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Short Note

Results of Tieman's conversion of common-midpoint to common-source point slant stacks on synthetic data

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INTRODUCTION

I became aware of Hans Tieman's curious method of transforming slant stacks of midpoint gathers into slant stacks of common shot gathers in January of 1996 during a presentation he gave to SEP. Inspiration to write this paper also comes from William Harlan and Jon Claerbout who expressed interest in Tieman's work. Harlan and Claerbout presented a derivation of the Tieman transform equations during an SEP seminar soon after Tieman's visit(Harlan and Claerbout, 1996). Tieman's transformation holds promise in providing an accurate method of evaluating dips in shot gathers with out having to apply slant stack transforms directly in the shot gathers themselves. A method such as this, that can accurately evaluate dips in shot gathers can be useful in plane wave decomposition which ultimately can be useful for downward continuation and migration. In this paper I use synthetic data to examine the contrast between direct application of slant stacks to shot gathers vs. the application of Tiemen's method of transforming slant stacks of cmp's to slant stacks of shot gathers. Tieman's method shows improvement over the direct application of slant stacks to shot gathers for synthetic data that is coarsely sampled along the offset axis.

PLANE WAVES & SNELL WAVES

Plane waves have the desirable property that they can be downward continued simply by time shifting in negative increments. In the Fourier domain this is particularly simple since this operation is limited to multiplication by a time shift operator. Unfortunately wave fields produced by shots, vibroseis, or airguns produce spherical wave fields and cannot be as simply downward continued. Plane wave theory ap-

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plied to reflection seismic work should not be abandoned though; any wave field can be decomposed into a collection of plane waves of differing orientations in space, analogous to the the way Fourier synthesis can be used to represent an arbitrary function(Claerbout, 1984). Since seismic data is not recorded as plane wave sets, the problem exists of how to decompose the seismic wave field into constitute plane waves. One approach is to consider the angle that energy arrives at the receivers in a given shot gather. Assuming that the near surface velocity is not drastically variable, energy arriving with the same angle at regular time intervals along a shot gather can be associated with a plane wave. The angle of incidence at the surface can be simply related to the slowness, or the Snells parameter, of the incident energy by the equation:

$$\frac{sin(\theta)}{v} = \frac{\partial t_0}{\partial h} \tag{1}$$

where θ is the angle of the ray with respect to the vertical, v is the velocity of the shallow subsurface, t_0 is the time at which the wave is incident on the surface and h is the offset. The right side of this equation is simply the dip of events in shot gather coordinates. It follows from this relation of plane waves to dips in shot gathers that the dip information obtainable by slant stacks of shot gathers can be a key tool in plane wave decomposition(Claerbout, 1984).

Discussion of plane waves in media of spatially variant velocity, such as the Earth, is at best a crude approximation. For the special case of a media of stratified velocities, a model which can be considered in many cases a good approximation of the Earth, plane waves have a simple derivative. A plane wave in a media of constant velocity incident at an acute angle to a media of stratified velocities will be distorted upon entering the region of stratified velocities. In the stratified media, the former plane wave will retain memory of its former existence and maintain a distinctive character, one important characteristic being the value of the velocity of the wavefront along a horizontal plane, such as the Earths surface. Waves with these characteristics are known as Snells waves. The inverse of the horizontal velocity of the wavefront is known as Snells parameter, or the slowness, which is the linear shift factor encounter in the slant stack transformation equation, $\tau = t - ph$, where p is Snells parameter, τ is the retarded time, t is the travel time in the shot gather and h is the offset in the shot gather. The important characteristic of Snell waves to practitioners is not only that the Snells parameter is constant but also that it is an observable(Claerbout, 1984).

