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Velocity space interpolation of aliased seismic data by preconditioning

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ABSTRACT

Sparsity and irregularity of spatial sampling are common problems in seismic data. Irregularity severely limits the types of processes which may be applied to a data set, and any process will likely fail on data which are overly sparse, as data and operator aliasing become a crippling problem. Interpolation schemes seek to dealias data, but are themselves challenged by aliasing, because it is difficult for an algorithm to pick exclusively the correct dip or dips to interpolate. Transformation to velocity (or slowness) space is an attractive basis for an interpolation algorithm, because the operator is limited to 'reasonable' directions, in that it operates only along centered hyperbolas. Since data can be organized so that it is symmetric, and largely hyperbolic, in CMP gathers, this type of interpolation should greatly reduce the risk of interpolating incorrect dips. However, the velocity transform is not an exact forward/inverse-transform pair, and the smoothness and/or noisiness of the estimated velocity spectrum presents a new and serious pitfall. While the original data space may be remodeled exactly, large artifacts are likely to appear in alternate similar data spaces, such as are appropriate for interpolation or regularization. By preconditioning the inversion, the model may be made more parsimonious, resulting in improved remodeling into a new data space.

INTRODUCTION

Irregular acquisition of seismic data challenges processing. Frequency domain processes are difficult or impossible to apply. Kirchhoff methods adapt easily, but are likely to suffer from poor amplitude behavior and other problems; an acquisition 'footprint' is often visible on the final images (Chemingui and Biondi, 1996). If data can first be regularized, and large gaps interpolated, processing results will improve, and otherwise unavailable frequency domain processes will be made usable.

Sparsity of data, and the data aliasing which results, create a serious challenge to

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interpolation algorithms. Given a signal which is spatially aliased, it is difficult for an interpolation algorithm to determine the correct dip at which to model missing events. Transformation to velocity space is an attractive option, because the operator stacks and sprays data only along centered hyperbolas. Thus only reasonable dips are possible in the interpolated data. This type of interpolation does, however, have another pitfall. The optimum velocity spectrum, as estimated by least squares inversion, is likely to be too smooth to make an effective interpolation.

Least squares solutions to inverse problems are often smoother than desired (Nichols, 1994) (Crawley, 1995). Because squared errors are minimized, large noise bursts have much more impact on the final solution than smooth, low-amplitude, erroneous variations. However, in some applications these smooth variations are as serious a problem as large noise bursts. Numerous geophysical topics have been approached from the standpoint of finding parsimonious or 'spikey' solutions, including deconvolution (Ulrych and Walker, 1982), velocity analysis (Vries and Berkhout, 1984), and balancing (Crawley, 1995). A number of projects at SEP have sought to find spikey velocity space representations. Nichols (1994) showed results of L_p norm solutions to velocity space inversions, and presented methods for solving L_p problems using iteratively reweighted least squares (IRLS). Berryman (Berryman, 1996) discusses formulation of L_p norm problems, and gives some important mathematical basis for selection of p. In his PhD thesis, Ji (1994) showed that missing near offset traces in marine cmp gathers can be interpolated via application of a modified operator.

In this paper, I discuss and demonstrate the application of a preconditioned algorithm to interpolation beyond aliasing of 2D synthetic and real CMP gathers, and discuss the relationship between IRLS, Ji's approach, and preconditioning.

INTERPOLATION AND LEAST SQUARES

The basis of the interpolation scheme is very simple. Data, which may be irregular, poorly sampled, have missing offsets, etc. are transformed to coordinates of zero-offset time and velocity. This velocity spectrum is then used to model data at the desired offsets.

This may be formulated simply, but not effectively, as follows (Ji, 1994a). Estimation of the velocity spectrum for a given CMP gather is done by solving the equation

$$d = Hm \tag{1}$$

where d is the recorded data, m the model in velocity space, and H the forward operator which creates a hyperbola in the data space from a spike in the model space. Interpolating in this manner does not yield satisfying results. The reason is in the null space of the operator H.

