Stanford Exploration Project, Report 92, November 12, 1997, pages 79–86

Short Note

Modeling heterogeneous reservoirs using the first order Born Approximation

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keywords: *modeling, amplitudes, reflection coefficients, AVO, scattering*

INTRODUCTION

This short-note is intended as a companion paper to Biondo et al. (1996). It aims to examine the validity of some of the assumptions necessary for the Born elastic scattering method which we used to create the synthetic seismograms, especially the assumption of weak scatterers which allows the problem to be treated as linear.

Born generated synthetic seismograms for plane layer models are compared with Zoeppritz reflection coefficients, and can be seen to give good qualitative agreement. The results of this modeling are used to aid the interpretation of multi-offset results from the main paper.

METHOD

Plane-layer reflectivity or Zoeppritz based modeling techniques cannot capture the effects of heterogeneities in the subsurface. On the other hand, full finite difference methods are too computationally intensive for modeling large 3-D multi-offset surveys. This led to the need to use a scattering method such as the first order Born approximation as developed by Wu and Aki (1985).

For a model that consists of weak scatterers embedded in a smoothly varying background medium, the first Born approximation is that the scatterers act independently. The total wavefield is then the linear sum of the scattered field from each diffractor. A more complete discussion of the method, which includes its applicability to both forward modeling and inverse problems, as well as a review of the literature is given by Beydoun and Mendes (1989).

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The model was parameterized in terms of the Lamé parameters and density. Small perturbations $\delta\rho$, $\delta\lambda$ and $\delta\mu$ were considered in a slowly varying background medium described by ρ_0, λ_0 and μ_0. The scattered wavefield from each point diffractor is then proportional to a linear combination of relative perturbations,

$$\frac{\delta\rho}{\rho_0},\ \frac{\delta\lambda}{\lambda_0+2\mu_0}\ \text{and}\ \frac{\delta\mu}{\lambda_0+2\mu_0}.$$

Care was taken to use the correct scattering characteristics, so that the amplitudes of the modeled events would be meaningful as long as the initial assumptions were valid.

The amplitudes of these relative perturbations in ρ and μ had r.m.s. values < 0.05, which fits into the $\ll 1$ regime. However, the perturbations in λ were larger, having an r.m.s. value about 0.2, with individual scatterers larger than 0.5.

COMPARISON WITH ZOEPPRITZ

Quantitative studies of the validity of first Born approximation have been made by various authors (Hudson and Heritage, 81; Malin and Phinney, 1985; Beydoun and Tarantola, 88), but we need to have a feel for the accuracy we may expect given the amplitude of the perturbations we have in our case.

To do this, results of the scattering method were compared with plane wave reflection coefficients calculated using Zoeppritz equations. The first set of models tested in this way were the three interfaces described by Rutherford and Williams (1989). These correspond to a variety of different shale/gas-sand AVO signatures. The relevant medium parameters are shown in Table 1, and the changes in elastic parameters across the interface are comparable with our case.

	Model 1		Model 2		Model 3	
	shale	sand	shale	sand	shale	sand
V_p	3.30	4.20	2.96	3.49	2.73	2.02
V_s	1.70	2.70	1.38	2.29	1.24	1.23
ρ	2.35	2.49	2.43	2.14	2.35	2.13
λ	12.0	7.72	12.0	3.62	10.3	2.24
μ	6.79	18.15	4.63	11.2	3.61	3.22

Table 1. Elastic constants for Rutherford and Williams (1989) interfaces.

Figure 1 shows the results of modeling these interfaces. The upper panels show results of Born modeling of a layer 1 km deep; corrections have been made for geometrical spreading and geophone directionality. The lower panels show the reflection coefficients for the same interfaces. The horizontal axes are half-offset in both cases, so comparisons can be made.