Shot gathers are an observation of a propagating wave event and can be modeled directly using the wave equation. Slant stacks applied to shot gathers reveal information about a the physical state of the propagating wave field for values of Snells parameter and time. Unfortunately slant stacks of common shot gathers can be plagued by aliasing effects when applied to sparsely sampled data due to the presence of conflicting dips. On the other hand, common midpoint gathers are much less prone to having severely conflicting dips because all the hyperbolic events are aligned along the zero offset axis. Common midpoint gathers are not a coherent representation of a wave field, rather a cmp gather is a fragmentation of many separate shot events and cannot be used to evaluate characteristics of the wavefield such as Snells parameter.

TIEMANS'S TRANSFORM

Tieman's sequence of transformation equations ideally applies to a world defined by continuous infinite functions of space and time. The use of two dimensional Fourier transforms in this method, both in the forward and inverse sense, indicate that there may be resolution questions and artifacts to deal with when this transform is applied to finite spaces, not to mention boundary effects inevitable with the slant stack. Precision of the two Fourier transformed axis, the retarded time axis, τ, and the midpoint axis, y, is important for the Tieman method because the transformation involves a shift of the third axis, the Snells parameter axis, p, as a function of the Fourier transformed axis. This is evident in the relation of Snells parameter in the shot domain to Snells parameter in the midpoint domain. The following is the Tieman transformation where the   represents a Fourier transform and Y and S represent the slant stack of the the cmp gather and the slant stack of the shot gather respectively. This notation follows the convention of Harlan and Claerbout(1996).

$$\tilde{\tilde{Y}}(k_y, p_y, f_y) \quad = \quad \tilde{\tilde{S}}(k_s = k_y, p_s = p_y - \frac{k_y}{2f_y}, f_s = f_y) \tag{2}$$

Dense sampling of the time axis is often achieved in real data situations, but the midpoint axis and offset axis will, in most cases be sampled much less. Coarse sampling in the midpoint axis could be problematic, especially if the sampling is such that dips evaluated by the $\frac{k_y}{2f_y}$ factor in the Tieman transformation are aliased. If the transformation is applied to unmigrated data, diffraction events could create confusing dips which could be significant to the shift equation. Aliasing due to coarse sampling along the offset axis of the cmp gather will also limit the accuracy of the transform, though not to the degree that aliasing would effect the direct application of a slant stack to the corresponding shot gather (Harlan and Claerbout, 1996).

THE SIMULATION

I applied Tieman's method on two different examples of synthetic data. The goal here is to compare the effects of sampling on the quality of the Tieman shot gather slant stacks with quality of standard slant stacks applied to shot gathers. The data sets were generated by an algorithm that generates a given number of regularly spaced constant fold cmp gathers with given offset spacing over a set of seventy five dipping reflectors. The reflectors are chosen to be randomly dipping in ranges from $-45° < dip < +45°$ at randomly chosen depths measured from the center of the midpoint array. I chose a large number of random reflectors to maximize the aliasing effects that occur on the shot gathers in hopes that the limits of either method, the Tieman or the standard slant stack, in distinguishing dips would be tested. Figure 1 is a constant offset section of the first example data set which depicts the geometry of this reflector model.