Solving this problem by conjugate gradients yields a model which predicts the recorded data very well. However, even for perfect hyperbolas, the final model will not be exact. Spikes will tend to be smoothed, and some noise is inevitable. This difference does not impact the model's ability to predict the data because, as Nichols (1994) points out, the difference between the exact model and the inversion result is in the null space of the operator. However, changing the data space, which me must do in order to interpolate or regularize, changes the null space. Thus, the slight artifacts in the velocity space, which model nothing in the original data space and thus do not contribute to the residual (which resides in the original data space), will model nonzero energy into a different data space, creating artifacts.

Figure 1 shows a simple example of a null space changing as data space changes. On the top left of Figure 1 is a velocity space with two spikes in it. The top right is the velocity space estimated by inversion. The bottom left is the difference between the two top panels used to model into the same data space as was used in the inversion. The bottom right is again modeling using the difference between the top two panels, but this time into a new, finely-sampled data space. Note that much more energy has appeared in the new data space. The max values in the bottom right panel are 100 times those in the bottom left panel. The null space of the operator has changed. The large amplitudes in the bottom right are about one third the amplitude of the spikes in the model space (top left of the figure), and of the hyperbola amplitudes in data space. This should represent an unacceptable interpolation artifact.

PRECONDITIONING

Interpolation seems to call for as spikey a model as possible. One way to a spikier model is a change of variables in the optimization, or preconditioning (Claerbout and Nichols, 1994). Define a new model n, such that:

$$m = Wn. \tag{2}$$

The earlier optimization can be rewritten as

$$d = HWn, \tag{3}$$

and after solving for n the original model m is regained by applying equation 2. The question is what operator W makes this added transformation worthwhile. After Nichols (1994) and Ji (1994), I choose W to be a diagonal weighting matrix. The desire to reduce smoothness suggests that the weights be chosen to raise the model to some power. In fact it turns out that choosing the power 1.5 corresponds to minimizing the L_1 norm of the model; one can choose an L_p norm by raising the model to the $(2-p)/2$ power (Nichols, 1994).

With W chosen, the gradient direction of the solution in n is

$$\Delta n = W'H'r \tag{4}$$

where r is the residual and prime notation connotes an adjoint operator. Of course, since W is a diagonal matrix, $W = W'$.

