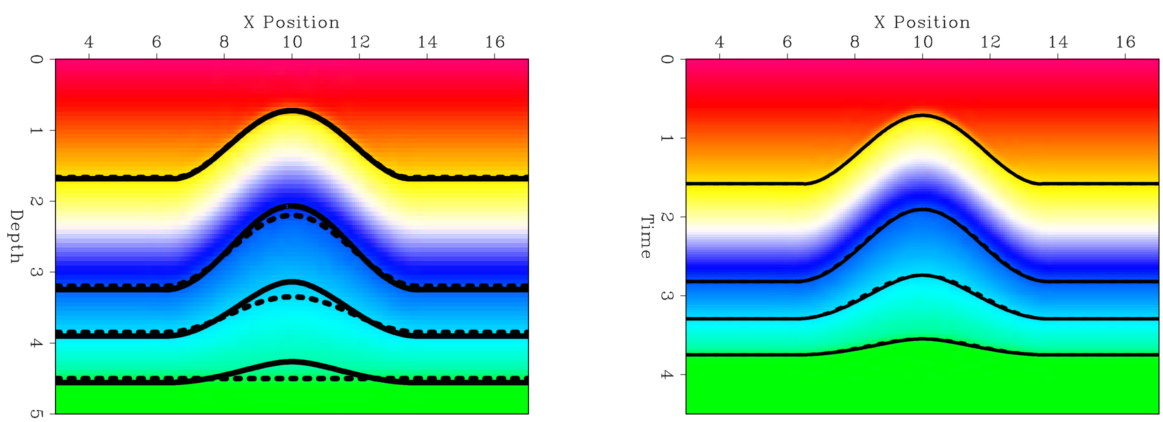

Figure 9: Left, depth velocity model with the correct reflector position (dashed) and the estimated reflector position using the initial guess at the velocity model (solid). Right, (τ,ξ) velocity model with the same reflectors superimposed. Note how reflector movement is significantly less than in the (τ,ξ) case. bob2-movement [CR]

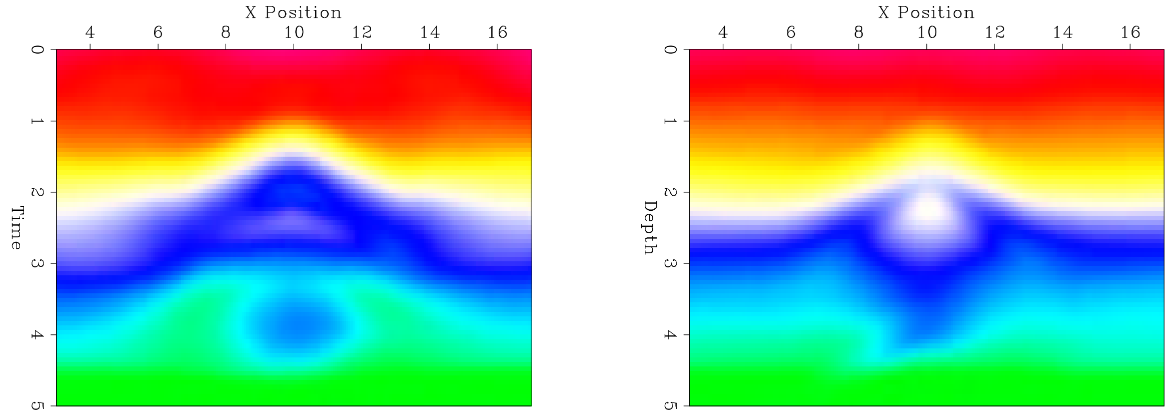

Figure 10: Velocity estimate by performing tomography in (τ,ξ) space and then mapping back into (z,x) space compared to our velocity estimate in depth, both after two non-linear iterations. bob2-tau [CR]

analyze the advantages of working in (τ,ξ) space we performed two non-linear iterations of tomography and remapped the resulting velocity model back into (z,x) space (Figure 10). Comparing the results of tomography in the time domain (Figure 10), with the results of tomography in the depth domain (Figure 8), we can notice how the doublet, seen in the depth tomography case, has significantly decreased and how the dome shape is much more prominent.

FUTURE WORK

There are several areas of future work on this project. First, more non-linear iterations need to be done for both the (x,z) and (τ,ξ) approaches. A layer stripping approach is also warranted. The w pattern on the third and fourth reflector lead a doublet in our velocity estimate. By resolving the velocity above the second reflector before attempting the global problem we might be able to avoid the doublet. In addition the procedure needs to be tested on a real 2-D data set and then extended and tested in 3-D.

CONCLUSIONS

By introducing geologic based smoothing filters into our tomography problem we can obtain solutions that are more geologically reasonable and avoid some of the non-linear traps that isotropic smoothers can fall into. Further improvement is obtained by separating the velocity and reflector position estimation problem by formulating our inversion in $(\tau,\ \xi)$ space. Overall the methodology holds promise in producing realistic velocity structures in areas of fairly complex geology.

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