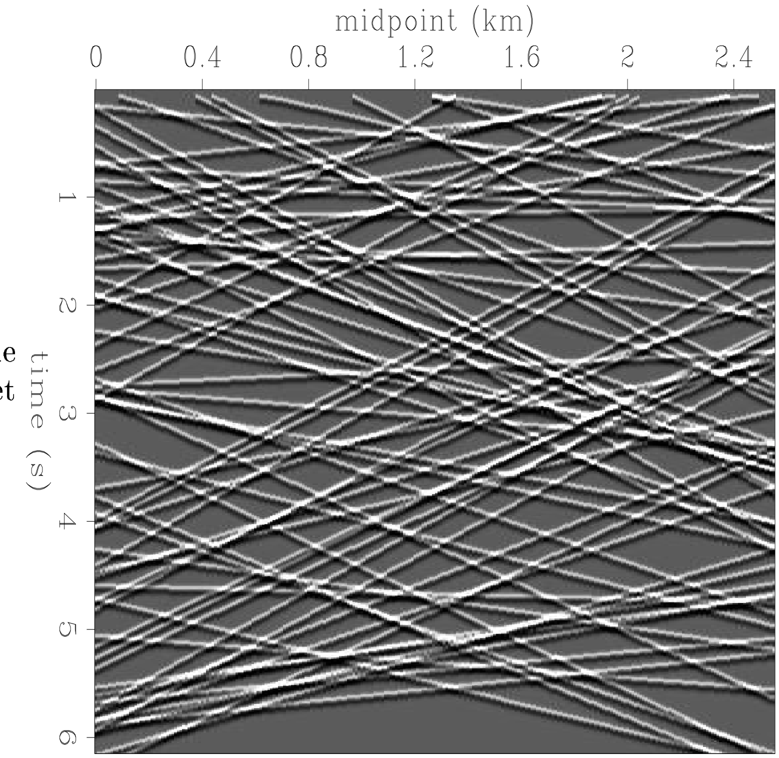

Figure 1: Reflector model in time and midpoint for a constant offset of two km curt-earth [CR]

The sampling of the time axis and midpoint axis for both test data sets are 512 an 256 respectively while the sampling of the offset axis in the first test is 80 and 20 for the second test. In the following Figure 2 are examples of a cmp gather and a shot gather of this data set.

The shots were modeled as being shot offend, a situation commonly encountered in marine surveys, thus the gathers only have positive offsets. The disparity in symmetry between the hyperbolic events in the cmp gather vs. the asymmetric events in the shot gather is clearly evident.

80 SAMPLE TEST

The Figures above, Figure 1 and Figure 2, were generated using 80 point sampling along the offset axis. Tiemans method was applied to the cmp gather in Figure 2 while the standard shot gather stack was applied to the shot gather in Figure 2. A comparison of the results reveals that they are almost indistinguishable. Minor

../curt/./Fig/gatheraniso.ps

Figure 2: Simulated gathers: (left) cmp gather, (right) shot gather

differences in overall contrast and clarity can be attributed to the difference in number of Fourier transforms applied. A slight improvement in precision may may favor the Tieman transformed data. The striking similarity of the two images confirms the accuracy of the Tieman transformation.

20 SAMPLE TEST

The second example illustrates a significant difference in the resulting slant stacks. The cmp and shot gathers that the transforms were applied to are illustrated in the 4. A comparison of these two stacks reveal a few apparently different features worthy of mention. The hyperbolic events on the cmp gather are all approximately tangential to a general hyperbolic trend. These events interfere over large ranges, creating long trends of high and low amplitude bands which extend in a coherent way along the hyperbolas . Even so, aliasing can be expected over short ranges. Careful scrutiny of the shot gather, on the other hand, reveals a more confused arrangement of energy in the region of the hyperbolic asymptotes. Sharply conflicting dips of the off axis hyperbolas confuse the hyperbolic forms, causing much more of the gather to be aliased than in the case of the cmp gather.

A comparison of the Tieman method applied to the cmp gathers and the direct application of slant stacks to the shot gathers reveals a significantly different result.