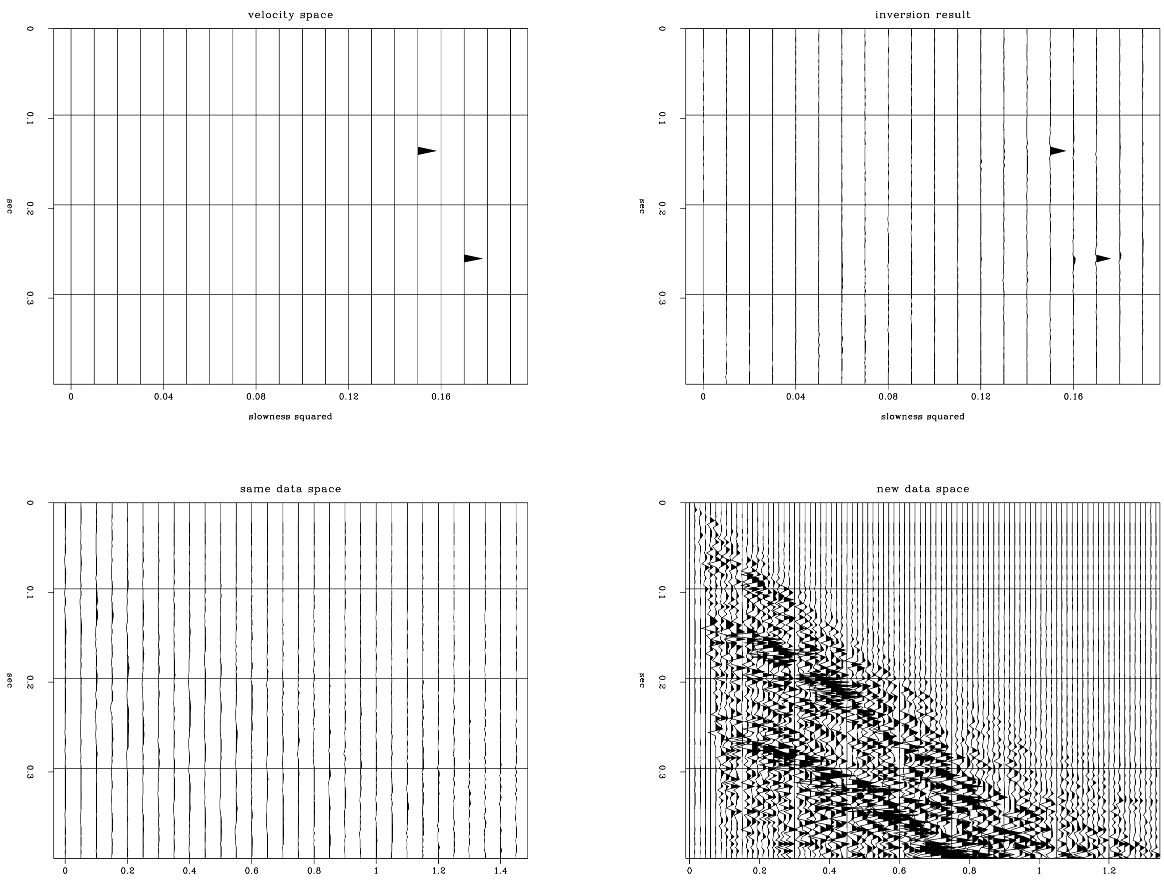

Figure 1: Null space effects: Top left and right are original and estimated velocity spaces. Bottom left is difference between top figures modeled into original data space. Bottom right is difference between top figures modeled into different data space. sean1-null-space [ER]

The use of a power of the model as its own weighting function introduces nonlinearity into the problem. The problem can be solved in steps, using a given weighting function for several iterations, or the weighting function can be updated with every iteration.

Preconditioning, IRLS, and Ji's interpolation

The preconditioning operator in this example is a diagonal weighting matrix, but any operator may be used as a preconditioner, the choice depends on the application. Often a preconditioner is chosen to speed up an inversion (Karrenbach, 1994), though here the weighting matrix is chosen to impart a desired property to the model.

In a manner analogous to weighting the model, the residual can be weighted. This results in a problem formulated as (Nichols, 1994)

$$Wd = WHm. \tag{5}$$

Both sides of the equation are weighted in data space. This is the approach known as iteratively reweighted least squares.

Ji's (1994) approach is similar to preconditioning, in that he applies an iterative reweighting to the model space. However, it is not applied as a change of variables. That is, the weighting is not reapplied to the final solution. This approach is like defining a new operator

$$\tilde{H} = WH \tag{6}$$

SYNTHETIC EXAMPLE

A 2D synthetic CMP gather was modeled using SEP's Kir_Mod program, and a model made up of point diffractors, to generate data with many dips, and non-hyperbolic 'cheops pyramid' events. This was an important part of this synthetic test: hyperbolas should collapse to points in velocity space, but flat-topped hyperbolas can not collapse perfectly. The synthetic was used as input to the algorithm, with the goal of simply refining the offset spacing, and thus dealiasing the gather. The input synthetic is shown in Figure 2, along with its frequency and velocity spectra; it can be seen that the gather is aliased, and in several regions the aliasing and crossing dips make for a confusing picture, even to the eye.

The gather was first input to the original L_2 inversion, without preconditioning. The estimated velocity spectrum was then remodeled into the original data space, and into a resampled data space. The remodeled outputs are shown in Figure 3. Remodeling into the original data space is perfect; crossing, non-hyperbolic events present no trouble. However, the null space effect is painfully obvious in the resampled data space. The new data space has been resampled by a factor of two; every other trace is interpolated, and every other trace has terrible artifacts. Notice that the