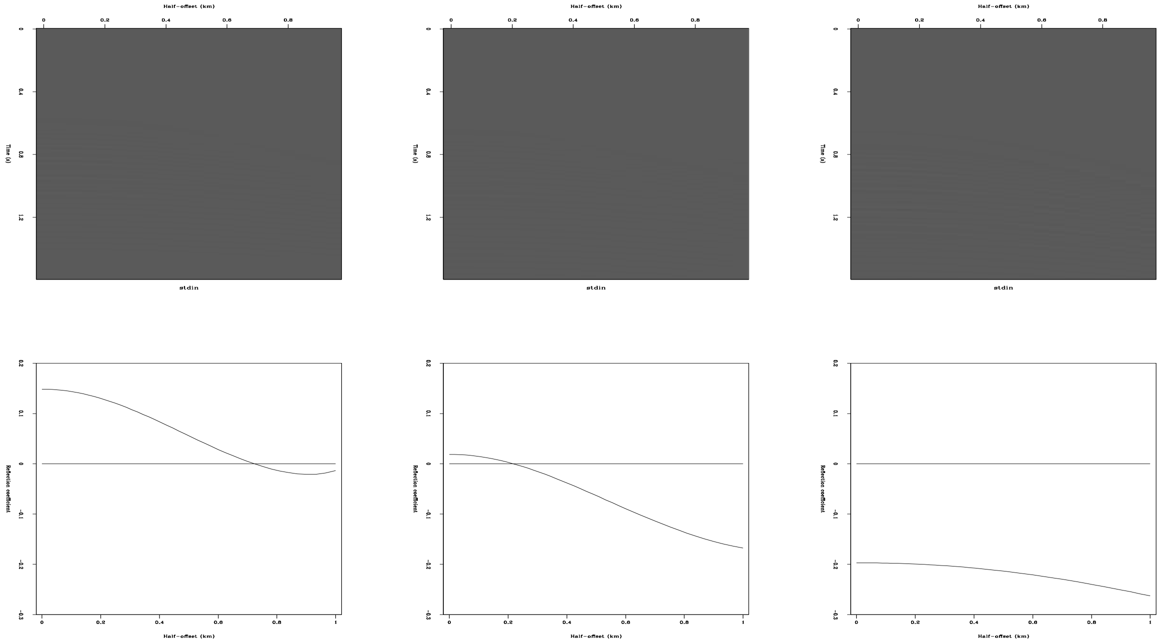

Figure 1: The upper panels show results of Born modeling of interfaces at 1 km depth; corrections have been made for geometrical spreading and geophone directionality. The lower panels show the Zoeppritz reflection coefficients for the same interfaces. The horizontal axes are half-offset in both cases. Left panels are for a Class 1 interface, center panels are for a Class 2 interface and right panels are for a Class 3 interface. james2-rw [CR]

The two modeling techniques are based upon totally different assumptions - Zoeppritz matches plane wave boundary conditions across a plane interface, and Born treats the interface as a series of independent weak scatterers: therefore it would be unrealistic the results to be in exact agreement. However, despite this, they show very good *qualitative* agreement: the phases are all correct, the phase reversals happen all at approximately the right offset, and the amplitudes increase or decrease in the right way. The choice to display one set of results as raster plots and the second set of results as graphs was made to emphasize that we were not expecting perfect agreement.

The second set of interfaces that were tested corresponded to boundaries between lithologies used in the reservoir modeling project. This gave understanding of the levels of accuracy we obtained for our seismic modeling, and it also provides insight into AVO signatures we may expect when analyzing the results.

Figure 2 shows Born modeling and Zoeppritz reflection coefficients for shale/gas-sand, shale/oil-sand and shale/water-sand contacts with elastic parameters taken from Biondi et al.(1996). The same corrections have been applied as in Figure 1 to facilitate qualitative comparison. Again the two modeling techniques give good agreement.