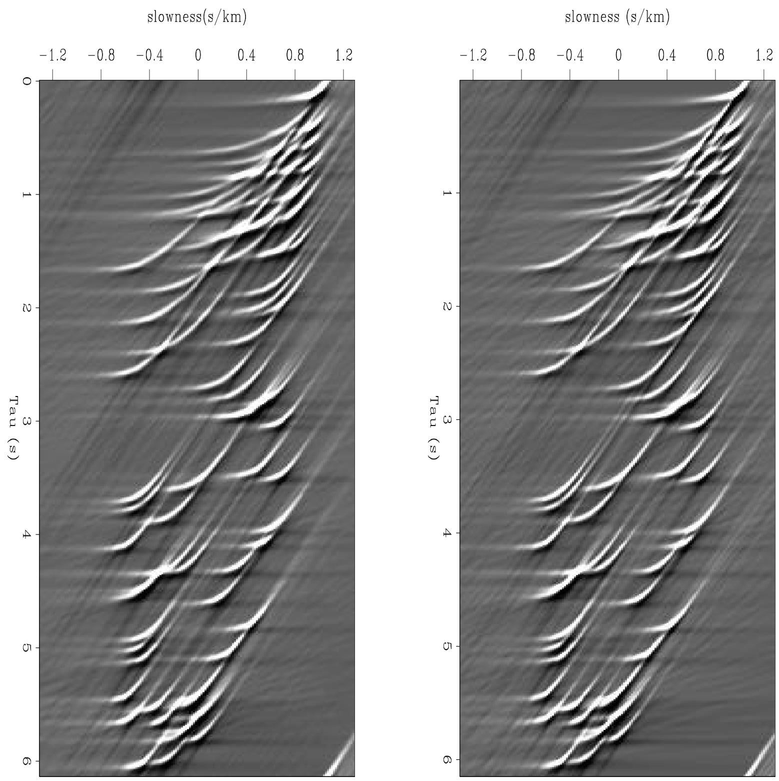

Figure 3: (left) Results of Tiemans method applied to data sampled 80 times along the offset axis; (right) Results of direct application of slant stacks on shot gathers for offset sampled 80 times curt-80aniso [CR]

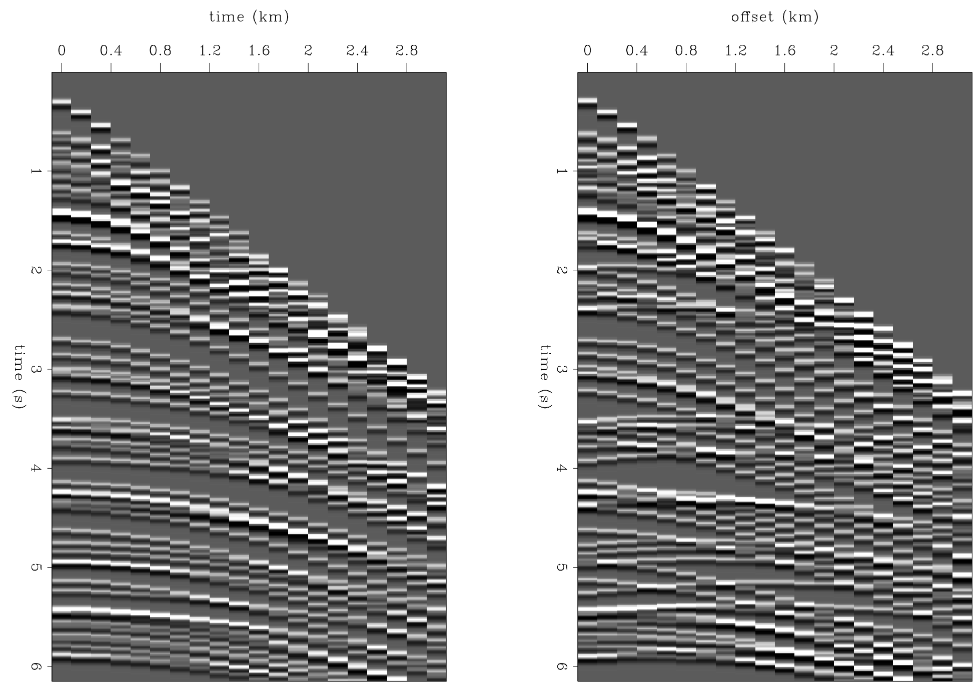

Figure 4: (left) cmp gather for 20 point offset sampling, (right) shot gather for 20 point offset sampling curt-20gathersaniso [CR]