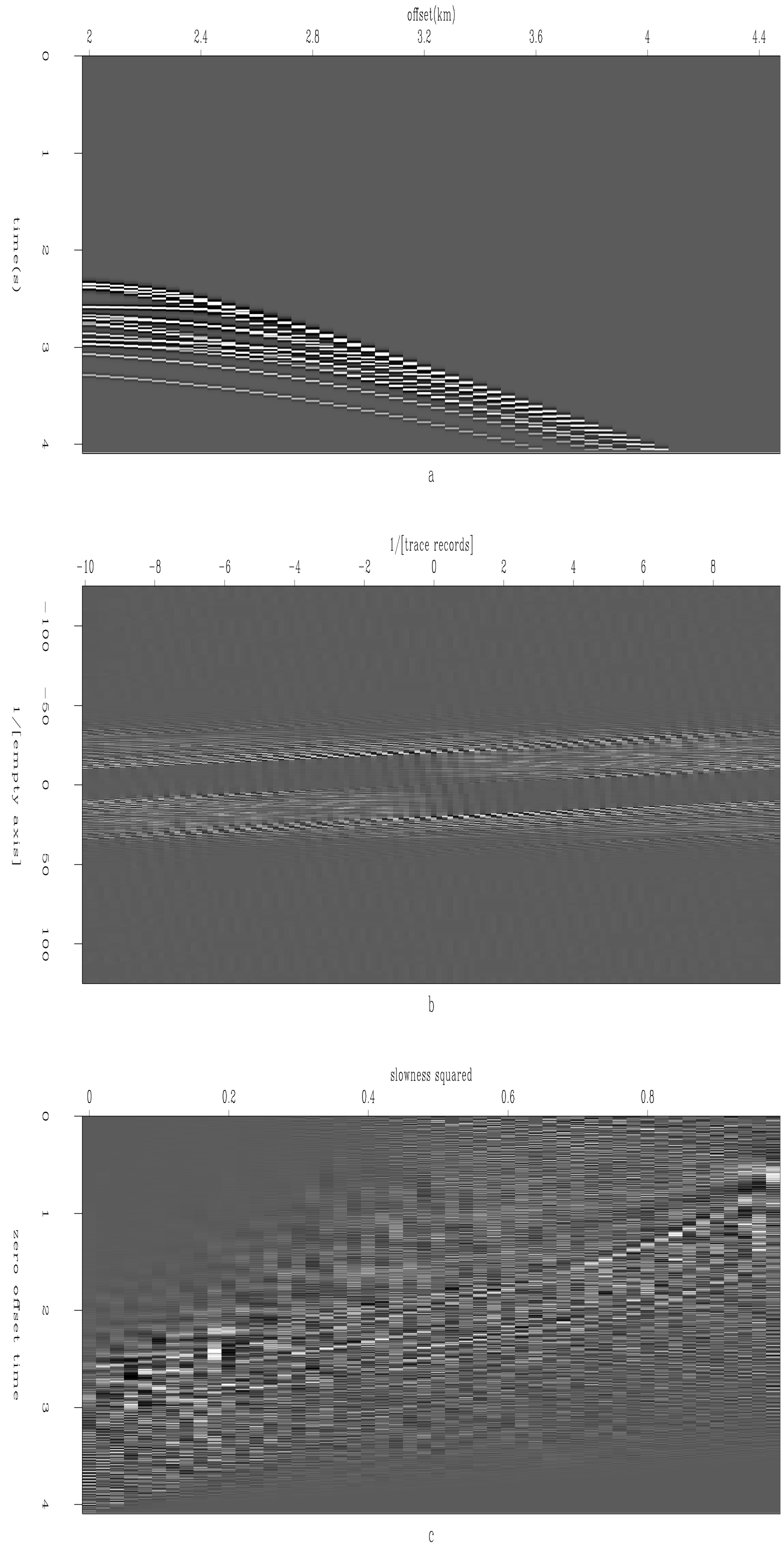

Figure 2: Input synthetic CMP gather (a), 2D Fourier transform (b), estimated velocity spectrum (c). sean1-synthin [NR]

velocity spectrum is fairly chaotic. This raises some doubt as to whether lowering the norm of the model will, in fact, help, because it is not obvious which values should be spiked.