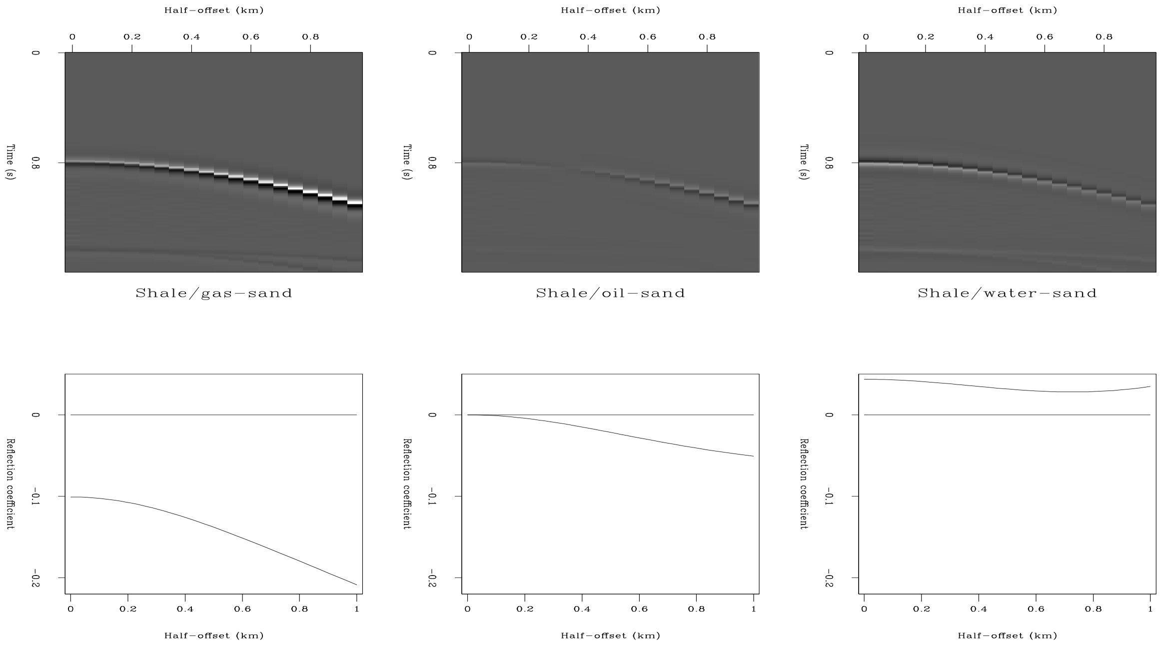

Figure 2: The upper panels show results of Born modeling of interfaces at 1 km depth; corrections have been made for geometrical spreading and geophone directionality. The lower panels show the reflection coefficients for the same interfaces. The horizontal axes are half-offset in both cases. Left panels are for a shale/gas-sand interface, center panels are for a shale/oil-sand interface and right panels are for a shale/water-sand interface. james2-geope [CR]

Both the shale/oil-sand and shale/gas-sand interfaces show increasing reflection coefficient with angle; but as could be expected, this effect is significantly larger for the shale/gas-sand interface.

RESULTS

Depth migration results from Biondi et al. (1996) are shown in Figure 3. These correspond to an in-line slice through the model at $Y = 2.27$ km, which intersects two water injection wells.

The left panels are zero-offset and the right panels have a source-receiver offset of 3000 m. The top row corresponds to $t = 0$, the middle row corresponds to $t = 3$ years and the bottom row is the difference between the two.

At $t = 0$, a gas cap extends from the top of the reservoir to approximately 1.72 km depth. After three years, the water injectors cause the gas cap to be unevenly depleted. This effect is clearly visible in the difference sections.

With the insight gained from the plane layer studies above, meaningful, but qualitative, comparisons between the two offsets can be made. The gain applied to the results from both zero- and far-offset surveys was the same, enabling direct comparisons be made between the two.

The clearest differences between the surveys at different offsets are that the large-offset sections have lower resolution, but larger amplitudes than the zero-offset sections. The decrease in resolution is an inherent effect of the migration operator. The increase in amplitudes is due to the fact that the reservoir is mostly oil/gas saturated, and the previous results show that the reflection coeffiecient for both the oil-sands and the gas-sands increase with offset.

The most noticable difference between the two ‘difference’ seismograms, is that the change in amplitude due to the water injection and disappearance of the gas cap is much larger at large offset than in the near-offset sesctions. This highlights the result from the plane layer-modeling that the large-offset section is most sensitive to the presence of gas.

Other differences between the two offsets are harder to interpret; although an ‘AVO cube’ may help this process. This shows some of the difficulties in drawing conclusions from variation in amplitude with offset. Heterogeneities in the subsurface are imaged with different resolution for different offset, and even with the simple lithologies that we have created, it is difficult to identify the source of anomalies.