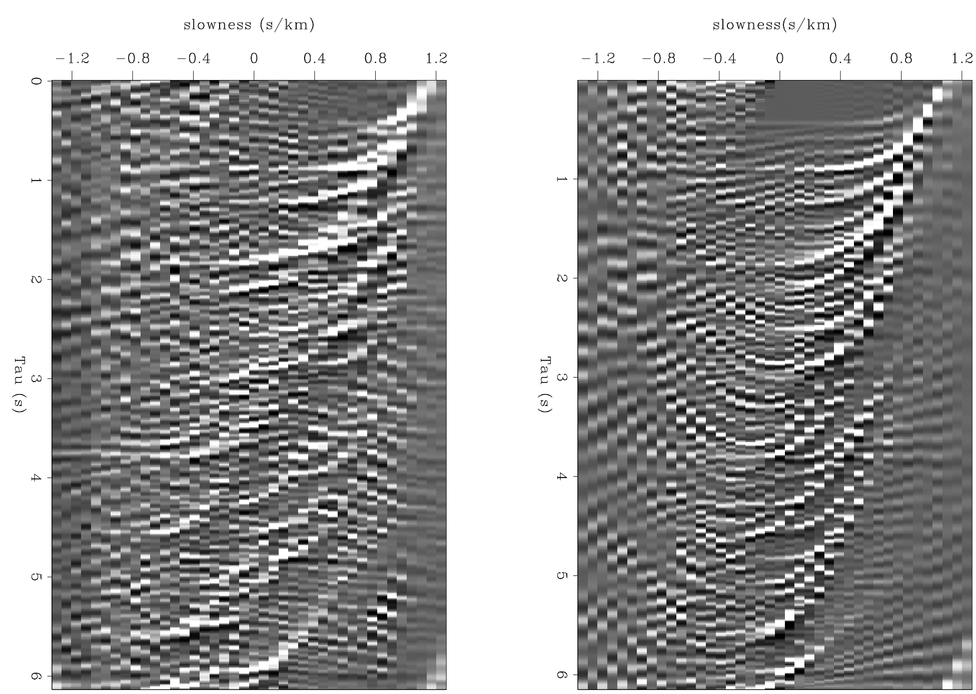

Figure 5: (left) Result of Tiemans method applied to the cmp gather; Results of slant stack applied directly to Shot gather curt-20aniso [CR]

Coherent energy is distributed over a larger range of slowness values in the case of the Tieman transformed stacks than in the case of the stacks applied directly to the shot gather. Both methods have similar results for large slowness values at earlier retarded times, but this would be expected due to the similarity in dip and energy of the earlier events in both gathers. The similarity of the two results changes promptly as later times are considered. The negative dips in the case of the directly applied slant stacks are washed out in a cloud of aliased energy, where as in the case of Tieman's transform, the negative slowness events are distinguishable to around $-0.5s/km$. A large band void of energy exists in the directly applied slant stack case from earlier times down to the latest times for relatively steep dips, while Tieman's method retains energy with distinguishable structure in this region. A curious arcing pattern exists on the directly applied stacks which do not in the Tieman converted stacks which also does not appear in the previous test with 80 point sampling.

CONCLUSION

The results from these two simulation examples must be considered in a very limited sense. Without a test on real data and/or more sophisticated synthetic cases, which may include more geological reflector models, noise, statics etc, the Tieman method cannot be satisfactorily tested. I tested the case in which there are many events of apparently different character in dip and in depth, creating a multitude of confused hyperbolas on a common shot gather. These two applications confirm the following:

- 1) that the Tieman method does work, which is apparent when the the striking similarity of the two transforms in the 80 point sample case are considered

- 2) that it handles stacking of sparsely sampled offsets better than the slant stack direcly applied to shot point data.

The other extreme would be to test Tieman's method and a direct shot gather slant stack on a model with a few reflectors of gradually changing dip. Also I did not deal with sparse sampling along midpoints which could be the source of inaccuracy for the Tieman method. Finally tests on real data are necessary to fully examine the applicability of this method. These preliminary results are promising and indicate that a better method of evaluating dips in shot gathers may exist in the Tieman method.

ACKNOWLEDGMENT

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