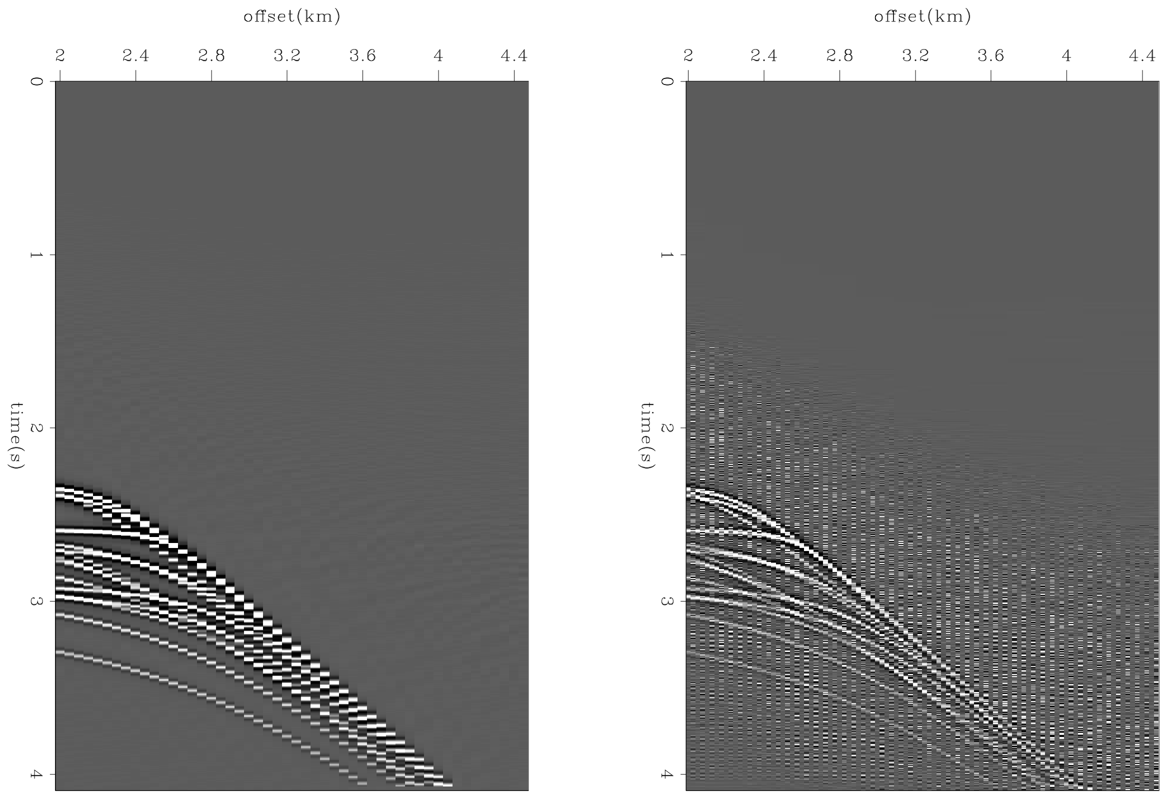

Figure 3: Non-preconditioned inversion results: data remodeled in the original data space (left), and a resampled data space (right). sean1-l2out [CR]

This same gather was also used as input to the preconditioned inversion, and the gather again remodeled into the same and resampled data spaces. The output is still not free from artifacts, but the artifacts' amplitude is greatly reduced. Further, almost all of the noise that remains is very high frequency and easily removable. The final interpolated output and its spectrum are displayed in Figure 4. Notice that the velocity spectrum is very different from the earlier result. It is pleasing that the non-hyperbolic events do have a parsimonious representation in velocity space. This is good news for the applicability of this type of interpolation.

REAL DATA EXAMPLE

As a real data test, an irregular CMP gather was used as input to the algorithm. This gather comes from a collection of profiles published in Ozdagan Yilmaz' book, *Seismic Data Processing* (1987). The input cmp gather and velocity spectrum estimated with the preconditioned inversion are displayed in Figure 5. As can be seen from the figure, the data are severely aliased, and it is difficult to identify many events. The