CONCLUSIONS

In order to verify that the first-order Born scattering approach used by Biondi et al. (1996) gave correct amplitudes, Born synthetic seismograms were compared with

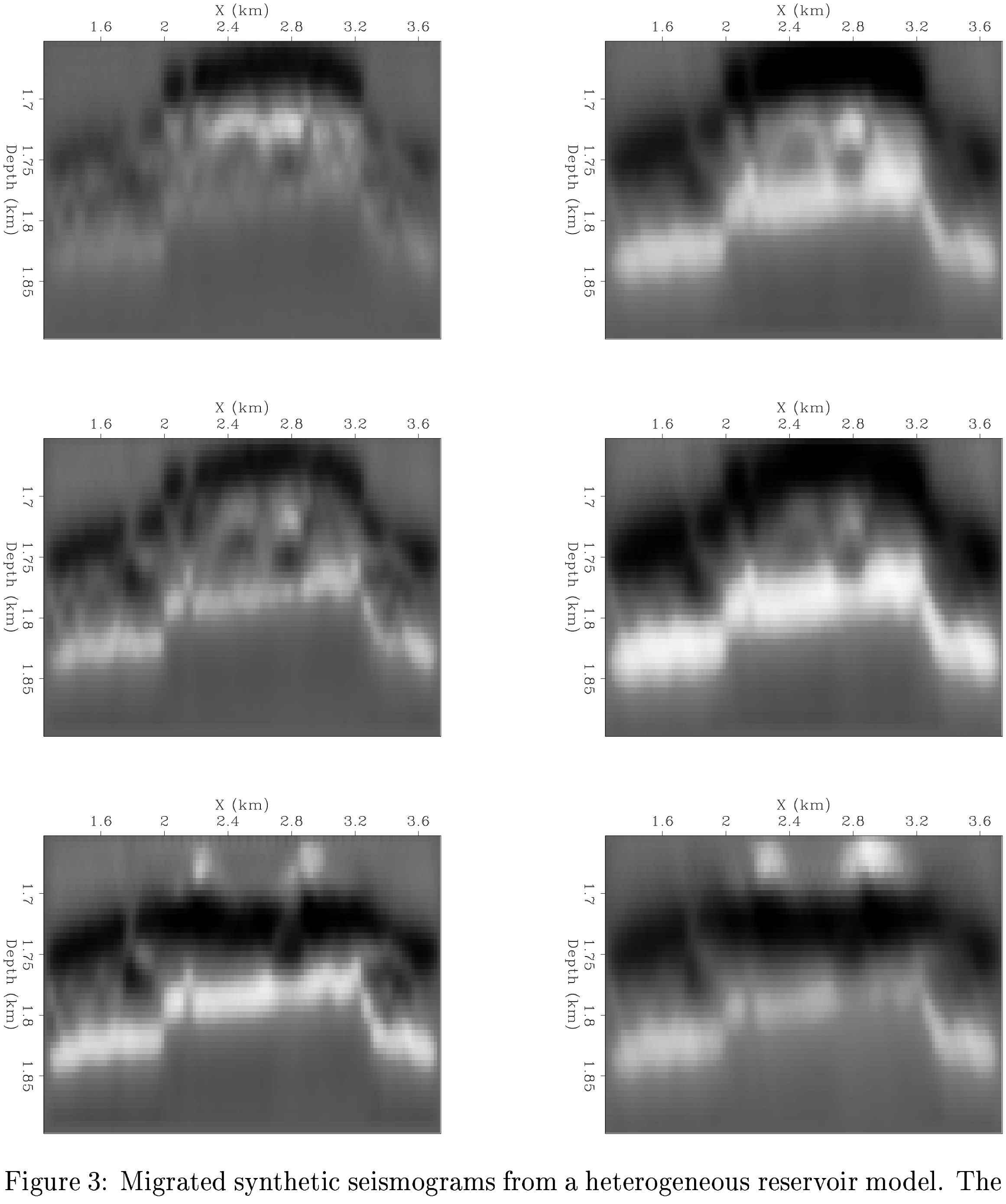

Figure 3: Migrated synthetic seismograms from a heterogeneous reservoir model. The left panels are zero-offset and the right panels have a source-receiver offset of 3000 m. The top row corresponds to $t = 0$, the middle row corresponds to $t = 3$ years and the bottom row is the difference between the two. james2-results [NR]

plane-layer reflection coefficients. Results showed good qualitative agreement between the two approaches. Two sets of results were tested in this way. The first set covered a range of possible AVO signatures, and the second set covered the possible shale/sand interfaces in the heterogeneous reservoir model described in the main paper. The plane-layer models were then used to gain an initial insight into the AVO response of the heterogeneous model.

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