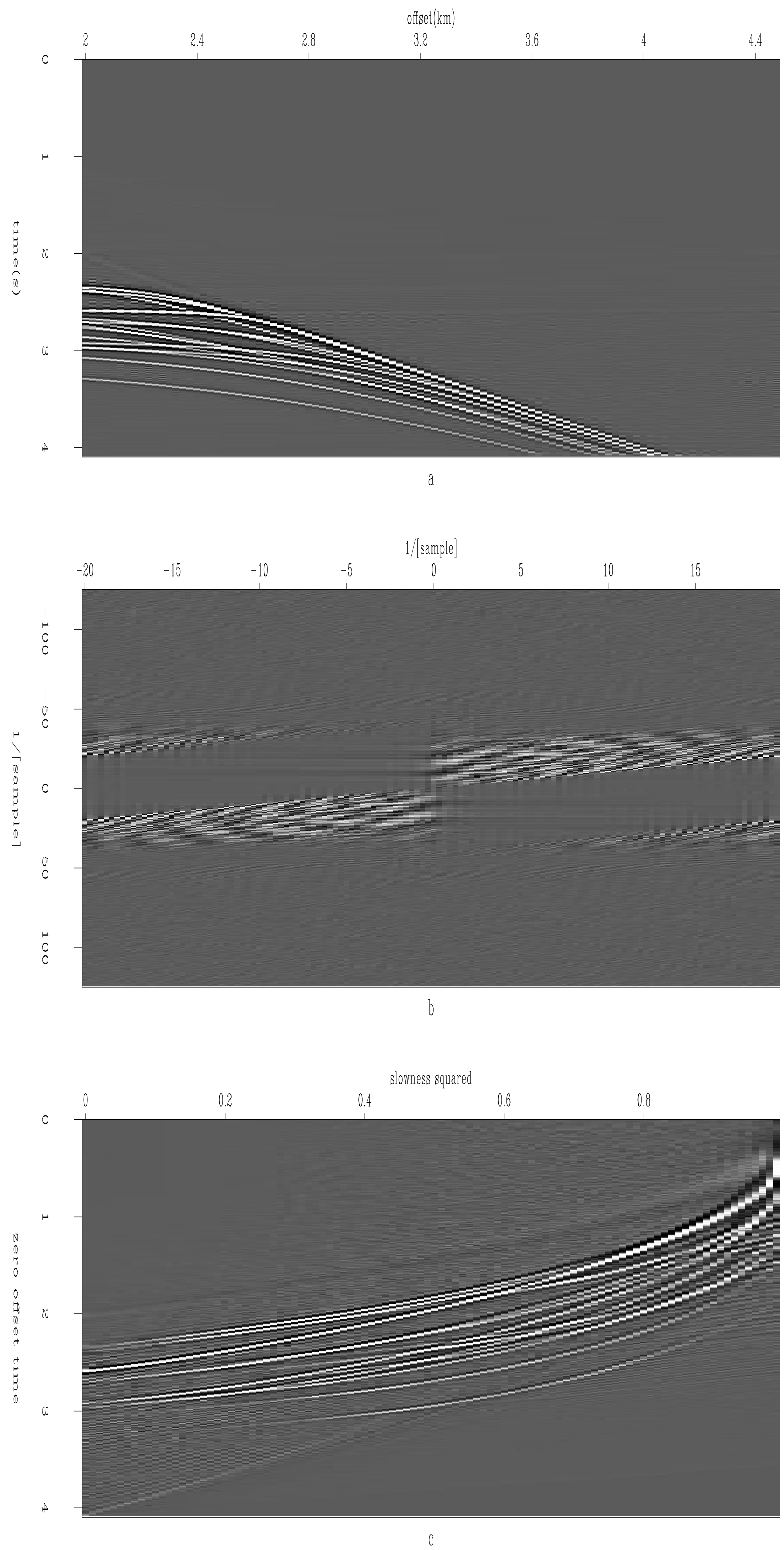

Figure 4: Preconditioned inversion results: data remodeled in resampled data space (a), 2D Fourier transform of remodeled data (b), and estimated velocity spectrum (c). Most of the previously aliased energy is now unaliased. sean1-l1out [CR]

remodeled output is displayed in Figure 6. The data are definitely less aliased. Some artifacts have appeared, mostly above the mute zone. A close look at the input reveals plane waves above the first breaks, which are like hyperbolas with negative zero-offset travel times, and which are thus probably difficult to model with the velocity space transform. Simply muting the input above the first breaks likely removes most of the artifacts.

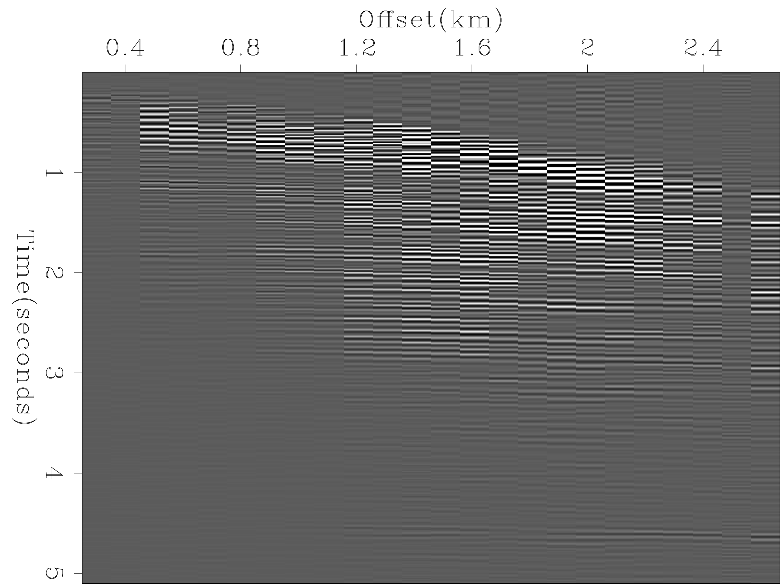

Figure 5: An aliased CMP gather used as input to the preconditioned inversion (left), and its estimated velocity spectrum (right). sean1-realin [NR]

CONCLUSION

Density is a desirable property in the input to most seismic processing. Sparse or irregular data leads to edge effects and aliasing; Kirchhoff methods will leave acquisition footprints or generate spurious events, frequency domain and finite-difference methods may be unstable or totally unapplicable. Velocity space may be used to interpolate irregular or sparse CMP gathers, but the success of such a method depends greatly on the spikiness of the model. This is because the null space in the operator changes when the data space changes, which is how the interpolation and/or regularization is accomplished. A spike in velocity space models a hyperbola to any data space, but some other shape in velocity space is likely to work only for a single data space. Using preconditioning, the model may be made spikier and interpolation results greatly enhanced.

This method gave good results on the examples studied here. Especially pleasing is the method's ability to handle non-hyperbolic moveouts, such as from point scatterers. Interesting questions arise with the next step, which is a move to 3D. How should azimuth be dealt with? Some subsequent processes will want offset and azimuth

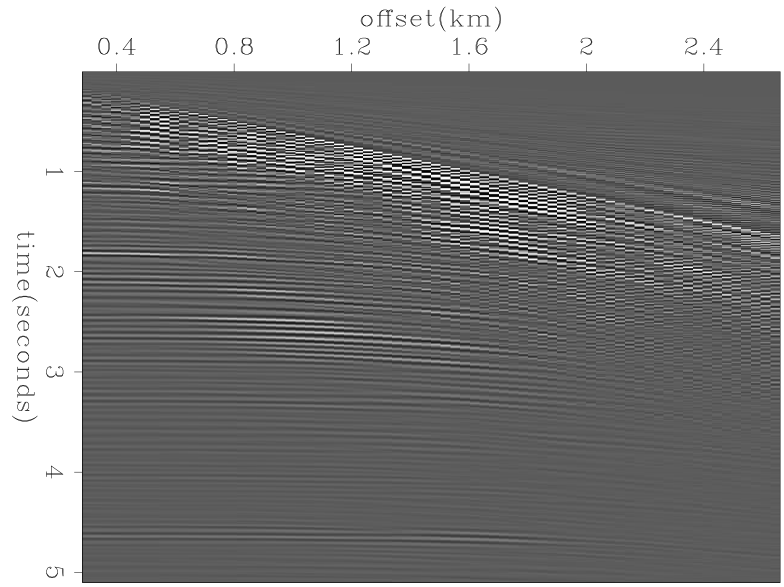

Figure 6: The estimated velocity spectrum remodeled into a more finely sampled data space. sean1-realout [CR]

coordinates, some x and y. Regardless, Kirchoff methods do best with regularly spaced data, where amplitudes may be properly calculated. Will preconditioning be able to regularize as well as interpolate, or will the moving null spaces sink it?

Further, though the synthetic example interpolated aliased data without creating events, and appeared to be as effective in a real data test, it is worth a more exhaustive test to be certain that the preconditioning will not actually create new events in the data space.

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